Mathematical modeling of the strategy of the early Islamic wars using game theory with emphasis on the battle of Siffin

Hojjat Allah Ebadizadeh[1]✉ . Alinaghi Lezgi[2]

[1] Basic Science of Imam Ali University, Tehran, Iran

[2] Lecturer at Imam Ali's Officer's College and Master of Science in Quran and Hadith

✉ ebadizadeh.h@gmail.com

Abstract Mathematics has been applied to all sciences; and religious and military sciences are no exception, and mathematics can be used highly to design different war operations and solve battlefield equations to gain relative or absolute superiority over the enemy. We can also see clearly the application of mathematics in the Game Theory of war in abundance. In this applied research, conducted in a library method, the challenges between the army of Amir al-Mu'minin, ʿAlī ibn Abī Ṭālib (as) and the army of Muʿāwiya ibn Abī Sufyān in the Battle of Siffin have been modeled using Game Theory and the strategies of each of these two fronts are compared. Finally, in order to examine the war utility, we seek to find the strategic equilibrium point between them. So, we obtain and discuss the utility function for each party's set of strategies in the battle of Siffin.

1. Introduction

Mathematical Modeling has always been a must in reviewing and analyzing real systems. Models can be very effective in knowing the behavior of the system at the same time or future moments and even modeling real systems. Models have always been and are needed to design new processes and analyze available processes in the engineering sciences. One of the most popular topics in recent years is the modeling of war games (Lezgi *et al.*, 1398).

Game Theory is a branch of applied mathematics used in the social sciences, economics, marketing, politics, international relations and computer sciences, military strategies and many other disciplines. Game Theory tries to find a mathematical formula to model a strategic situation (Akramizadeh *et al.*, 1398).

Rational behavior; the principle of Game Theory is that the behavior of players is rational. Being rational means that each player is only looking to maximize his profits and each player knows how to maximize his profit, so guessing his behavior will be based on a cost-benefit graph.

In addition to applying Game Theory in war-related analysis (such as modeling the Cold War or the Cuban nuclear crisis), some scholars today have used Games Theory to solve terrorism issues, such as modeling terrorist behavior.

While this study will attempt to extract prominent indices from the text of the history of the early wars of Islam with the focus on the battle of Siffin, and to be compared with the concepts and algorithms of the War Game, it is hoped that this effort will provide a framework for similar further studies; but it is clear to note that Game Theory is the result of human experiences and computations, and naturally has conceptual and computational errors, and sometimes it may not be completely mixable with hidden criteria in the religious teachings that are based on fixed and unchangeable divine traditions. In describing the profits and losses, what exists in the literature of this field is different from what is derived from the religious doctrine of the concept of profit and loss (especially with regard to the concept of eternal reward).

What will be explored in this research lies in the range of the sīrat Alawite and the teachings of Nahj al-Balāghah. We do not intend to view the material presented as a whole religion, but in terms of the conditions and circumstances of Amir al-Mu'minin's life, this text will be one of the most comprehensive and helpful guidelines.

The research of narratives on topics covered by predefined narrow keywords can, to a certain extent, clarify the dimensions of the problem. But in areas similar to what will be dealt with in this research, it is not possible to find a point simply by looking for keywords, but rather we should look at it in a scientific-analytical way.

Strategic science approaches with the book of war and jihad by Amir al-Mu'minin (AS) can open new horizons on this subject. What have been the origins of the wars, what strategies did Amir (AS) use, and what fundamental assumptions have shaped the prophet's strategy and security doctrine, are these teachings analyzable in the form of Games Theory as a human and conventional artifact? Can it support the strategic decision-making and decision-helping in the war?

The main purpose of the present study is to provide a model that paves the way for future research in the field of Game Theory and its applications in political and international relations based on Islamic teachings.

2. Theoretical foundations and recognition of research history

The recognition of the strategy concept

Strategy is a governmental issue that has exited from the fence of military force and power and includes politics, culture, economics and society. In other words, strategy in general means the technique of harmonizing the cultural, political, economic, religious, psychological, military capabilities of a nation or country and its application to realize the national goals; therefore, in any time and place conditions it can be mentioned and applied. (Azghandi, 1394).

The military strategy, according to Colonel Arthur Lykke, is to set military goals, set up military theories to achieve goals, and use military resources to execute theories. When each of these basic and fundamental elements conflict with other elements, national security will be at risk (Roshandel, 1373).

There is no consensus among the scholars of international relations and sociologists in the division of all kinds of military strategies. Some divide these strategies in relation to the historical evolution of the strategy into traditional1 and modern strategies. Some, according to the battlefield, divide it into terrestrial, aerospace, and marine strategies. The third group also identifies strategies with five characteristics, namely: direct threat, indirect2 threat, advance or indirect actions through surprise, traditional destruction and partisan strategy (Azghandi, 1394).

Game Theory in the occurrence of war

Game Theory has been created from the community of logic and mathematics and in some way relies on the abstract reasoning. The centrality of this theory has been established on the rationality of the actors involved in the game. That is, players compete in the game and try to get the highest points from the other party (meaning that people act due to the profit and cost basis). However, individuals can act irrationally and emotionally in conflict situations. But game theorists believe that people who participate in the game act by rational calculation and by guessing the actions of the other party and try to gain points from the other side (EbadiZadeh, 1397).

In Game Theory, each player seeks a situation in which each of the competing parties performs evaluations and measures to prevent the success of the other party. The difference between a game and a real conflict situation is that the game is based on predetermined conditions, while in a conflict situation there is not necessarily a predetermined situation (Ibid, 1397).

Player: people who influence each other's decisions are called players, people who feel that there is an interdependence between their decisions and their actions and they are aware of that interdependence. Players who interact with a particular player are called the "opponent" of that player.

---

1 Namely terrestrial strategies

2 Using a variety of tools, such as diplomacy or business relations, to limit competitor's freedom of action.

The reasons for the occurrence of the Siffin battle

When Amir al-Mu'minin (AS) came to the caliphate, Muʿāwiya ibn Abī Sufyān was governor of Syria. The second caliph had appointed him as the governor of Syria in 18 AH (Ibn Athir, 1408), and during the caliphate of the third caliph, Muʿāwiya remained in the position of governor of the Syria (Ibn Asakir, 1421). Imam Ali (AS) decided to send Abdullah ibn Abbas as the governor of Syria; so he wrote a letter to Muʿāwiya in which he asked him to come to Medina with the rich of Syria and make allegiance to Imam (as), but Muʿāwiya refused to make allegiance to Imam Ali (AS); rather he mentioned the avenge of ''Uthman's blood.

Amir al-Mu'minin Ali (AS) in a letter addressed Muʿāwiya and wrote:

"The people's allegiance to me is a public allegiance, whether those who were present in Medina at the time of allegiance or those who were in Syria, Basra and other cities. Everyone knows that I haven't kill 'Uthman to deserve retaliation, and 'Uthman's heirs deserve to revenge for his blood more than you, but you are one of those who opposed him and at the time he asked your help and you could help him, you didn't help him until he was killed." (Ibn Abi al-Hadīd)

Muʿāwiya did not respond to Imam's letter (Balādhurī, 1996) and he gathered the people in the mosque and told them: "I am the governor of ʿUmar ibn al-Khaṭṭāb and also ʿUthmān ibn ʿAffān over the people of Syria. I am the legal guardian for the blood of the caliph who was killed oppressed. What do you think about the blood of ʿUthmān? All the people declared their support in revenge of the blood of the Third Caliph, and this was a practical response from Muʿāwiya to the Imam. After the end of the battle of the Jamel war, Imam Ali (AS) moved the center of the caliphate to Kufa and tried to persuade Muʿāwiya to obey Imam Ali (AS) (Ibn 'A'tham, 1411), but after he was convinced Muʿāwiya has no intention of allegiance and obedience, and the elders of Kufa are defending Imam in the war against Muʿāwiya, he invited the people to jihad in a public sermon.

Collecting the Corps

Amir al-Mu'minin (AS) called on the elders of the Companions, who were from Anṣār and Muhājirān (i.e. emigrants), and asked them to comment on going to Syria, and wrote a letter to Ibn Abbas, the Governor of Basra, to invite the people of Basra to accompany him, then many people of Basra came to Kufa with Ibn 'Abbas at the invitation of the Imam (AS). He also wrote a letter to Mikhnaf ibn Sulaym, the ruler of Isfahan, and called him to join the Imam's army (Ibn Muzāḥim, 1403) and Imam also ordered the fighters to move to Nakhliya, a military camp in Kufa3. Muʿāwiya, who had also decorated the pulpit of the Syria Mosque with the bloody clothing of the Third Caliph, while many were crying around him, prepared the Syria people to confront the Iraqi army.

---

[3] A number of Kufi women were also present in the battle of Siffin.

Siege of Euphrates

At first the Muʿāwiyaʿs army reached the Euphrates River and denied the Kufa army access to the Euphrates to weaken and destroy them. Imam (AS) sent Musayyib b. Rabi' al-Riyahi and Sa'sa'a b. Sawhan, saying: "go to Muʿāwiya and tell him, 'your soldiers have stood between us and the water and have blocked our access to water, if we were to get water earlier than you and build our camp there, we would not deny your access to the water, Stop blocking the water so that our army and yours use it equally, otherwise we have to fight over water (Ibn 'A'tham, 1411).

In a conversation between Muʿāwiya and the messengers of Imam Ali (as), Muʿāwiya became angry and said: "Ali will have no share of this water. God shall not give water to Muʿāwiya and his father from the Pound of Kawthar, if I let Ali (a) or his companions drink from the water of Euphrates; except, they win the war."(Ibid, 1397)

Finally, the war began and the Euphrates was seized by the army of Kufa. Then Imam Ali (AS), contrary to the opinion of some companions who believed that water should not be allowed to the Syria Corps, ordered to let the water free for everyone and no one should prevent the Muʿāwiyaʿs army to take water (Ibid, 1397).

Laylat al-Harir

Laylat al-Harir was a difficult and decisive night. The two armies fought in that night, many soldiers of both sides were killed. According to Minqari4, in that night, there was no sound heard except the sound of hitting the swords (Ibn Muzāḥim, 1403). Ibn Miskawayh described that night: "They fought in that night, so hard that spears broke and no arrow was left in quivers and then they began fighting with swords." (Ibn Miskawayh, 1379). The soldiers of Imam Ali (a) were close to victory, until Ash'ath5 rose among the warriors of Kindis tribe, gave a sermon and with a peace-making tone, asked for stopping more bloodshed. According to hadiths, as soon as Muʿāwiya was informed about the sermon of Ash'ath, ordered to put the Qur'ans on the spears and

---

4 Naṣr b. Muzāḥim b. Sayyār ʿAṭṭār Minqari is from Iraq and one of the Shiite historians in the second century. Sheikh Tūsī believes that he is one of the companions of Imam Bāqir (AS).

5 Ashʿath ibn Qays kindi was the head of the Kindah tribe and the governor of Amir al-Muʾminin (AS) in Azerbaijan. Ashʿath was in the army of Imam in the battle of Siffin but he had relationship with Muʿāwiya during the war and on the night of Laylat al-Harir, when Imam was on the verge of victory, he began to urge soldiers stop the battle and prepared the ground for Muʿāwiya to put the Quran on the spears. Ashʿath had an important role in imposing the conversation to Imam and he also forced Imam to choose Abu Musa Ashʿari, who was an idiot person, for the conversation. He also played an influential role in inciting the Khawarij against Imam (Balādhurī, 1966 and Tabari, 1967) Also his daughter was Jaʿadeh, the wife of Imam Hasan (AS), who poisoned martyred him. (Saduq, 1395) His son was Muhammad ibn Ash'ath who handed Muslim ibn Aqil over to Ibn Ziyad and he was in the army of Kufa and stood in front of Imam Hussein (AS) (Abu Mikhnaf, 1417)

shouted the motto of "no verdict except the verdict of Allah". (Ibn Muzāḥim, 1403 and Dinvari, 1960).

Imam Ali's (AS) reaction to his troops

After the motto of "no verdict except the verdict of Allah" was issued in the battlefield, the people demanded Imam (AS) to answer them, while Malik Ashtar, army's commander, had no idea but to continue the war (Ibn Muzāḥim, 1403). Imam Ali (AS) arose and said, "I always wanted to carry on my job with you and be with you ... yesterday I was your commander and today I am your follower, yesterday I was banning you and today you are banning me. You are interested in the fleeting world, and I cannot make you do something that is unpleasant to you." (Ibid, 1397).

Historians disagree on the death toll of the two corps in the battle of Siffin, some have said: Seventy thousand people have been killed, including forty-five thousand men from the Muʿāwiyaʿs army and twenty-five thousand from Imam Ali's army (Ibn Abi al-Hadid).

The battle of Siffin	
Year 37 AH	Time
A region called Siffin	Place
The end of the war by accepting arbitration	Result
The disloyalty and unacceptance of the caliphate of Imam Ali (AS) by Muʿāwiya and declaration of independence in the governorate of Syria	Reason of the war
Army of Imam Ali (AS) - Army of Muʿāwiya ibn Abu Sufyān	Fighters

3. Research methodology

This research is a library research, but from an objective point of view, the research is practical, as it tries to put the results of this research into practical use and, with the help of its results, solve the problems of the organization.

The Strategies of Muʿāwiya and Imam Ali (AS)

We put the strategies of Muʿāwiya in set S1 and the strategies of Imam Ali (AS) in set S2:

$A_1 = S_1$) blackmail Imam Ali (as) to take over the rule of Syria and Egypt,

A_2) pretending oppressed and avenging the blood of ʿUthmān

A_3) an attempt to deceive public opinion,

A_4) bribing the commanders of Imam Ali's army,

A_5) direct war,

A_6) an attempt to isolate Imam Ali (AS) in the minds of the people,

A_7) the use of the sacred and the Qur'an on bayonet,

A_8) applying the influence in imposing the blind negotiator on Imam Ali (as) for the judgment,

A_9) making a discord among the troops of Imam Ali (as),

A_{10}) a psychological war.

$I_1 = S_2$) Rejecting Muʿāwiyaʿs conditions for governing on Syria and Egypt,

I_2) trying to seduce public opinion,

I_3) trying not to enter into war,

I_4) public jihad against Muʿāwiya,

I_5) summoning Basra and Isfahan troops,

I_6) warlike proper decoration of the corps,

I_7) motivating many in the army with fiery speeches,

I_8) using resourceful, bravery and warrior commanders,

I_9) direct and brave presence on the battlefields to stimulate the sentiments of the soldiers,

I_{10}) observing the ethics in all war scenes,

I_{11}) acting rightly during the war.

Consequences:

The amount of win or loss and what the players earn at the end of a game is called "consequence." The consequence can be money, income, privilege, and so on. At this stage, the possible consequences of each option (strategy) must be estimated. The key question in each game is that which strategy should be used by each player to achieve the most possible consequence; the answer is the "balance point" of the game. But what is the "balance point "?

The balance:

Whenever a player uses a strategy that is the best answer to the strategy chosen by other players, then the combination of the chosen strategies is called "game balance". The balance point of the game is a combination of players' strategy that happens in practice. In balance, players do not necessarily get the most consequences. This can be due to conflict of interests or lack of knowledge of players. In fact, one of the main goals of Game Theory is to find the same point of balance for each game.

The utility function for game theory

The ith player, who is involved in decision making, must fight against his rival for maximum revenue. We measure the income of ith player based on the utility function for that player. In this study, utility function model is calculated based on the cost-benefit analysis.

According to the rational decision-making theory, using the cost-benefit analysis model is nowadays common in macro-political decision-making. In this way, all the costs and benefits of a decision are evaluated and analyzed, and then they reach a decision (EbadiZadeh, 1397).

There are many definitions for cost-benefit analysis. For Bremen, this type of analysis is an indicator weighting method for decision making and acts as a scale. That is, all the positive elements of one project (benefits) have been put in one balance sheet, and all the negative elements and costs (costs) have been put in the other one, and the heavier will be the winner. Cost-benefit analysis is a decision-making logic that operates on the basis of calculating the possible outcomes of different decision options (Ibid, 1397). Cost-benefit analysis answers five basic questions:

1) How much direct benefits will be obtained from this expense?

2) What indirect benefits will come from this cost?

3) Whether the obtained benefits are greater than the spent costs?

4) What will happen if the costs are not done?

5) Is there a less expensive option if this is necessary?

This analytical approach helps to determine the best decision for maximizing benefits against minimum costs.

Modeling the utility function

In 2011, Berman presented the following formula for utility function in the following form:

For the utility function can quantitatively predict the everyday war, this function considers three aspects: Human Cost (H), Economic Cost (E) The gained or lost socio-political Influence Value (I) (Berman *et al.*, 2011).

Missing costs include:

Human resources:

- Losing the loyalty of the corps' forces of the loser side

- Paying pensions to the families of those killed in the war

- Elimination of effective human resources in social and defense activities

Economic aspect:

- losing Taxes on areas dominated by the Kufa Corps including the territories of Hejaz, Yemen, Arab Iraq, Iran and Egypt, and Syria for the army of Syria.

- Problems arising from the imposed cost of war

Socio-political influences:

- Losing loyalty of the dominated territories, which was Syria for the Syria Corps, and Hijaz lands for the Kufa Corps, including Mecca and Medina, the Yemeni land, the Arab Iraqi land including Kufa and Basra, the Iranian land including Azerbaijan, Tabaristan, Khorasan, Ray, Sistan, Isfahan and Ahwaz as well as the land of Egypt.

- Reducing the geographical-political boundaries of the loser

Expenses incurred include:

Human resources:

- Obtaining the loyalty of the loser corps forces

- Receiving taxation from the new areas under control

- Gaining effective human resources in social and defense activities

Economic aspect:

- Receiving taxation from areas under control which was for Kufa Corps in the territories of Hejaz, Yemen, Arab Iraq, Iran and Egypt, and for the Syria Corps in the Syria.

- Obtaining war trophies.

Socio-political influences:

- Obtaining the loyalty of the dominated territories which was Syria for Syria; and for Kufa it was Hejaz including Mecca and Medina, Yemen, Arab Iraq including Kufa and Basra, Iran including Azerbaijan, Tabaristan, Khorasan, Ray, Sistan, Isfahan, Ahwaz and the land of Egypt.

- Expanding the geo-political boundaries of the victorious side and thus becoming a global superpower.

According to what Berman and his colleagues have done (2011), the benefits of war can be calculated by:

$$u_i = \alpha_H(H_G - H_L) + \alpha_E(E_G - E_L) + (1 - \alpha_H - \alpha_E)(I_G - I_L)$$

In relation (1), H_L and H_G respectively represent the gained and lost costs of human resources resulting from the war.

E_L and E_G represent the economic value gained and lost by the war, I_L and I_G are the value of socio-political influences gained and lost by the war. α_E and α_H, respectively, represent the weight of economic and human factors in the decision making of the ith player, with values ranging from zero to one: $0 \leq \alpha_H, \alpha_E \leq 1$

Two things that should be considered in the model are that, first, the calculations toward time must be dynamic with respect to changes in the unit of time, and second, there must be a decision maker's opinion's consequent. Accordingly, and for dynamism toward time, we consider a coefficient for different times, indicating the importance of each time in the range 0 to t. The coefficient δ is assumed between 0 and 1 for this aim and if the exponentiation t is considered for this coefficient, the coefficient value for the utility function will be different and time dependent. In addition, the coefficient β is considered as the risk coefficient to reach the utility (Sherman, 2015).

In 2015, Sherman, in his article, concluded that to make an important decision, the opinions of different people, each aiming to maximize revenue, are influential. Therefore, the ith player must choose a weight for each of the ideas and consider it in the calculations. With this description, the utility value is calculated from the following relation:

$$U_i(t) = \left(\sum_{t=1}^{T} \delta^{t-1} \left[\frac{\sum_{j=1}^{N} \gamma_j \alpha_{Hj}}{N} (H_{Gt} - H_{Lt}) + \frac{\sum_{j=1}^{N} \gamma_j \alpha_{Ej}}{N} (E_{Gt} - E_{Lt}) \right. \right.$$
$$\left. \left. + \frac{\sum_{j=1}^{N} \gamma_j (1 - \alpha_{Hj} - \alpha_{Ej})}{N} (I_{Gt} - I_{Lt}) \right] \right)^{1-\beta}$$

Another point to keep in mind is that each of the six economic values mentioned can have M subset and have different dimensions. As a result, relation (2) is rewritten as follows:

$$U_i(t) = (\sum_{t=1}^{T} \delta^{t-1} [\frac{\sum_{j=1}^{N} \gamma_j \alpha_{Hj}}{N} \sum_{k=1}^{M} (H_{Gkt} - H_{Lkt}) + \frac{\sum_{j=1}^{N} \gamma_j \alpha_{Ej}}{N} \sum_{k=1}^{M} (E_{Gkt} - E_{Lkt}) + \frac{\sum_{j=1}^{N} \gamma_j (1-\alpha_{Hj}-\alpha_{Ej})}{N} \sum_{k=1}^{M} (I_{Gkt} - I_{Lkt})])^{1-\beta}$$

Although the ith player is more commonly known as one side of the conflict, he can be any player or group. Different actors differ significantly in how we evaluate the weights of α_H, α_E and $\alpha_H - \alpha_E - 1$ in terms of human life value, economic value and influence value, respectively.

Utility table gained for hypothetical war:

In the table below, we check the utility function in the states of victory of either Kufa or Syria (V), their defeat (F), peace and acceptance of the terms and conditions of the parties' treaty (P), and finally staying in the former position and continuing political disputes (C).

The utility gained for Kufa corps (player 2)	The utility gained for Syria corps (player 1)	
$\left\{ \sum_{t=1}^{T} \left(\delta^{t-1}_{2} [\alpha_{H2}(-H_{Lt}) + \alpha_{E2}(-E_{Lt}) + (1-\alpha_{H2}-\alpha_{E2})(-I_{Lt})] \right) \right\}^{1-\beta_2}$	$\left\{ \sum_{t=1}^{T} \left(\delta^{t-1}_{1} [\alpha_{H1}(H_{Gt} - H_{Lt}) + \alpha_{E1}(E_{Gt} - E_{Lt}) + (1-\alpha_{H1}-\alpha_{E1})(I_{Gt})] \right) \right\}^{1-\beta_1}$	V_1 F_2
$\left\{ \sum_{t=1}^{T} \left(\delta^{t-1}_{2} [\alpha_{H2}(H_{Gt} - H_{Lt}) + \alpha_{E2}(E_{Gt} - E_{Lt}) + (1-\alpha_{H2}-\alpha_{E2})(I_{Gt} - I_{Lt}) \right) \right\}^{1-\beta_2}$	$\left\{ \sum_{t=1}^{T} \left(\delta^{t-1}_{1} [\alpha_{H1}(H_{Gt} - H_{Lt}) + \alpha_{E1}(E_{Gt} - E_{Lt}) + (1-\alpha_{H1}-\alpha_{E1})(I_{Gt} - I_{Lt})] \right) \right\}^{1-\beta_1}$	P
$\left\{ \sum_{t=1}^{T} \left(\delta^{t-1}_{2} [\alpha_{H2}(H_{Gt} - H_{Lt}) + \alpha_{E2}(E_{Gt} - E_{Lt}) + (1-\alpha_{H2}-\alpha_{E2})(I_{Gt} - I_{Lt})] \right) \right\}^{1-\beta_2}$	$\left\{ \sum_{t=1}^{T} \left(\delta^{t-1}_{1} [\alpha_{H1}(-H_{Lt}) + \alpha_{E1}(-E_{Lt}) + (1-\alpha_{H1}-\alpha_{E1})(-I_{Lt})] \right) \right\}^{1-\beta_1}$	F_1 V_2
0	0	C

α_{H_1}, β_1, δ_1^{t-1}, α_{E_1} respectively, are the coefficient of human value, coefficient of economic value, coefficient of different times' importance, and coefficient of war risk for the Syria Corps (Player 1). Indicators 2 are used for the Kufa Corps (Player 2). Given that in the event of a war between the two armies and defeat, the loser will not gain any human, economic, and socio-political value, so the value of H_{Gt}, E_{Gt} and I_{Gt} is set at zero. Situations 1 to 3 occur when one of the two corps prefers the strategy of starting a war to the strategy of not starting a war. Also, case 4 represents a situation in which the

two armies consider the strategy of not starting a war rational. If the strategy of starting a war, for one of the states 1 to 3, has a favorable outcome for one of the parties, the player will adopt one of the strategies for starting or not starting the war, depending on the likelihood of that happening. These possibilities can determine the political positions of the players in the Islamic society of that time regarding their military power. We conclude from these states that if a player wants to avoid a war, he or she must try to keep the favorable value of starting the war for the opposite side in each of the 1 to 3 as negative, or keep the likelihood of negative favorability in the event of a war for the opposite side (the negative utility would be one of the states 1 to 3). It is also important to note that the end of a war can be terminated in a state of equilibrium that is a prelude to the start of the next war, or that one of the parties involved in the war reaches a level of negative desirability that justifies a new war for him.

4. Conclusion

The most important thing to do when modeling utility function is to estimate the benefits and losses resulted from starting and even continuing a war. What prevents the start of a war is called the cost of war, and what drives the opposition and its politicians toward war is the benefit of the war. So we should try to estimate the real costs and benefits. Therefore, it is necessary to estimate the costs and benefits of war in a scientific context. War involves the costs and benefits of human, economic, or other profits that will exist for the winner (loser) if he wins (loses). Estimating each of these points is essential to starting or continuing a particular war.

References

1. Abu Mikhnaf Kufi, Lut ibn Yahya, (1417). AH. Waqaa al-Taf, Scholar and Editor: Muhammad Hadi Yousefi Gharavi, Third Edition, Islamic Publications Office, Qom.

2. Akramizadeh et al. (1398). H.S. Strategic Gam Theory with an Islamic Approach with Emphasis on the Early Islamic Battles. AJA Strategic Studies Center.

3. Azghandi, S.A. (1394). H.S. War and Peace (Reviewing the Contemporary Military and Strategic Issues), Eighth Edition, Qom.

4. Balādhurī, ʾAḥmad ibn Yaḥyā, (1996). A.D. Jamal min Ansab al-Ashraf, Inquiry: Soheil Zakar and Riyadh Zarkeli, *Dār* al-Fikr, Beirut.

5. Berman, E., Shapiro, J. N., & Felter, J. H. (2011). Can hearts and minds be bought? The economics of counterinsurgency in Iraq. *Journal of Political Economy*, *119*(4), 766-819.

6. B. O'neill. nd. "Game Theory Models of Peace and War"

7. EbadiZadeh, H. (1379). H.S. Mathematical Modeling of Interactions and Strategies of the Islamic Republic of Iran and Saudi Arabia Using Game Theory, *The Scientific-Research Journal of Strategic Knowledge of Interdisciplinary Studies*. Higher National Defense University.

8. Ibn Abi al-Hadid, nd. Description of Nahj al-Balāghah, research by Muhammad Abu al-Fazl Ibrahim, *Dār Īhyā al-Turāth al-'Arabī*, Beirut.

9. Ibn Asākir, Ali ibn Hassan. (1421). AH. History of Damascus, Inquiry by Abi Abdullah Ali Ashur al-Junoubi, Beirut, *Dār Īhyā al-Turāth al-'Arabī*.

10. Ibn A'tham. (1411). A.H. Al-*Futūḥ*, Beirut, *Dār* al-Nadveh.

11. Ibn Athir, 'Izz al-Din Abu al-Hassan. (1408). AH. *Al-Kāmil fī al-tārīkh*, Beirut, *Dār Īhyā al-Turāth al-'Arabī*.

12. Ibn Miskawayh, Abū ʿAlī, (1379). H.S. Tajārib al-Umam, Research by Abu al-Ghasem Emami, Soroush, Second T, Tehran.

13. Ibn Muzāḥim, Nasr. (1403). A.H. The Battle of Siffin, Inquiry by Abdul Salam Mohammad Aaron, Second Edition, Qom, Manshurat Maktabah al-Mar'ashi al-Najafi.

14. Lezgi, A.N., et al. (1398). H.S. Mathematical Modeling of Gossip Propagation with a Defensive Approach by Looking at Verses and Narratives, *Scientific-Research Journal of Strategic Knowledge Interdisciplinary Studies*, No. 34.

15. Roshandel, J. (1373). H.S. Military Strategy, *Journal of Law and Political Science Faculty*, University of Tehran. No. 31.

16. Sadūq, Muhammad. (1395). A.H. Kamal al-Din and Tamam al-Nimah, Researcher and Editor: Ali Akbar Ghaffari, Second Edition, Dar al-Kutub al-Islamiyah, Tehran.

17. Sharman, J. C. (2015). War, selection, and micro-states: Economic and sociological perspectives on the international system. *European Journal of International Relations*, *21*(1), 194-214.

18. Ṭabarī, Muḥammad ibn Jarīr, Tarikh al-Umam wa al-Muluk, (1967). A.D. Research: Muhammad Abu al-Fazl Ibrahim, *Dār al-Turāth*, Beirut.

19. Vukovic, S. (2014). A.D. "International mediation as a distinct form of conflict management", *International Journal of Conflict Management*, Vol. 25 No. 1, pp. 61-80.