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Average operators based on spherical cubic fuzzy number and their application in multi-attribute decision making

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Abstract Since spherical fuzzy sets and interval-valued spherical fuzzy sets are better methods to deal with fuzziness and uncertainty. In this paper, therefore, we present the definition of spherical cubic fuzzy sets in which the degree of membership, degree of neutral membership and the degree of non-membership are cubic fuzzy numbers that satisfy the conditions that the square sum of its degree of membership, neutral and non-membership is less than or equal to one . We describe some fundamental operators and we establish score and accuracy functions to compare two spherical cubic fuzzy numbers. The distance between two spherical cubic fuzzy numbers is defined as well. spherical cubic fuzzy weighted average (SCFWA), spherical cubic fuzzy ordered weighted average (SCFOWA) and spherical cubic fuzzy hybrid weighted average (SCFHWA) operators are proposed based on the specified operators. We discuss some of the existing operators' operational laws and propose a multi-criteria decision-making (MCDM) approach based on developed operators. In addition, comparison to existing methods, the methods and operators introduced in this paper provide more general, more precise and accurate results since these methods and operators generalize their existing methods. In addition, the multi-criteria decision-making approach based on these proposed operators has been developed and the operational processes have been presented in detail as well. Finally, in order to illustrate validity, practicality and efficiency, an illustrative example is given to display the decision-making steps in detail of these proposed methods and operators.

Keywords Spherical cubic fuzzy set; SCFWA operator; SCFOWA operator; SCFHWA operator; Multi-attribute decision-making

1. Introduction

Multiple-attribute decision-making (MADM) means that the best alternative is chosen according to the multiple attribute from the limited alternatives set, that can be said to be cognitive processing. Decision-making multi criteria (MCDM) is an important branch of the decision-making theory that has been widely used in practices involving humans (Kou *et al.*, 2016). The assessment information is generally fuzzy because the real decision-making issues have always been created from the complicated context. In general, there are two types of fuzzy information: one quantitative and one qualitative. Fuzzy set (FS) (L.A. Zadeh,1965), intuitionistic fuzzy set (IFS) (K.T. Atanassov,1986), Pythagorean fuzzy fuzzy set (PFS) (R.R. Yager,2013), picture fuzzy set (Cuong and Kreinovich, 2013), spherical fuzzy set (Ashraf *et al.*, 2019) and so on can express quantitative fuzzy data. The FS theory suggested by Zadeh (L.A. Zadeh, 1965) was used to explain fuzzy quantitative information containing only a degree of membership. On this basis, Atanassov (K.T. Atanassov,1986) presented IFS, which consists of a degree of membership and a degree of non-membership that complies with the restriction form that the sum of two degrees is less than equal to one. Sometimes, however, the two degrees do not reach the criterion, but the sum of the squares of the two degrees is less than or equal to one. The PFS was introduced by Yager (R.R. Yager,2013) in which the square sum of degrees of membership and non-membership is equal to or less than one. In this case, the PFS is more capable of expressing fuzzy information than the IFS. For instance, if an expert provides the membership with the support to support the problem is 0.8 and the opposing non-membership is 0.6. IFS is obviously unable to explain this decision data, but PFS can describe it effectively. The IFS and PFS now struggle to produce any acceptable result in a case where the neutral membership degree determines independently in real-life problems. Based on these conditions, Cuong (Cuong and Kreinovich, 2013)introduced the concept of the picture fuzzy set to resolve this situation (P_cFS).Three indexes (membership degree $\tilde{P}(x)$, neutral membership degree $\tilde{I}(x)$ and non-membership degree $\tilde{N}(x)$) were used in (P_cFS) with a condition of $0 \le \tilde{P}(x) + \tilde{I}(x) + \tilde{N}(x) \le 1$. Obviously, to deal with fuzziness and vagueness, (P_cFSs) are more appropriate than IFS and PFS.

Sometimes, we face several problems in real life that can not be solved by using (P_cFS) such as when $\tilde{P}(x) + \tilde{I}(x) + \tilde{N}(x) > 1$. In those cases, ($P_c$FS) has no potential to produce any acceptable outcome. To state this condition, we offer an example:1/6,3/6 and 3/6 respectively are for help and against the degree of membership of an alternative. This satisfies the condition that their sum exceeds one for (P_cFS) and is not approved. The definition of spherical fuzzy sets (SFSs) is viewed as a generalization of (P_cFS) based on these circumstances. In SFS, membership degrees satisfy the case, $0 \le \tilde{P}^2(x) + \tilde{I}^2(x) + \tilde{N}^2(x) \le 1$.

In the Pythagorean fuzzy set, (Peng *et al.*,2015) introduced some new properties, which are division, subtraction and other significant properties. To understand the multi-criteria decision-making problems in the Pythagorean fuzzy setting, writers deal with the techniques of dominance and dependency ranking. (Khan *et al.*,2018) Prioritised

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aggregation operators were designed for multi-criteria decision-making based on Pythagorean fuzzy sets. (Qiyas *et al.*,2020) utilizing linguistic picture fuzzy aggregation operators for multiple-attribute decision-making problems. Many researchers (Qiyas *et al.*,2019) presented the application decision making by using the idea of triangular picture fuzzy linguistic ordered weighted induced aggregation operators. In 2019, (Rafiq *et al.*,2019) define a new approach of spherical fuzzy sets by utilizing the cosine similarity measure and their application in decision making. (Ashraf *et al.*,2019) presented the application of GRA method on choquent integral by utilizing the concept of spherical fuzzy set. In 2020 (Ashraf *et al.*,2019, Ashraf *et al.*,2019) many researches presented the idea of spherical fuzzy set by defining the symmetric sum and aggregation operators. In (Ashraf *et al.*,2019) presented a new representation of t-norm and t-conorm and their application in decision support system. (Jin *et al.*,2019) given the concept of spherical fuzzy logarithmic aggregation operators by utilizing the idea of entropy measure. In 2020 (Ashraf *et al.*,2020) given a new idea of spherical Dombi aggregation operators and their application in group decision problems. In (Ashraf and Abdullah, 2019) presented the various aggregation operators using the spherical fuzzy set and their application in decision making problems. (Ashraf *et al.*,2019) given the idea of logarithmic hybrid operators by utilizing the single valued neutrosophic sets and application in decision support systems. In (Jin *et al.*,2019) presented the new concept of linguistic spherical fuzzy aggregation operators and their application in multi-attribute decision-making. In 2020 (Ashraf *et al.*,2020) gave latest research on emergency decision for COVID-19 by using the spherical fuzzy sets. Many reseachers (Ashraf *et al.*,2019, Ashraf and Abdullah,2020, Ashraf *et al.*,2020) discussed various real-life applications by utilizing the spherical fuzzy sets, neutrosophic fuzzy set (Zeng *et al.* ,2019) presented covering based spherical roughness by utilizing TOPSIS method and their application in multi-attribute decision making. In 2019 (Barukab *et al.*,2019) presented the concept of fuzzy TOPSIS based on spherical fuzzy sets and entropy measure and application in decision-making.

Many researchers given the advanced concept of combining the fuzzy sets, intuitionistic fuzzy set, picture fuzzy set with cubic set theory. (Abdullah,2019) presented a new idea of intuitionistic cubic fuzzy sets and their application in supplier's selection problem. (Kaur and Garg, 2018) given the new approach of cubic intuitionistic fuzzy with Bonferroni mean operators and their application in muti-attribute decision-making. (Abdullah and Aslam,2020) discussed the application of decision support systems of hydropower plant location by utilizung the intuitionistic cubic fuzzy set. (Ashraf *et al.*, 2018) presented the more generalize concept of picture cubic set and in (Ashraf and Abdullah,2020) given the various aggregation operators by defining the idea cubic picture fuzzy sets and their application in decision-making. To generalize the concept of intuitionistic cubic fuzzy sets and picture fuzzy sets (Khan *et al.*,2019) presented the idea of Pythagorean cubic fuzzy aggregation operators and theor application in decision making problems. Khan et al. presented the new approach of Pythagorean cub fuzzy set by Extended TOPSIS method. To generalize the idea of intuitionistic cubic, picture cubic and Pythagorean cubic fuzzy set, in 2020 (Ayaz *et al.*,2020) presented the new approach of spherical cubic fuzzy set by defining the Hamacher aggregation operators and their

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application in evaluation of enterprise performance. Many researchers has great contribution in T-spherical fuzzy sets and discussed their application in multi-attribute decision making problem. (Ali *et al.*,2019) presented the idea of Complex T-spherical fuzzyaggregation operators with application to multi-attribute decision making. (Garg *et al.*,2018) given T-spherical fuzzy algorithm on improved interactive aggregation operators and their application in multi-aatribute decision-making. After that Zeng et al. [36] presented a new approach of T-spherical fuzzy set by defining Einstein interactive aggregation operators and their application in evaluation of photovoltaic cells. In [38], Garg et al. given the idea of T-spherical power aggregation operators and their application in multi-attribute decision making. (Liu *et al.*,2021) presented the new concept of normal T-spherical fuzzy aggregation operators and application in decision-making. (Ashraf and Abdullah, 2020) given the new concept and highlight the resolution of recent pandamic issue of COVID-19 under the spherical fuzzy sets. After that, (Ashraf et al.,2020) presented a new emergency response of spherical intelligent fuzzy decision process to diagonse of COVID-19. In (Ullah *et al.*,2020) presented the concept of evaluation of the performance of search and Rescue Robots utilizing T-spherical Hamacher aggregation operators. (Ullah *et al.*,2020), given the new approach of T-spherical fuzzy set by defining correlation coefficients and their application in clustering and multi-attribute decision making.

In this article, we present the definition of the spherical cubic fuzzy set (SCFS), which is the simplification of the spherical fuzzy set based on the restriction that the square sum of supremum is equal to 1. Here, we present the concept of the spherical cubic fuzzy set (PCFS), which is the general concept of the interval-valued spherical fuzzy set. We will dicuss some SCFS properties. To compare spherical cubic fuzzy numbers, we define the score and degree of accuracy of the spherical cubic fuzzy numbers (SCFNs). The distance measure between Pythagorean cubic fuzzy numbers is also defined. The spherical cubic fuzzy set satisfy the condition that the supremum's square sum to the degrees of its membership is less than or equal to one. Aggregation operators, namely spherical cubic fuzzy weighted averaging (SCFWA), spherical cubic fuzzy ordered weighted averaging (SCFOWA), spherical cubic fuzzy hybrid weighted averaging (SCFHWA) operators. In addition, these operators are then used for decision-making problems in which experts preferences in the spherical cubic fuzzy knowledge in order to illustrate the new approach's practicality and efficiency.

2. Preliminaries

Let us briefly recall in this section the basic definition of fuzzy set, intuitionistic fuzzy set, intuitionistic cubic fuzzy set, cubic fuzzy set of Pythagorean, spherical cubic fuzzy set. These definitions are going to be used here in the following analysis.

Definition 2.1. (L.A. Zadeh,1965)Let $\breve{\mathbf{X}}$ be a fixed set. A fuzzy set (FS) $\breve{\mathbf{F}}$ is defined as:

$$\breve{\mathbf{F}} = \left\{\langle \breve{\mathbf{x}}, \tilde{\mu}_{\breve{\mathbf{F}}}(\breve{\mathbf{x}})\rangle \mid \breve{\mathbf{x}} \in \breve{\mathbf{X}}\right\} \quad (1)$$

where $\tilde{\mu}_{\breve{F}}: \breve{X} \rightarrow [0,1]$ and $\tilde{\mu}_{\breve{F}}(\breve{x})$ is known as membership degree of $\breve{x}$ in $\breve{X}$.

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Definition 2.2. (K.T. Atanassov,1986)Let $\breve{\mathbf{X}}$ be a fixed set. An intuitionistic fuzzy set (IFS) $\breve{\mathbf{I}}$ is defined as:

$$\breve{\boldsymbol{I}} = \{\langle \breve{\boldsymbol{x}}, \tilde{\boldsymbol{\mu}}_{\breve{I}}(\breve{\boldsymbol{x}}), \tilde{\boldsymbol{\vartheta}}_{\breve{I}}(\breve{\boldsymbol{x}})\rangle \mid \breve{\boldsymbol{x}} \in \breve{\boldsymbol{X}}\}, \tag{2}$$

where $\tilde{\mu}_{\breve{I}}: \breve{X} \rightarrow [0,1]$ and $\tilde{\vartheta}_{\breve{I}}: \breve{X} \rightarrow [0,1]$ under the specified condition

$0 \leq \tilde{\mu}_{\breve{I}}(\breve{x}) + \tilde{\vartheta}_{\breve{I}}(\breve{x}) \leq 1$

for all $\breve{x} \in \breve{X}$, $\tilde{\mu}_{\breve{I}}(\breve{x})$ represent the membership degree and $\tilde{\vartheta}_{\breve{I}}(\breve{x})$ represent the non-membership degree of $\breve{x}$ in $\breve{X}$.

Definition 2.3. (Abdullah and Aslam, 2020)Let $\breve{X}$ be a fixed set. A cubic set (IFS) $\bar{C}$ is defined as:

$$\bar{\mathbf{C}} = \{\langle \breve{\mathbf{x}}, \tilde{\boldsymbol{\mu}}_{\bar{C}}(\breve{\mathbf{x}}), \tilde{\boldsymbol{\vartheta}}_{\bar{C}}(\breve{\mathbf{x}})\rangle \mid \breve{\mathbf{x}} \in \breve{\mathbf{X}}\}, \tag{3}$$

where $\tilde{\mu}_{\bar{C}}(\breve{x})$ represent the interval-valued fuzzy set and $\tilde{\vartheta}_{\bar{C}}(\breve{x})$ represent the simple fuzzy set in $\breve{X}$.

Definition 2.4. (Abdullah and Aslam, 2020)Let $\breve{\mathbf{X}}$ be a fixed set. An intuitionistic cubic fuzzy set (ICFS) $\breve{\mathbf{I}}_{\mathbf{c}}$ is defined as:

$$\breve{\mathbf{I}}_{\mathbf{c}} = \{\langle \breve{\mathbf{x}}, \mathbf{c}_{\breve{\mathbf{I}}_{\mathbf{c}}}, \dot{\mathbf{c}}_{\breve{\mathbf{I}}_{\mathbf{c}}}\rangle \mid \breve{\mathbf{x}} \in \breve{\mathbf{X}}\}, \tag{4}$$

where $c_{\breve{I}_c} = \langle [\tilde{a}^-, \tilde{a}^+], \tilde{\mu}\rangle$ represent the membership degree and $\dot{c}_{\breve{I}_c} = \langle [\tilde{b}^-, \tilde{b}^+], \tilde{\vartheta}\rangle$ represent the non-membership degree of $\breve{I}_c$. Where $[\tilde{a}^-, \tilde{a}^+] \subseteq [0,1]$, $[\tilde{b}^-, \tilde{b}^+] \subseteq [0,1]$and $\tilde{\mu}: \breve{X} \rightarrow [0,1]$, $\tilde{\vartheta}: \breve{X} \rightarrow [0,1]$ under the specified condition

$0 \leq \sup[\tilde{a}^-, \tilde{a}^+] + \sup[\tilde{b}^-, \tilde{b}^+] \leq 1$ and $0 \leq \tilde{\mu} + \tilde{\vartheta} \leq 1$.

for all $\breve{x} \in \breve{X}$, $\tilde{\mu}(\breve{x})$ represent the membership degree and $\tilde{\vartheta}(\breve{x})$ represent the non-membership degree of $\breve{x}$ in $\breve{X}$.

Definition 2.5. (Khan *et al.*,2019) Let $\breve{\mathbf{X}}$ be a fixed set. A Pythagorean cubic fuzzy set (PCFS) $\tilde{\mathbf{P}}_{\mathbf{c}}$ is defined as:

$$\tilde{\mathbf{P}}_{\mathbf{c}} = \{\langle \breve{\mathbf{x}}, \mathbf{c}_{\tilde{\mathbf{P}}_{\mathbf{c}}}, \dot{\mathbf{c}}_{\tilde{\mathbf{P}}_{\mathbf{c}}}\rangle \mid \breve{\mathbf{x}} \in \breve{\mathbf{X}}\}, \tag{5}$$

where $c_{\tilde{P}_c} = \langle [\tilde{a}^-, \tilde{a}^+], \tilde{\mu}\rangle$ represent the membership degree and $\dot{c}_{\tilde{P}_c} = \langle [\tilde{b}^-, \tilde{b}^+], \tilde{\vartheta}\rangle$ represent the non-membership degree of $\tilde{P}_c$. Where $[\tilde{a}^-, \tilde{a}^+] \subseteq [0,1]$, $[\tilde{b}^-, \tilde{b}^+] \subseteq [0,1]$and $\tilde{\mu}: \breve{X} \rightarrow [0,1]$, $\tilde{\vartheta}: \breve{X} \rightarrow [0,1]$ under the specified condition

$0 \leq \sup[\tilde{a}^-, \tilde{a}^+] + \sup[\tilde{b}^-, \tilde{b}^+] \leq 1$ and $0 \leq \tilde{\mu} + \tilde{\vartheta} \leq 1$.

for all $\breve{x} \in \breve{X}$, $\tilde{\mu}(\breve{x})$ represent the membership degree and $\tilde{\vartheta}(\breve{x})$ represent the non-membership degree of $\breve{x}$ in $\breve{X}$.

Definition 2.6. (Ayaz *et al.*,2020) Let $\breve{\mathbf{X}}$ be a fixed set. A spherical cubic fuzzy set (SCFS) $\tilde{\mathbf{S}}_{\mathbf{c}}$ is defined as:

$$\tilde{\mathbf{S}}_{\mathbf{c}} = \{\langle \breve{\mathbf{x}}, \mathbf{c}_{\tilde{\mathbf{S}}_{\mathbf{c}}}, \dot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c}}}, \ddot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c}}} \rangle \mid \breve{\mathbf{x}} \in \breve{\mathbf{X}}\}, \tag{6}$$

where $c_{\tilde{S}_c} = \langle [\tilde{a}^-, \tilde{a}^+], \tilde{\mu} \rangle$ represent the membership degree, $\dot{c}_{\tilde{S}_c} = \langle [\tilde{n}^-, \tilde{n}^+], \tilde{\delta} \rangle$ represent the neutral degree and $\ddot{c}_{\tilde{S}_c} = \langle [\tilde{b}^-, \tilde{b}^+], \tilde{\vartheta} \rangle$ represent the non-membership degree of $\tilde{S}_c$. Where $[\tilde{a}^-, \tilde{a}^+] \subseteq [0,1]$, $[\tilde{n}^-, \tilde{n}^+] \subseteq [0,1]$, $[\tilde{b}^-, \tilde{b}^+] \subseteq [0,1]$ and $\tilde{\mu}: \breve{X} \to [0,1]$, $\tilde{\delta}: \breve{X} \to [0,1]$, $\tilde{\vartheta}: \breve{X} \to [0,1]$ under the specified condition

$$0 \leq (\sup[\tilde{a}^-, \tilde{a}^+])^2 + (\sup[\tilde{n}^-, \tilde{n}^+])^2 + \left(\sup[\tilde{b}^-, \tilde{b}^+]\right)^2 \leq 1 \text{ and } 0 \leq \tilde{\mu}^2 + \tilde{\delta}^2 + \tilde{\vartheta}^2 \leq 1.$$

for all $\breve{x} \in \breve{X}$, $\tilde{\mu}(\breve{x})$ represent the membership degree, $\tilde{\delta}(\breve{x})$ represent the neutral degree and $\tilde{\vartheta}(\breve{x})$ represent the non-membership degree of $\breve{x}$ in $\breve{X}$.

The degree of indeterminacy of (SCFS) can be defined as:

$$\tilde{\pi}_{\tilde{S}_c} = \langle \sqrt{1 - \left((\sup[\tilde{a}^-, \tilde{a}^+])^2 + (\sup[\tilde{n}^-, \tilde{n}^+])^2 + \left(\sup[\tilde{b}^-, \tilde{b}^+]\right)^2\right)}, \sqrt{1 - (\tilde{\mu}^2 + \tilde{\delta}^2 + \tilde{\vartheta}^2)} \rangle.$$

For our convenience, we call $\tilde{S}_c = \langle c_{\tilde{S}_c}, \dot{c}_{\tilde{S}_c}, \ddot{c}_{\tilde{S}_c} \rangle$ spherical cubic fuzzy number (SCFN).

Definition 2.7. (Ashraf and Abdullah,2020) Let $\tilde{\mathbf{S}}_{\mathbf{c_1}} = \langle \mathbf{c}_{\tilde{\mathbf{S}}_{\mathbf{c_1}}}, \dot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_1}}}, \ddot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_1}}} \rangle, \tilde{\mathbf{S}}_{\mathbf{c_2}} = \langle \mathbf{c}_{\tilde{\mathbf{S}}_{\mathbf{c_2}}}, \dot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_2}}}, \ddot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_2}}} \rangle$ and $\tilde{\mathbf{S}}_{\mathbf{c}} = \langle \mathbf{c}_{\tilde{\mathbf{S}}_{\mathbf{c}}}, \dot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c}}}, \ddot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c}}} \rangle$ be three spherical cubic fuzzy numbers, then the following operational laws hold:

1. $$\tilde{S}_{c_1} \oplus \tilde{S}_{c_2} = \left\{ \begin{matrix} \left(\begin{matrix} \left[\sqrt{(\tilde{a}_1^-)^2 + (\tilde{a}_2^-)^2 - (\tilde{a}_1^-)^2(\tilde{a}_2^-)^2}, \sqrt{(\tilde{a}_1^+)^2 + (\tilde{a}_2^+)^2 - (\tilde{a}_1^+)^2(\tilde{a}_2^+)^2}\right], \\ \sqrt{(\tilde{\mu}_1)^2 + (\tilde{\mu}_2)^2 - (\tilde{\mu}_1)^2(\tilde{\mu}_2)^2} \end{matrix} \right), \\ \left([\tilde{n}_1^-\tilde{n}_2^-, \tilde{n}_1^+\tilde{n}_2^+], \tilde{\delta}_1\tilde{\delta}_2\right), \left([\tilde{b}_1^-\tilde{b}_2^-, \tilde{b}_1^+\tilde{b}_2^+], \tilde{\vartheta}_1\tilde{\vartheta}_2\right) \end{matrix} \right\};$$
2. $$\tilde{S}_{c_1} \oplus \tilde{S}_{c_2} = \left\{ \begin{matrix} \left([\tilde{a}_1^-\tilde{a}_2^-, \tilde{a}_1^+\tilde{a}_2^+], \tilde{\mu}_1\tilde{\mu}_2\right), \left([\tilde{n}_1^-\tilde{n}_2^-, \tilde{n}_1^+\tilde{n}_2^+], \tilde{\delta}_1\tilde{\delta}_2\right), \\ \left(\begin{matrix} \left[\sqrt{(\tilde{b}_1^-)^2 + (\tilde{b}_2^-)^2 - (\tilde{b}_1^-)^2(\tilde{b}_2^-)^2}, \sqrt{(\tilde{b}_1^+)^2 + (\tilde{b}_2^+)^2 - (\tilde{b}_1^+)^2(\tilde{b}_2^+)^2}\right], \\ \sqrt{(\tilde{\vartheta}_1)^2 + (\tilde{\vartheta}_2)^2 - (\tilde{\vartheta}_1)^2(\tilde{\vartheta}_2)^2} \end{matrix} \right) \end{matrix} \right\};$$

3. $$\tilde{S}_c^{\ \tau} = \left\{ \begin{matrix} ([(\tilde{a}^-)^\tau, (\tilde{a}^+)^\tau], (\tilde{\mu})^\tau), ([(\tilde{n}^-)^\tau, (\tilde{n}^+)^\tau], (\tilde{\delta})^\tau), \\ \left(\left[\sqrt{1-\left(1-(\tilde{b}^-)^2\right)^\tau}, \sqrt{1-\left(1-(\tilde{b}^+)^2\right)^\tau}\right], \sqrt{1-\left(1-(\tilde{\vartheta})^2\right)^\tau}\right) \end{matrix} \right\};$$

4. $$\tau.\tilde{S}_c = \left\{ \begin{matrix} \left(\left[\sqrt{1-(1-(\tilde{a}^-)^2)^\tau}, \sqrt{1-(1-(\tilde{a}^+)^2)^\tau}\right], \sqrt{1-(1-(\tilde{\mu})^2)^\tau}\right), \\ ([(\tilde{n}^-)^\tau, (\tilde{n}^+)^\tau], (\tilde{\delta})^\tau), ([(\tilde{b}^-)^\tau, (\tilde{b}^+)^\tau], (\tilde{\vartheta})^\tau) \end{matrix} \right\}.$$

Definition 2.8. (Ayaz *et al.*,2020) Let $\tilde{\mathbf{S}}_\mathbf{c} = \left(\langle[\tilde{\mathbf{a}}^-, \tilde{\mathbf{a}}^+], \tilde{\boldsymbol{\mu}}\rangle, \langle[\tilde{\mathbf{n}}^-, \tilde{\mathbf{n}}^+], \tilde{\boldsymbol{\delta}}\rangle, \langle[\tilde{\mathbf{b}}^-, \tilde{\mathbf{b}}^+], \tilde{\boldsymbol{\vartheta}}\rangle\right)$ be a spherical cubic fuzzy number. Then the score function is defined as:

$$\bar{\mathbf{S}}(\tilde{\mathbf{S}}_\mathbf{c}) = \frac{(\tilde{\mathbf{a}}^- + \tilde{\mathbf{a}}^+ + \tilde{\boldsymbol{\mu}})^2 + (\tilde{\mathbf{n}}^- + \tilde{\mathbf{n}}^+ + \tilde{\boldsymbol{\delta}})^2 - (\tilde{\mathbf{b}}^- + \tilde{\mathbf{b}}^+ + \tilde{\boldsymbol{\vartheta}})^2}{\mathbf{9}} \tag{7}$$

where $\bar{S}(\tilde{S}_c) \in [-1,1]$.

Definition 2.9. (Ayaz *et al.*,2020) Let $\tilde{\mathbf{S}}_\mathbf{c} = \left(\langle[\tilde{\mathbf{a}}^-, \tilde{\mathbf{a}}^+], \tilde{\boldsymbol{\mu}}\rangle, \langle[\tilde{\mathbf{n}}^-, \tilde{\mathbf{n}}^+], \tilde{\boldsymbol{\delta}}\rangle, \langle[\tilde{\mathbf{b}}_\mathbf{1}^-, \tilde{\mathbf{b}}^+], \tilde{\boldsymbol{\vartheta}}\rangle\right)$ be a spherical cubic fuzzy number. Then the accuracy function is defined as:

$$\bar{\mathbf{A}}(\tilde{\mathbf{S}}_\mathbf{c}) = \frac{(\tilde{\mathbf{a}}^- + \tilde{\mathbf{a}}^+ + \tilde{\boldsymbol{\mu}})^2 + (\tilde{\mathbf{n}}^- + \tilde{\mathbf{n}}^+ + \tilde{\boldsymbol{\delta}})^2 + (\tilde{\mathbf{b}}^- + \tilde{\mathbf{b}}^+ + \tilde{\boldsymbol{\vartheta}})^2}{\mathbf{9}} \tag{8}$$

where $\bar{A}(\tilde{S}_c) \in [0,1]$.

Definition 2.10. (Ayaz *et al.*,2020) Let $\tilde{\mathbf{S}}_{\mathbf{c}_1} = \left(\langle[\tilde{\mathbf{a}}_1^-, \tilde{\mathbf{a}}_1^+], \tilde{\boldsymbol{\mu}}_1\rangle, \langle[\tilde{\mathbf{n}}_1^-, \tilde{\mathbf{n}}_1^+], \tilde{\boldsymbol{\delta}}_1\rangle, \langle[\tilde{\mathbf{b}}_1^-, \tilde{\mathbf{b}}_1^+], \tilde{\boldsymbol{\vartheta}}_1\rangle\right)$ and $\tilde{\mathbf{S}}_{\mathbf{c}_2} = \left(\langle[\tilde{\mathbf{a}}_2^-, \tilde{\mathbf{a}}_2^+], \tilde{\boldsymbol{\mu}}_2\rangle, \langle[\tilde{\mathbf{n}}_2^-, \tilde{\mathbf{n}}_2^+], \tilde{\boldsymbol{\delta}}_2\rangle, \langle[\tilde{\mathbf{b}}_2^-, \tilde{\mathbf{b}}_2^+], \tilde{\boldsymbol{\vartheta}}_2\rangle\right)$ be two spherical cubic fuzzy numbers. Then the comparison analysis is given below:

1. If $\bar{S}(\tilde{S}_{c_1}) > \bar{S}(\tilde{S}_{c_2}) \Rightarrow \tilde{S}_{c_1} > \tilde{S}_{c_2}$;
2. If $\bar{S}(\tilde{S}_{c_1}) < \bar{S}(\tilde{S}_{c_2}) \Rightarrow \tilde{S}_{c_1} < \tilde{S}_{c_2}$;
3. If $\bar{S}(\tilde{S}_{c_1}) = \bar{S}(\tilde{S}_{c_2})$ then
   a. If $\bar{A}(\tilde{S}_{c_1}) > \bar{A}(\tilde{S}_{c_2}) \Rightarrow \tilde{S}_{c_1} > \tilde{S}_{c_2}$;
   b. If $\bar{A}(\tilde{S}_{c_1}) < \bar{A}(\tilde{S}_{c_2}) \Rightarrow \tilde{S}_{c_1} < \tilde{S}_{c_2}$;
   c. If $\bar{A}(\tilde{S}_{c_1}) = \bar{A}(\tilde{S}_{c_2}) \Rightarrow \tilde{S}_{c_1} = \tilde{S}_{c_2}$.

3. Spherical cubic fuzzy aggregation operators

In this section, we presented some weighted averaging operators for a collection of spherical cubic fuzzy numbers (SCFNs), and and analyze some of its characteristics.

3.1 Spherical cubic fuzzy weighted averaging aggregation operators

This section examines weighted averaging aggregation operators based on specified operational characteristics of SCFNs.

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Definition 3.1.1. Let $\tilde{\mathbf{S}}_{\mathbf{c_i}} = \langle \mathbf{c}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \dot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \ddot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}} \rangle$ $(\mathbf{i} = \mathbf{1}, \mathbf{2}, \dots, \mathbf{n})$ be any collection of SCFNs and $\mathbf{SCFWA}: \mathbf{SCFN^n} \rightarrow \mathbf{SCFN}$, then spherical cubic fuzzy weighted average (SCFWA) operator is defined as:

$$\mathbf{SCFWA}_{\bar{\boldsymbol{\omega}}}\left(\tilde{\mathbf{S}}_{\mathbf{c_1}}, \tilde{\mathbf{S}}_{\mathbf{c_2}}, \dots, \tilde{\mathbf{S}}_{\mathbf{c_n}}\right) = \sum_{\mathbf{i=1}}^{\mathbf{n}} \bar{\boldsymbol{\omega}}_{\mathbf{i}} \tilde{\mathbf{S}}_{\mathbf{c_i}}, \tag{9}$$

where $\bar{\omega}_i = (\bar{\omega}_1, \bar{\omega}_2, \dots, \bar{\omega}_n)$ represent the weight vector of $\tilde{S}_{c_i}$ and $\bar{\omega}_i \geq 0$, $\sum_{i=1}^{n} \bar{\omega}_i = 1$.

Theorem 3.1.2. Let $\tilde{\mathbf{S}}_{\mathbf{c_i}} = \langle \mathbf{c}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \dot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \ddot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}} \rangle$ $(\mathbf{i} = \mathbf{1}, \mathbf{2}, \dots, \mathbf{n})$ be any collection of SCFNs. Then by using the Definition [9] and the operational properties of SCFNs the following results can be obtained.

$$\mathrm{SCFWA}_{\bar{\omega}}\left(\tilde{S}_{c_1}, \tilde{S}_{c_2}, \dots, \tilde{S}_{c_n}\right)$$
$$= \left\{ \begin{array}{c} \left(\left[\sqrt{1 - \prod_{i=1}^{n}(1 - (\tilde{a}_i^-)^2)^{\bar{\omega}_i}}, \sqrt{1 - \prod_{i=1}^{n}(1 - (\tilde{a}_i^+)^2)^{\bar{\omega}_i}}\right], \sqrt{1 - \prod_{i=1}^{n}(1 - (\tilde{\mu}_i)^2)^{\bar{\omega}_i}}\right), \\ \left(\left[\prod_{i=1}^{n}(\tilde{n}_i^-)^{\bar{\omega}_i}, \prod_{i=1}^{n}(\tilde{n}_i^+)^{\bar{\omega}_i}\right], \prod_{i=1}^{n}\left(\tilde{\delta}_i\right)^{\bar{\omega}_i}\right), \left(\left[\prod_{i=1}^{n}\left(\tilde{b}_i^-\right)^{\bar{\omega}_i}, \prod_{i=1}^{n}\left(\tilde{b}_i^+\right)^{\bar{\omega}_i}\right], \prod_{i=1}^{n}\left(\tilde{\vartheta}_i\right)^{\bar{\omega}_i}\right) \end{array} \right\}$$

where $\bar{\omega}_i = (\bar{\omega}_1, \bar{\omega}_2, \dots, \bar{\omega}_n)$ represent the weight vector of $\tilde{S}_{c_i}$ and $\bar{\omega}_i \geq 0$, $\sum_{i=1}^{n} \bar{\omega}_i = 1$.

Proof. We proved this by using the mathematical induction method. So we follow as

(a) For $\mathbf{n} = \mathbf{2}$,

$$\bar{\omega}_1 \tilde{S}_{c_1} = \left\{ \begin{array}{c} \left(\left[\sqrt{1 - (1 - (\tilde{a}_1^-)^2)^{\bar{\omega}_1}}, \sqrt{1 - (1 - (\tilde{a}_1^+)^2)^{\bar{\omega}_1}}\right], \sqrt{1 - (1 - (\tilde{\mu}_1)^2)^{\bar{\omega}_1}}\right), \\ \left([(\tilde{n}_1^-)^{\bar{\omega}_1}, (\tilde{n}_1^+)^{\bar{\omega}_1}], \left(\tilde{\delta}_1\right)^{\bar{\omega}_1}\right), \left(\left[\left(\tilde{b}_1^-\right)^{\bar{\omega}_1}, \left(\tilde{b}_1^+\right)^{\bar{\omega}_1}\right], \left(\tilde{\vartheta}_1\right)^{\bar{\omega}_1}\right) \end{array} \right\}$$

and

$$\bar{\omega}_2 \tilde{S}_{c_2} = \left\{ \begin{array}{c} \left(\left[\sqrt{1 - (1 - (\tilde{a}_2^-)^2)^{\bar{\omega}_2}}, \sqrt{1 - (1 - (\tilde{a}_2^+)^2)^{\bar{\omega}_2}}\right], \sqrt{1 - (1 - (\tilde{\mu}_2)^2)^{\bar{\omega}_2}}\right), \\ \left([(\tilde{n}_2^-)^{\bar{\omega}_2}, (\tilde{n}_2^+)^{\bar{\omega}_2}], \left(\tilde{\delta}_2\right)^{\bar{\omega}_2}\right), \left(\left[\left(\tilde{b}_2^-\right)^{\bar{\omega}_2}, \left(\tilde{b}_2^+\right)^{\bar{\omega}_2}\right], \left(\tilde{\vartheta}_2\right)^{\bar{\omega}_2}\right) \end{array} \right\}$$

Then

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$$
\begin{aligned}
&\mathrm{SCFWA}_{\bar{\omega}}\left(\tilde{S}_{c_1}, \tilde{S}_{c_2}\right) = \bar{\omega}_1 \tilde{S}_{c_1} + \bar{\omega}_2 \tilde{S}_{c_2} \\
&= \left\{\begin{array}{c}\left(\left[\sqrt{1-(1-(\tilde{a}_1^-)^2)^{\bar{\omega}_1}}, \sqrt{1-(1-(\tilde{a}_1^+)^2)^{\bar{\omega}_1}}\right], \sqrt{1-(1-(\tilde{\mu}_1)^2)^{\bar{\omega}_1}}\right), \\ \left([(\tilde{n}_1^-)^{\bar{\omega}_1}, (\tilde{n}_1^+)^{\bar{\omega}_1}], (\tilde{\delta}_1)^{\bar{\omega}_1}\right), \left(\left[(\tilde{b}_1^-)^{\bar{\omega}_1}, (\tilde{b}_1^+)^{\bar{\omega}_1}\right], (\tilde{\vartheta}_1)^{\bar{\omega}_1}\right)\end{array}\right\} \\
&+ \left\{\begin{array}{c}\left(\left[\sqrt{1-(1-(\tilde{a}_2^-)^2)^{\bar{\omega}_2}}, \sqrt{1-(1-(\tilde{a}_2^+)^2)^{\bar{\omega}_2}}\right], \sqrt{1-(1-(\tilde{\mu}_2)^2)^{\bar{\omega}_2}}\right), \\ \left([(\tilde{n}_2^-)^{\bar{\omega}_2}, (\tilde{n}_2^+)^{\bar{\omega}_2}], (\tilde{\delta}_2)^{\bar{\omega}_2}\right), \left(\left[(\tilde{b}_2^-)^{\bar{\omega}_2}, (\tilde{b}_2^+)^{\bar{\omega}_2}\right], (\tilde{\vartheta}_2)^{\bar{\omega}_2}\right)\end{array}\right\}
\end{aligned}
$$

$$
= \left\{\begin{array}{c}
\left(\begin{array}{c}\left[\begin{array}{c}\sqrt{(1-(1-(\tilde{a}_1^-)^2)^{\bar{\omega}_1}) + (1-(1-(\tilde{a}_2^-)^2)^{\bar{\omega}_2}) - (1-(1-(\tilde{a}_1^-)^{\mathbf{2}})^{\bar{\omega}_1})(1-(1-(\tilde{a}_2^-)^2)^{\bar{\omega}_2})}, \\ \sqrt{(1-(1-(\tilde{a}_1^+)^2)^{\bar{\omega}_1}) + (1-(1-(\tilde{a}_2^+)^2)^{\bar{\omega}_2}) - (1-(1-(\tilde{a}_1^+)^2)^{\bar{\omega}_1})(1-(1-(\tilde{a}_2^+)^2)^{\bar{\omega}_2})}\end{array}\right], \\ \sqrt{(1-(1-(\tilde{\mu}_1)^2)^{\bar{\omega}_1}) + (1-(1-(\tilde{\mu}_2)^2)^{\bar{\omega}_2}) - (1-(1-(\tilde{\mu}_1)^2)^{\bar{\omega}_1})(1-(1-(\tilde{\mu}_2)^2)^{\bar{\omega}_2})}\end{array}\right), \\
\left([(\tilde{n}_1^-)^{\bar{\omega}_1}.(\tilde{n}_2^-)^{\bar{\omega}_2}, (\tilde{n}_1^+)^{\bar{\omega}_1}.(\tilde{n}_2^+)^{\bar{\omega}_2}], (\tilde{\delta}_1)^{\bar{\omega}_1}.(\tilde{\delta}_2)^{\bar{\omega}_2}\right), \\
\left(\left[(\tilde{b}_1^-)^{\bar{\omega}_1}.(\tilde{b}_2^-)^{\bar{\omega}_2}, (\tilde{b}_1^+)^{\bar{\omega}_1}.(\tilde{b}_2^+)^{\bar{\omega}_2}\right], (\tilde{\vartheta}_1)^{\bar{\omega}_1}.(\tilde{\vartheta}_2)^{\bar{\omega}_2}\right)
\end{array}\right\}
$$

$$
= \left\{\begin{array}{c}
\left(\left[\begin{array}{c}\sqrt{1-(1-(\tilde{a}_1^-)^2)^{\bar{\omega}_1}(1-(\tilde{a}_2^-)^2)^{\bar{\omega}_2}}, \sqrt{1-(1-(\tilde{a}_1^+)^2)^{\bar{\omega}_1}(1-(\tilde{a}_2^+)^2)^{\bar{\omega}_2}} \\ \sqrt{1-(1-(\tilde{\mu}_1)^2)^{\bar{\omega}_1}(1-(\tilde{\mu}_2)^2)^{\bar{\omega}_2}}\end{array}\right],\right), \\
\left([(\tilde{n}_1^-)^{\bar{\omega}_1}.(\tilde{n}_2^-)^{\bar{\omega}_2}, (\tilde{n}_1^+)^{\bar{\omega}_1}.(\tilde{n}_2^+)^{\bar{\omega}_2}], (\tilde{\delta}_1)^{\bar{\omega}_1}.(\tilde{\delta}_2)^{\bar{\omega}_2}\right), \\
\left(\left[(\tilde{b}_1^-)^{\bar{\omega}_1}.(\tilde{b}_2^-)^{\bar{\omega}_2}, (\tilde{b}_1^+)^{\bar{\omega}_1}.(\tilde{b}_2^+)^{\bar{\omega}_2}\right], (\tilde{\vartheta}_1)^{\bar{\omega}_1}.(\tilde{\vartheta}_2)^{\bar{\omega}_2}\right)
\end{array}\right\}
$$

$$
= \left\{\begin{array}{c}
\left(\left[\sqrt{1-\prod_{i=1}^{2}(1-(\tilde{a}_{\mathbf{i}}^-)^2)^{\bar{\omega}_i}}, \sqrt{1-\prod_{i=1}^{2}(1-(\tilde{a}_{\mathbf{i}}^+)^2)^{\bar{\omega}_i}}\right], \sqrt{1-\prod_{i=1}^{2}(1-(\tilde{\mu}_i)^2)^{\bar{\omega}_i}}\right), \\
\left(\left[\prod_{i=1}^{2}(\tilde{n}_i^-)^{\bar{\omega}_i}, \prod_{i=1}^{2}(\tilde{n}_i^+)^{\bar{\omega}_i}\right], \prod_{i=1}^{2}(\tilde{\delta}_i)^{\bar{\omega}_i}\right), \left(\left[\prod_{i=1}^{2}(\tilde{b}_i^-)^{\bar{\omega}_i}, \prod_{i=1}^{2}(\tilde{b}_i^+)^{\bar{\omega}_i}\right], \prod_{i=1}^{2}(\tilde{\vartheta}_i)^{\bar{\omega}_i}\right)
\end{array}\right\}
$$

(b) Suppose the result is true for $n = k$ i.e.

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$$\mathrm{SCFWA}_{\overline{\omega}}\left(\tilde{S}_{c_1}, \tilde{S}_{c_2}, \dots, \tilde{S}_{c_k}\right)$$
$$= \left\{\begin{matrix} \left(\left(\left[\sqrt{1-\prod_{i=1}^{k}(1-(\tilde{a}_i^-)^2)^{\overline{\omega}_i}}, \sqrt{1-\prod_{i=1}^{k}(1-(\tilde{a}_i^+)^2)^{\overline{\omega}_i}}\right], \sqrt{1-\prod_{i=1}^{k}(1-(\tilde{\mu}_i)^2)^{\overline{\omega}_i}}\right),\right. \\ \left.\left(\left[\prod_{i=1}^{k}(\tilde{n}_i^-)^{\overline{\omega}_i}, \prod_{i=1}^{k}(\tilde{n}_i^+)^{\overline{\omega}_i}\right], \prod_{i=1}^{k}\left(\tilde{\delta}_i\right)^{\overline{\omega}_i}\right), \left(\left[\prod_{i=1}^{k}\left(\tilde{b}_i^-\right)^{\overline{\omega}_i}, \prod_{i=1}^{k}\left(\tilde{b}_i^+\right)^{\overline{\omega}_i}\right], \prod_{i=1}^{k}\left(\tilde{\vartheta}_i\right)^{\overline{\omega}_i}\right)\right) \end{matrix}\right\}$$

(c) Now we have to show that the result is valid for $n = k + 1$, using (a) & (b)

$$\mathrm{SCFWA}_{\overline{\omega}}\left(\tilde{S}_{c_1}, \tilde{S}_{c_2}, \dots, \tilde{S}_{c_k}, \tilde{S}_{c_{k+1}}\right) = \sum_{i=1}^{k} \overline{\omega}_i \tilde{S}_{c_i} + \overline{\omega}_{k+1}\tilde{S}_{c_{k+1}}$$

$$\mathrm{SCFWA}_{\overline{\omega}}\left(\tilde{S}_{c_1}, \tilde{S}_{c_2}, \dots, \tilde{S}_{c_k}, \tilde{S}_{c_{k+1}}\right) =$$
$$= \left\{\begin{matrix} \left(\left(\left[\sqrt{1-\prod_{i=1}^{k}(1-(\tilde{a}_{\mathbf{i}}^-)^2)^{\overline{\omega}_i}}, \sqrt{1-\prod_{i=1}^{k}(1-(\tilde{a}_{\mathbf{i}}^+)^2)^{\overline{\omega}_i}}\right], \sqrt{1-\prod_{i=1}^{k}(1-(\tilde{\mu}_i)^2)^{\overline{\omega}_i}}\right),\right. \\ \left.\left(\left[\prod_{i=1}^{k}(\tilde{n}_i^-)^{\overline{\omega}_i}, \prod_{i=1}^{k}(\tilde{n}_i^+)^{\overline{\omega}_i}\right], \prod_{i=1}^{k}\left(\tilde{\delta}_i\right)^{\overline{\omega}_i}\right), \left(\left[\prod_{i=1}^{k}\left(\tilde{b}_i^-\right)^{\overline{\omega}_i}, \prod_{i=1}^{k}\left(\tilde{b}_i^+\right)^{\overline{\omega}_i}\right], \prod_{i=1}^{k}\left(\tilde{\vartheta}_i\right)^{\overline{\omega}_i}\right)\right) \end{matrix}\right\}$$
$$+ \left\{\begin{matrix} \left(\left(\left[\sqrt{1-\left(1-\left(\tilde{a}_{k+1}^-\right)^2\right)^{\overline{\omega}_{k+1}}}, \sqrt{1-\left(1-\left(\tilde{a}_{k+1}^+\right)^2\right)^{\overline{\omega}_{k+1}}}\right], \sqrt{1-(1-(\tilde{\mu}_{k+1})^2)^{\overline{\omega}_{k+1}}}\right)\right. \\ \left.\left([(\tilde{n}_{k+1}^-)^{\overline{\omega}_{k+1}}, (\tilde{n}_{k+1}^+)^{\overline{\omega}_{k+1}}], \left(\tilde{\delta}_{k+1}\right)^{\overline{\omega}_{k+1}}\right), \left(\left[\left(\tilde{b}_{k+1}^-\right)^{\overline{\omega}_{k+1}}, \left(\tilde{b}_{k+1}^+\right)^{\overline{\omega}_{k+1}}\right], \left(\tilde{\vartheta}_{k+1}\right)^{\overline{\omega}_{k+1}}\right)\right) \end{matrix}\right.$$

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$$\left(\begin{array}{c}\left[\begin{array}{l}\sqrt{\left(1-\prod_{i=1}^{k}(1-(\tilde{a}_{i}^{-})^{2})^{\varpi_i}\right)+\left(1-\left(1-\left(\tilde{a}_{k+1}^{-}\right)^{2}\right)^{\varpi_{k+1}}\right)-\left(1-\prod_{i=1}^{k}(1-(\tilde{a}_{i}^{-})^{2})^{\varpi_i}\right)\left(1-\left(1-\left(\tilde{a}_{k+1}^{-}\right)^{2}\right)^{\varpi_{k+1}}\right)},\\ \sqrt{\left(1-\prod_{i=1}^{k}(1-(\tilde{a}_{i}^{+})^{2})^{\varpi_i}\right)+\left(1-\left(1-\left(\tilde{a}_{k+1}^{+}\right)^{2}\right)^{\varpi_{k+1}}\right)-\left(1-\prod_{i=1}^{k}(1-(\tilde{a}_{i}^{+})^{2})^{\varpi_i}\right)\left(1-\left(1-\left(\tilde{a}_{k+1}^{+}\right)^{2}\right)^{\varpi_{k+1}}\right)}\end{array}\right],\\ \sqrt{\left(1-\prod_{i=1}^{k}(1-(\tilde{\mu}_{i})^{2})^{\varpi_i}\right)+(1-(1-(\tilde{\mu}_{k+1})^{2})^{\varpi_{k+1}})-\left(1-\prod_{i=1}^{k}(1-(\tilde{\mu}_{i})^{2})^{\varpi_i}\right)(1-(1-(\tilde{\mu}_{k+1})^{2})^{\varpi_{k+1}})}\end{array}\right),$$

$$\left(\left[\prod_{i=1}^{k}(\tilde{n}_{i}^{-})^{\varpi_i}.(\tilde{n}_{k+1}^{-})^{\varpi_{k+1}},\prod_{i=1}^{k}(\tilde{n}_{i}^{+})^{\varpi_i}.(\tilde{n}_{k+1}^{+})^{\varpi_{k+1}}\right],\prod_{i=1}^{k}\left(\tilde{\delta}_{i}\right)^{\varpi_i}.\left(\tilde{\delta}_{k+1}\right)^{\varpi_{k+1}}\right),$$
$$\left(\left[\prod_{i=1}^{k}\left(\tilde{b}_{i}^{-}\right)^{\varpi_i}.\left(\tilde{b}_{k+1}^{-}\right)^{\varpi_{k+1}},\prod_{i=1}^{k}\left(\tilde{b}_{i}^{+}\right)^{\varpi_i}.\left(\tilde{b}_{k+1}^{+}\right)^{\varpi_{k+1}}\right],\prod_{i=1}^{k}\left(\tilde{\vartheta}_{i}\right)^{\varpi_i}.\left(\tilde{\vartheta}_{k+1}\right)^{\varpi_{k+1}}\right)$$

$$=\left\{\begin{array}{c}\left(\begin{array}{c}\left[\sqrt{1-\prod_{i=1}^{k}(1-(\tilde{a}_{i}^{-})^{2})^{\varpi_i}\left(1-\left(\tilde{a}_{k+1}^{-}\right)^{\mathbf{2}}\right)^{\varpi_{k+1}}},\sqrt{1-\prod_{i=1}^{k}(1-(\tilde{a}_{i}^{+})^{2})^{\varpi_i}\left(1-\left(\tilde{a}_{k+1}^{+}\right)^{2}\right)^{\varpi_{k+1}}}\right],\\ \sqrt{1-\prod_{i=1}^{k}(1-(\tilde{\mu}_{i})^{2})^{\varpi_i}(1-(\tilde{\mu}_{k+1})^{2})^{\varpi_{k+1}}}\end{array}\right),\\ \left(\left[\prod_{i=1}^{k}(\tilde{n}_{i}^{-})^{\varpi_i}.(\tilde{n}_{k+1}^{-})^{\varpi_{k+1}},\prod_{i=1}^{k}(\tilde{n}_{i}^{+})^{\varpi_i}.(\tilde{n}_{k+1}^{+})^{\varpi_{k+1}}\right],\prod_{i=1}^{k}\left(\tilde{\delta}_{i}\right)^{\varpi_i}.\left(\tilde{\delta}_{k+1}\right)^{\varpi_{k+1}}\right),\\ \left(\left[\prod_{i=1}^{k}\left(\tilde{b}_{i}^{-}\right)^{\varpi_i}.\left(\tilde{b}_{k+1}^{-}\right)^{\varpi_{k+1}},\prod_{i=1}^{k}\left(\tilde{b}_{i}^{+}\right)^{\varpi_i}.\left(\tilde{b}_{k+1}^{+}\right)^{\varpi_{k+1}}\right],\prod_{i=1}^{k}\left(\tilde{\vartheta}_{i}\right)^{\varpi_i}.\left(\tilde{\vartheta}_{k+1}\right)^{\varpi_{k+1}}\right)\end{array}\right\}$$

$$=\left\{\begin{array}{l}\left(\left(\left[\sqrt{1-\prod_{i=1}^{k+1}(1-(\tilde{a}_i^-)^2)^{\bar{\omega}_i}},\sqrt{1-\prod_{i=1}^{k+1}(1-(\tilde{a}_i^+)^2)^{\bar{\omega}_i}}\right],\sqrt{1-\prod_{i=1}^{k+1}(1-(\tilde{\mu}_i)^2)^{\bar{\omega}_i}}\right),\right.\\ \left.\left(\left[\prod_{i=1}^{k+1}(\tilde{n}_i^-)^{\bar{\omega}_i},\prod_{i=1}^{k+1}(\tilde{n}_i^+)^{\bar{\omega}_i}\right],\prod_{i=1}^{k+1}(\tilde{\delta}_i)^{\bar{\omega}_i}\right),\left(\left[\prod_{i=1}^{k+1}(\tilde{b}_i^-)^{\bar{\omega}_i},\prod_{i=1}^{k+1}(\tilde{b}_i^+)^{\bar{\omega}_i}\right],\prod_{i=1}^{k+1}(\tilde{\vartheta}_i)^{\bar{\omega}_i}\right)\right)\end{array}\right\}$$

The result is true for $n = k + 1$. The result is therefore satisfied for the entire n. Therefore,

$$\mathrm{SCFWA}_{\bar{\omega}}(\tilde{S}_{c_1},\tilde{S}_{c_2},\dots,\tilde{S}_{c_n})$$
$$=\left\{\begin{array}{l}\left(\left(\left[\sqrt{1-\prod_{i=1}^{n}(1-(\tilde{a}_i^-)^2)^{\bar{\omega}_i}},\sqrt{1-\prod_{i=1}^{n}(1-(\tilde{a}_i^+)^2)^{\bar{\omega}_i}}\right],\sqrt{1-\prod_{i=1}^{n}(1-(\tilde{\mu}_i)^2)^{\bar{\omega}_i}}\right),\right.\\ \left.\left(\left[\prod_{i=1}^{n}(\tilde{n}_i^-)^{\bar{\omega}_i},\prod_{i=1}^{n}(\tilde{n}_i^+)^{\bar{\omega}_i}\right],\prod_{i=1}^{n}(\tilde{\delta}_i)^{\bar{\omega}_i}\right),\left(\left[\prod_{i=1}^{n}(\tilde{b}_i^-)^{\bar{\omega}_i},\prod_{i=1}^{n}(\tilde{b}_i^+)^{\bar{\omega}_i}\right],\prod_{i=1}^{n}(\tilde{\vartheta}_i)^{\bar{\omega}_i}\right)\right)\end{array}\right\}$$

Properties: Some properties are obviously fulfilled by the SCFWA operator.

Idempotency: Let $\tilde{\mathbf{S}}_{\mathbf{c_i}} = \langle \mathbf{c}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \dot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \ddot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}} \rangle$ $(\mathbf{i} = \mathbf{1}, \mathbf{2}, \dots, \mathbf{n})$ be any collection of SCFNs. Then $\tilde{\mathbf{S}}_{\mathbf{c_i}} = \langle \mathbf{c}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \dot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \ddot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}} \rangle$ are identical, i.e.,

$$\mathrm{SCFWA}_{\bar{\omega}}(\tilde{S}_{c_1},\tilde{S}_{c_2},\dots,\tilde{S}_{c_n}) = \tilde{S}_c.$$

Boundary: Let $\tilde{\mathbf{S}}_{\mathbf{c_i}} = \langle \mathbf{c}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \dot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \ddot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}} \rangle$ $(\mathbf{i} = \mathbf{1}, \mathbf{2}, \dots, \mathbf{n})$ be any collection of SCFNs. Then

$$\tilde{S}^- \le \mathrm{SCFWA}_{\bar{\omega}}(\tilde{S}_{c_1},\tilde{S}_{c_2},\dots,\tilde{S}_{c_n}) \le \tilde{S}^+.$$

where

$$\tilde{S}^- = \{\langle[\min\tilde{a}_i^-,\min\tilde{a}_i^-],\min\tilde{\mu}_i\rangle,\langle[\min\tilde{n}_i^-,\min\tilde{n}_i^-],\min\tilde{\delta}_i\rangle,\langle[\max\tilde{b}_i^-,\max\tilde{b}_i^-],\max\tilde{\vartheta}_i\rangle\},$$
$$\tilde{S}^+ = \{\langle[\max\tilde{a}_i^-,\max\tilde{a}_i^-],\max\tilde{\mu}_i\rangle,\langle[\min\tilde{n}_i^-,\min\tilde{n}_i^-],\min\tilde{\delta}_i\rangle,\langle[\min\tilde{b}_i^-,\min\tilde{b}_i^-],\min\tilde{\vartheta}_i\rangle\}.$$

Monotonicity: Let $\tilde{\mathbf{S}}_{\mathbf{c_i}} = \langle \mathbf{c}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \dot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \ddot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}} \rangle$ $(\mathbf{i} = \mathbf{1}, \mathbf{2}, \dots, \mathbf{n})$ be any collection of SCFNs. Then

$$\mathbf{SCFWA}_{\bar{\boldsymbol{\omega}}}(\tilde{\mathbf{S}}_{\mathbf{c_1}},\tilde{\mathbf{S}}_{\mathbf{c_2}},\dots,\tilde{\mathbf{S}}_{\mathbf{c_n}}) = \mathbf{SCFWA}_{\bar{\boldsymbol{\omega}}}(\tilde{\mathbf{S}}^*_{\mathbf{c_1}},\tilde{\mathbf{S}}^*_{\mathbf{c_2}},\dots,\tilde{\mathbf{S}}^*_{\mathbf{c_n}}).$$

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3.2 Spherical cubic fuzzy ordered weighted averaging aggregation operators

This section examines ordered weighted averaging aggregation operators based on specified operational characteristics of SCFNs.

Definition 3.2.1. Let $\tilde{\mathbf{S}}_{\mathbf{c_i}} = \langle \mathbf{c}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \dot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \ddot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}} \rangle$ $(\mathbf{i} = \mathbf{1}, \mathbf{2}, \dots, \mathbf{n})$ be any collection of SCFNs and $\mathbf{SCFOWA}: \mathbf{SCFN^n} \to \mathbf{SCFN}$, then spherical cubic fuzzy ordered weighted average (SCFOWA) operator is defined as:

$$\mathbf{SCFOWA}_{\bar{\boldsymbol{\omega}}}(\tilde{\mathbf{S}}_{\mathbf{c_1}}, \tilde{\mathbf{S}}_{\mathbf{c_2}}, \dots, \tilde{\mathbf{S}}_{\mathbf{c_n}}) = \sum_{\mathbf{i=1}}^{\mathbf{n}} \bar{\boldsymbol{\omega}}_{\mathbf{i}}\, \tilde{\mathbf{S}}_{\mathbf{c}_{\tilde{\eta}(\mathbf{i})}}, \tag{10}$$

where i^{th} largest weighted value is $\tilde{S}_{c_{\tilde{\eta}(i)}}$ by total ordering$\tilde{S}_{c_{\tilde{\eta}(1)}} \geq \tilde{S}_{c_{\tilde{\eta}(2)}} \geq, \dots, \tilde{S}_{c_{\tilde{\eta}(n)}}$.Where $\bar{\omega}_i = (\bar{\omega}_1, \bar{\omega}_2, \dots, \bar{\omega}_n)$ represent the weight vector of $\tilde{S}_{c_i}$ and $\bar{\omega}_i \geq 0$, $\sum_{i=1}^{n} \bar{\omega}_i = 1$.

Theorem 3.2.2. Let $\tilde{S}_{c_i} = \langle c_{\tilde{S}_{c_i}}, \dot{c}_{\tilde{S}_{c_i}}, \ddot{c}_{\tilde{S}_{c_i}} \rangle$ $(i = 1,2, \dots, n)$ be any collection of SCFNs. Then by using the Definition [10] and the operational properties of SCFNs the following results can be obtained.

$$\mathrm{SCFOWA}_{\bar{\omega}}(\tilde{S}_{c_1}, \tilde{S}_{c_2}, \dots, \tilde{S}_{c_n})$$
$$= \left\{ \begin{array}{l} \left(\left(\left[\sqrt{1 - \prod_{i=1}^{n} \left(1 - (\tilde{a}^{-}_{\tilde{\eta}(i)})^2\right)^{\bar{\omega}_i}}, \sqrt{1 - \prod_{i=1}^{n} \left(1 - (\tilde{a}^{+}_{\tilde{\eta}(i)})^2\right)^{\bar{\omega}_i}} \right], \sqrt{1 - \prod_{i=1}^{n} \left(1 - (\tilde{\mu}_{\tilde{\eta}(i)}) \right.} \right.\right. \\ \left(\left(\left[\prod_{i=1}^{n} (\tilde{n}^{-}_{\tilde{\eta}(i)})^{\bar{\omega}_i}, \prod_{i=1}^{n} (\tilde{n}^{+}_{\tilde{\eta}(i)})^{\bar{\omega}_i} \right], \prod_{i=1}^{n} (\tilde{\delta}_{\tilde{\eta}(i)})^{\bar{\omega}_i} \right), \left(\left[\prod_{i=1}^{n} (\tilde{b}^{-}_{\tilde{\eta}(i)})^{\bar{\omega}_i}, \prod_{i=1}^{n} (\tilde{b}^{+}_{\tilde{\eta}(i)})^{\bar{\omega}_i} \right], \prod_{i=1}^{n} (\tilde{\vartheta} \right.\right. \end{array} \right.$$

where $\tilde{S}_{c_{\tilde{\eta}(i)}}$, i^{th} largest weighted value by total ordering $\tilde{S}_{c_{\tilde{\eta}(1)}} \geq \tilde{S}_{c_{\tilde{\eta}(2)}} \geq, \dots, \tilde{S}_{c_{\tilde{\eta}(n)}}$.

Proof. In the similar way, as in Theorem [3.1.2] uses the mathematical induction procedure at **n** and here the procedure is omitted.

Properties: Some properties are obviously fulfilled by the SCFOWA operator.

Idempotency: Let $\tilde{\mathbf{S}}_{\mathbf{c_i}} = \langle \mathbf{c}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \dot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \ddot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}} \rangle$ $(\mathbf{i} = \mathbf{1}, \mathbf{2}, \dots, \mathbf{n})$ be any collection of SCFNs. Then $\tilde{\mathbf{S}}_{\mathbf{c_i}} = \langle \mathbf{c}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \dot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \ddot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}} \rangle$ are identical, i.e.,

$$\mathrm{SCFOWA}_{\bar{\omega}}(\tilde{S}_{c_1}, \tilde{S}_{c_2}, \dots, \tilde{S}_{c_n}) = \tilde{S}_c.$$

Boundary: Let $\tilde{\mathbf{S}}_{\mathbf{c_i}} = \langle \mathbf{c}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \dot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \ddot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}} \rangle$ $(\mathbf{i} = \mathbf{1}, \mathbf{2}, \dots, \mathbf{n})$ be any collection of SCFNs. Then

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$\tilde{S}^{-} \leq \mathrm{SCFOWA}_{\bar{\omega}}\left(\tilde{S}_{c_1}, \tilde{S}_{c_2}, \ldots, \tilde{S}_{c_n}\right) \leq \tilde{S}^{+}.$

where

$$\tilde{S}^{-} = \left\{\langle[\min\tilde{a}_i^{-}, \min\tilde{a}_i^{-}], \min\tilde{\mu}_i\rangle, \langle[\min\tilde{n}_i^{-}, \min\tilde{n}_i^{-}], \min\tilde{\delta}_i\rangle, \langle\left[\max\tilde{b}_i^{-}, \max\tilde{b}_i^{-}\right], \max\tilde{\vartheta}_i\rangle\right\},$$

$$\tilde{S}^{+} = \left\{\langle[\max\tilde{a}_i^{-}, \max\tilde{a}_i^{-}], \max\tilde{\mu}_i\rangle, \langle[\min\tilde{n}_i^{-}, \min\tilde{n}_i^{-}], \min\tilde{\delta}_i\rangle, \langle\left[\min\tilde{b}_i^{-}, \min\tilde{b}_i^{-}\right], \min\tilde{\vartheta}_i\rangle\right\}.$$

Monotonicity: Let $\tilde{\mathbf{S}}_{\mathbf{c_i}} = \langle \mathbf{c}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \dot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \ddot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}} \rangle$ $(\mathbf{i} = \mathbf{1}, \mathbf{2}, \ldots, \mathbf{n})$ be any collection of SCFNs. Then

$\mathbf{SCFOWA}_{\bar{\boldsymbol{\omega}}}\left(\tilde{\mathbf{S}}_{\mathbf{c_1}}, \tilde{\mathbf{S}}_{\mathbf{c_2}}, \ldots, \tilde{\mathbf{S}}_{\mathbf{c_n}}\right) = \mathbf{SCFOWA}_{\bar{\boldsymbol{\omega}}}\left(\tilde{\mathbf{S}}^{*}_{\mathbf{c_1}}, \tilde{\mathbf{S}}^{*}_{\mathbf{c_2}}, \ldots, \tilde{\mathbf{S}}^{*}_{\mathbf{c_n}}\right).$

3.3 Spherical cubic fuzzy hybrid weighted averaging aggregation operators

This section examines hybrid weighted averaging aggregation operators based on specified operational characteristics of SCFNs. The spherical cubic fuzzy weighted average operator considers itself only to be essential for the aggregated spherical cubic fuzzy sets. The spherical cubic fuzzy ordered weighted average operator only concerns the position value of the aggregate spherical cubic fuzzy sets ranking order. The following spherical cubic fuzzy weighted hybrid averaging operator can be described in order to overcome the drawbacks of the above two spherical fuzzy aggregation operators.

Definition 3.3.1. Let $\tilde{\mathbf{S}}_{\mathbf{c_i}} = \langle \mathbf{c}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \dot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \ddot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}} \rangle$ $(\mathbf{i} = \mathbf{1}, \mathbf{2}, \ldots, \mathbf{n})$ be any collection of SCFNs and $\mathbf{SCFHWA}: \mathbf{SCFN}^{\mathbf{n}} \rightarrow \mathbf{SCFN}$, then spherical cubic fuzzy hybrid weighted average (SCFHWA) operator is defined as:

$$\mathbf{SCFHWA}_{\bar{\boldsymbol{\omega}}}\left(\tilde{\mathbf{S}}_{\mathbf{c_1}}, \tilde{\mathbf{S}}_{\mathbf{c_2}}, \ldots, \tilde{\mathbf{S}}_{\mathbf{c_n}}\right) = \sum_{\mathbf{i}=\mathbf{1}}^{\mathbf{n}} \bar{\boldsymbol{\omega}}_{\mathbf{i}}\, \tilde{\mathbf{S}}'_{\mathbf{c}_{\tilde{\eta}(i)}}, \tag{11}$$

where i^{th} largest weighted value is $\tilde{S}'_{c_{\tilde{\eta}(i)}}$ and $\left(\tilde{S}'_{c_{\tilde{\eta}(i)}} = n\,\bar{\omega}_i \tilde{S}_{c_i}\right)$ and $\bar{\omega}_i = (\bar{\omega}_1, \bar{\omega}_2, \ldots, \bar{\omega}_n)$ represent the weight vector of $\tilde{S}_{c_i}$ and $\bar{\omega}_i \geq 0$, $\sum_{i=1}^{n} \bar{\omega}_i = 1$. Also, $\bar{\omega}_i^{*} = (\bar{\omega}_1^{*}, \bar{\omega}_2^{*}, \ldots, \bar{\omega}_n^{*})$ is the associated weight vector and $\bar{\omega}_i^{*} \geq 0$, $\sum_{i=1}^{n} \bar{\omega}_i^{*} = 1$.

Theorem 3.3.2. Let $\tilde{\mathbf{S}}_{\mathbf{c_i}} = \langle \mathbf{c}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \dot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \ddot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}} \rangle$ $(\mathbf{i} = \mathbf{1}, \mathbf{2}, \ldots, \mathbf{n})$ be any collection of SCFNs. Then by using the Definition [2.7] and the operational properties of SCFNs the following results can be obtained.

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$$\mathrm{SCFHWA}_{\bar{\omega}}\left(\tilde{S}_{c_1}, \tilde{S}_{c_2}, \ldots, \tilde{S}_{c_n}\right)$$
$$= \left\{ \begin{array}{l} \left(\left[\sqrt{1-\prod_{i=1}^{n}\left(1-\left(\widetilde{a'}^{-}_{\tilde{\eta}(i)}\right)^{2}\right)^{\bar{\omega}_i}}, \sqrt{1-\prod_{i=1}^{n}\left(1-\left(\widetilde{a'}^{+}_{\tilde{\eta}(i)}\right)^{2}\right)^{\bar{\omega}_i}}\right], \sqrt{1-\prod_{i=1}^{n}\left(1-\left(\widetilde{\mu'}_{\tilde{\eta}}\right.\right.} \right. \\ \left(\left[\prod_{i=1}^{n}\left(\widetilde{n'}^{-}_{\tilde{\eta}(i)}\right)^{\bar{\omega}_i}, \prod_{i=1}^{n}\left(\widetilde{n'}^{+}_{\tilde{\eta}(i)}\right)^{\bar{\omega}_i}\right], \prod_{i=1}^{n}\left(\widetilde{\delta'}_{\tilde{\eta}(i)}\right)^{\bar{\omega}_i}\right), \left(\left[\prod_{i=1}^{n}\left(\widetilde{b'}^{-}_{\tilde{\eta}(i)}\right)^{\bar{\omega}_i}, \prod_{i=1}^{n}\left(\widetilde{b'}^{+}_{\tilde{\eta}(i)}\right)^{\bar{\omega}_i}\right], \prod_{i=1} \right. \end{array} \right.$$

where $\tilde{S}_{c_{\tilde{\eta}(i)}}$, i^{th} largest weighted value by total ordering $\tilde{S}_{c_{\tilde{\eta}(1)}} \geq \tilde{S}_{c_{\tilde{\eta}(2)}} \geq, \ldots, \tilde{S}_{c_{\tilde{\eta}(n)}}$, and i^{th} largest weighted value is $\widetilde{S'}_{c_{\tilde{\eta}(i)}}$, $\left(\widetilde{S'}_{c_{\tilde{\eta}(i)}} = n\bar{\omega}_i \tilde{S}_{c_i}\right)$.

Proof. In Theorem [3.1.2] uses the mathematical induction procedure at **n** and here the procedure is omitted.

Properties: Some properties are obviously fulfilled by the SCFHWA operator.

Idempotency: Let $\tilde{\mathbf{S}}_{\mathbf{c_i}} = \langle \mathbf{c}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \dot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \ddot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}} \rangle$ $(\mathbf{i} = \mathbf{1}, \mathbf{2}, \ldots, \mathbf{n})$ be any collection of SCFNs. Then $\tilde{\mathbf{S}}_{\mathbf{c_i}} = \langle \mathbf{c}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \dot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \ddot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}} \rangle$ are identical, i.e.,

$$\mathrm{SCFHWA}_{\bar{\omega}}\left(\tilde{S}_{c_1}, \tilde{S}_{c_2}, \ldots, \tilde{S}_{c_n}\right) = \tilde{S}_c.$$

Boundary: Let $\tilde{\mathbf{S}}_{\mathbf{c_i}} = \langle \mathbf{c}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \dot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \ddot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}} \rangle$ $(\mathbf{i} = \mathbf{1}, \mathbf{2}, \ldots, \mathbf{n})$ be any collection of SCFNs. Then

$$\tilde{S}^{-} \leq \mathrm{SCFHWA}_{\bar{\omega}}\left(\tilde{S}_{c_1}, \tilde{S}_{c_2}, \ldots, \tilde{S}_{c_n}\right) \leq \tilde{S}^{+}.$$

where

$$\tilde{S}^{-} = \left\{\langle [\min\tilde{a}_i^{-}, \min\tilde{a}_i^{-}], \min\tilde{\mu}_i \rangle, \langle [\min\tilde{n}_i^{-}, \min\tilde{n}_i^{-}], \min\tilde{\delta}_i \rangle, \langle \left[\max\tilde{b}_i^{-}, \max\tilde{b}_i^{-}\right], \max\tilde{\vartheta}_i \rangle\right\},$$

$$\tilde{S}^{+} = \left\{\langle [\max\tilde{a}_i^{-}, \max\tilde{a}_i^{-}], \max\tilde{\mu}_i \rangle, \langle [\min\tilde{n}_i^{-}, \min\tilde{n}_i^{-}], \min\tilde{\delta}_i \rangle, \langle \left[\min\tilde{b}_i^{-}, \min\tilde{b}_i^{-}\right], \min\tilde{\vartheta}_i \rangle\right\}.$$

Monotonicity: Let $\tilde{\mathbf{S}}_{\mathbf{c_i}} = \langle \mathbf{c}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \dot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}}, \ddot{\mathbf{c}}_{\tilde{\mathbf{S}}_{\mathbf{c_i}}} \rangle$ $(\mathbf{i} = \mathbf{1}, \mathbf{2}, \ldots, \mathbf{n})$ be any collection of SCFNs. Then

$$\mathrm{SCFHWA}_{\bar{\omega}}\left(\tilde{S}_{c_1}, \tilde{S}_{c_2}, \ldots, \tilde{S}_{c_n}\right) = \mathrm{SCFHWA}_{\bar{\omega}}\left(\tilde{S}^{*}_{c_1}, \tilde{S}^{*}_{c_2}, \ldots, \tilde{S}^{*}_{c_n}\right).$$

4. Multi-attribute process by spherical cubic fuzzy weighted averaging aggregation operators

This section proposes to use the spherical cubic weighted averaging aggregation operators to solve MADM problems. For a problem with MADM, suppose $A = \{a_1, a_2, \ldots, a_m\}$ be any m alternative finite set, $H = \{h_1, h_2, \ldots, h_n\}$ be any n attributes

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finite set and collection of q DMs are $\{t_1, t_2, \dots, t_q\}$. If the y^{th} $(y = 1,2, \dots, q)$ DM provides the alternative a_i $(i = 1,2, \dots, m)$, on the h_i $(i = 1,2, \dots, n)$ attribute set under any discrete term.

$$D = \langle \left[d_{\tilde{s}_{c_{jk}}}, \dot{d}_{\tilde{s}_{c_{jk}}} \right], \ddot{d}_{\tilde{s}_{c_{jk}}} \rangle = \left[\left(\langle [\tilde{a}_{jk}^-, \tilde{a}_{jk}^+], \tilde{\mu}_{jk} \rangle, \langle [\tilde{n}_{jk}^-, \tilde{n}_{jk}^+], \tilde{\delta}_{jk} \rangle, \langle [\tilde{b}_{jk}^-, \tilde{b}_{jk}^+], \tilde{\vartheta}_{jk} \rangle \right) \right]_{m \times n}$$

	h_1	h_2	$\cdots$	h_n
a_1	$\begin{pmatrix} \langle [\tilde{a}_{11}^-, \tilde{a}_{11}^+], \tilde{\mu}_{11} \rangle, \\ \langle [\tilde{n}_{11}^-, \tilde{n}_{11}^+], \tilde{\delta}_{11} \rangle, \\ \langle [\tilde{b}_{11}^-, \tilde{b}_{11}^+], \tilde{\vartheta}_{11} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [\tilde{a}_{12}^-, \tilde{a}_{12}^+], \tilde{\mu}_{12} \rangle, \\ \langle [\tilde{n}_{12}^-, \tilde{n}_{jk}^+], \tilde{\delta}_{12} \rangle, \\ \langle [\tilde{b}_{12}^-, \tilde{b}_{12}^+], \tilde{\vartheta}_{12} \rangle \end{pmatrix}$	$\cdots$	$\begin{pmatrix} \langle [\tilde{a}_{1n}^-, \tilde{a}_{1n}^+], \tilde{\mu}_{1n} \rangle, \\ \langle [\tilde{n}_{1n}^-, \tilde{n}_{1n}^+], \tilde{\delta}_{1n} \rangle, \\ \langle [\tilde{b}_{1n}^-, \tilde{b}_{1n}^+], \tilde{\vartheta}_{1n} \rangle \end{pmatrix}$
a_2	$\begin{pmatrix} \langle [\tilde{a}_{21}^-, \tilde{a}_{21}^+], \tilde{\mu}_{21} \rangle, \\ \langle [\tilde{n}_{21}^-, \tilde{n}_{21}^+], \tilde{\delta}_{21} \rangle, \\ \langle [\tilde{b}_{21}^-, \tilde{b}_{21}^+], \tilde{\vartheta}_{21} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [\tilde{a}_{22}^-, \tilde{a}_{22}^+], \tilde{\mu}_{22} \rangle, \\ \langle [\tilde{n}_{22}^-, \tilde{n}_{22}^+], \tilde{\delta}_{22} \rangle, \\ \langle [\tilde{b}_{22}^-, \tilde{b}_{22}^+], \tilde{\vartheta}_{22} \rangle \end{pmatrix}$	$\cdots$	$\begin{pmatrix} \langle [\tilde{a}_{2n}^-, \tilde{a}_{2n}^+], \tilde{\mu}_{2n} \rangle, \\ \langle [\tilde{n}_{2n}^-, \tilde{n}_{2n}^+], \tilde{\delta}_{2n} \rangle, \\ \langle [\tilde{b}_{2n}^-, \tilde{b}_{2n}^+], \tilde{\vartheta}_{2n} \rangle \end{pmatrix}$
	$\vdots$	$\vdots$	$\cdots$	$\vdots$
a_m	$\begin{pmatrix} \langle [\tilde{a}_{m1}^-, \tilde{a}_{m1}^+], \tilde{\mu}_{m1} \rangle, \\ \langle [\tilde{n}_{m1}^-, \tilde{n}_{m1}^+], \tilde{\delta}_{m1} \rangle, \\ \langle [\tilde{b}_{m1}^-, \tilde{b}_{m1}^+], \tilde{\vartheta}_{m1} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [\tilde{a}_{m2}^-, \tilde{a}_{m2}^+], \tilde{\mu}_{m2} \rangle, \\ \langle [\tilde{n}_{m2}^-, \tilde{n}_{m2}^+], \tilde{\delta}_{m2} \rangle, \\ \langle [\tilde{b}_{m2}^-, \tilde{b}_{m2}^+], \tilde{\vartheta}_{m2} \rangle \end{pmatrix}$	$\cdots$	$\begin{pmatrix} \langle [\tilde{a}_{mn}^-, \tilde{a}_{mn}^+], \tilde{\mu}_{mn} \rangle, \\ \langle [\tilde{n}_{mn}^-, \tilde{n}_{mn}^+], \tilde{\delta}_{mn} \rangle, \\ \langle [\tilde{b}_{mn}^-, \tilde{b}_{mn}^+], \tilde{\vartheta}_{mn} \rangle \end{pmatrix}$

The DM in which $\langle \left[d_{\tilde{s}_{c_{jk}}}, \dot{d}_{\tilde{s}_{c_{jk}}} \right], \ddot{d}_{\tilde{s}_{c_{jk}}} \rangle$ the set of SCFNs is presented and which represents the evaluation details of any alternative a_i $(i = 1,2, \dots, m)$, on the h_i $(i = 1,2, \dots, n)$ attribute set. If $\overline{\omega}_i = (\overline{\omega}_1, \overline{\omega}_2, \dots, \overline{\omega}_n)$ represent the weight vector of attribute with $\overline{\omega}_i \geq 0$, $\sum_{i=1}^{n} \overline{\omega}_i = 1$ and the weight vector of DMs is $\overline{\gamma} = (\overline{\gamma}_1, \overline{\gamma}_2, \dots, \overline{\gamma}_p)$ with $\overline{\gamma}_p \geq 0$, $\sum_{k=1}^{p} \overline{\gamma}_k = 1$.

The key technique for solving MADM problems is described below:

Step 1: We have two forms of criterion: one is said to be positive and the other to be negative. We need to convert the negative criteria into positive criteria for the negative criterion. In this stage we are constructing spherical cubic fuzzy matrices for decision-making, $D^t = \left[\overline{d}_{jk}^t \right]_{m \times n} = \langle \left[d_{\tilde{s}_{c_{jk}}}^t, \dot{d}_{\tilde{s}_{c_{jk}}}^t \right], \ddot{d}_{\tilde{s}_{c_{jk}}}^t \rangle$ $(t = 1,2, \dots, n)$. If there are two types of parameters, such as positive (benefit) and negative (cost), then the spherical cubic fuzzy decision matrices, $D^t = \langle \left[d_{\tilde{s}_{c_{jk}}}^t, \dot{d}_{\tilde{s}_{c_{jk}}}^t \right], \ddot{d}_{\tilde{s}_{c_{jk}}}^t \rangle$ the normalized spherical cubic fuzzy decision matrices can be transformed, $R^t = \left[\overline{r}_{jk}^t \right]_{m \times n} = \langle \left[r_{\tilde{s}_{c_{jk}}}^t, \dot{r}_{\tilde{s}_{c_{jk}}}^t \right], \ddot{r}_{\tilde{s}_{c_{jk}}}^t \rangle$ $(t = 1,2, \dots, n)$.

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$$\bar{r}_{jk}^{t} = \begin{cases} \bar{d}_{jk}^{t} \text{ for benefit criterion} \\ \bar{d}_{jk}^{*t} \text{ for cost criterion} \end{cases}$$

$\bar{d}_{jk}^{*t}$ shows the complement of $\bar{d}_{jk}^{t}$. If all the parameters have the same form, normalization is not required.

Step 2: We collect the SCFNs for any given decision maker, use the SCFWA operators discussed in Theorem [1] spherical cubic fuzzy aggregation operator. This allows us to select the best alternative in the selection of alternatives.

Step 3: Calculate value, accuracy, and certainty by using the Definition [8] respectively to rank the given function values alternatives.

Step 4: Arrange in the form of the ascending order the values determined using comparison methods in Definition [10] of all alternatives and select the highest value option. Alternative that has the highest benefit, which is our best result or suitable alternative in decision-making.

4.1 Numerical Results

The preference of suppliers in manufacturing is a critical component of the development process. Although the selection of the supplier is not convenient, the best decision would improve economic growth and product quality. The proposed model would be very helpful for choosing the supplier. The proposed strategy for selecting the supplier is based on the following arguments:

The marketing manager finds that the supplier purchases the components according to four criteria which are the organizational culture and action plan. The supplier approach is an important step in the industry's organization. The option of the best decision will maximize your company's production, but it is very difficult to choose a suitable supplier. The suggested model would then be used to determine and choose the most appropriate supplier for a business in eastern Pakistan. The proposed access to supplier selection was given as follows:

The plan to find the best supplier to purchase components. The decision maker takes the following four factors into account. The set of four parameters is denoted with $\{h_1, h_2, h_3, h_4\}$. The vector of weight of the four parameters is $\bar{\omega} = (0.35, 0.4, 0.25)^T$. A committee of three decision makers noticed that it was appropriate to further estimate four suppliers. The four providers category is $\{a_1, a_2, a_3, a_4\}$. The ranking criteria are required for the classification of the suppliers. Decision matrices are as defined in the following SCFNs.

Step 1:

The decisions of decision makers given their in Table $1 - 3$.

Table 1. Spherical cubic fuzzy information of 1st decision maker

	h_1	h_2	h_3	h_4
a_1	$\begin{pmatrix}\langle[0.1,0.3],0.4\rangle,\\ \langle[0.6,0.4],0.5\rangle,\\ \langle[0.2,0.1],0.2\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.2,0.4],0.4\rangle,\\ \langle[0.1,0.2],0.3\rangle,\\ \langle[0.2,0.3],0.3\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.4,0.6],0.6\rangle,\\ \langle[0.2,0.2],0.6\rangle,\\ \langle[0.4,0.6],0.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.2,0.5],0.5\rangle,\\ \langle[0.1,0.4],0.3\rangle,\\ \langle[0.4,0.6],0.5\rangle\end{pmatrix}$
a_2	$\begin{pmatrix}\langle[0.3,0.4],0.2\rangle,\\ \langle[0.2,0.3],0.3\rangle,\\ \langle[0.2,0.1],0.4\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.4,0.4],0.6\rangle,\\ \langle[0.1,0.2],0.3\rangle,\\ \langle[0.1,0.5],0.3\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.1,0.3],0.1\rangle,\\ \langle[0.2,0.4],0.3\rangle,\\ \langle[0.2,0.4],0.4\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.4,0.2],0.6\rangle,\\ \langle[0.1,0.5],0.4\rangle,\\ \langle[0.2,0.4],0.5\rangle\end{pmatrix}$
a_3	$\begin{pmatrix}\langle[0.1,0.3],0.2\rangle,\\ \langle[0.2,0.4],0.4\rangle,\\ \langle[0.4,0.3],0.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.3,0.6],0.2\rangle,\\ \langle[0.5,0.6],0.3\rangle,\\ \langle[0.3,0.4],0.2\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.3,0.4],0.2\rangle,\\ \langle[0.2,0.5],0.3\rangle,\\ \langle[0.3,0.4],0.8\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.1,0.2],0.2\rangle,\\ \langle[0.2,0.3],0.5\rangle,\\ \langle[0.2,0.5],0.5\rangle\end{pmatrix}$
a_4	$\begin{pmatrix}\langle[0.1,0.3],0.4\rangle,\\ \langle[0.1,0.4],0.4\rangle,\\ \langle[0.2,0.3],0.3\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.1,0.2],0.4\rangle,\\ \langle[0.3,0.5],0.5\rangle,\\ \langle[0.4,0.4],0.3\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.3,0.6],0.9\rangle,\\ \langle[0.2,0.2],0.3\rangle,\\ \langle[0.1,0.2],0.1\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.2,0.3],0.1\rangle,\\ \langle[0.1,0.2],0.2\rangle,\\ \langle[0.2,0.1],0.7\rangle\end{pmatrix}$

Table 2. Spherical cubic fuzzy information of 2nd decision maker

	h_1	h_2	h_3	h_4
a_1	$\begin{pmatrix}\langle[0.1,0.3],0.6\rangle,\\ \langle[0.2,0.4],0.3\rangle,\\ \langle[0.2,0.3],0.1\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.1,0.4],0.3\rangle,\\ \langle[0.2,0.3],0.2\rangle,\\ \langle[0.1,0.5],0.2\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.2,0.6],0.6\rangle,\\ \langle[0.2,0.1],0.1\rangle,\\ \langle[0.2,0.4],0.4\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.1,0.3],0.6\rangle,\\ \langle[0.4,0.5],0.6\rangle,\\ \langle[0.2,0.5],0.2\rangle\end{pmatrix}$
a_2	$\begin{pmatrix}\langle[0.1,0.2],0.4\rangle,\\ \langle[0.1,0.4],0.5\rangle,\\ \langle[0.1,0.5],0.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.2,0.4],0.3\rangle,\\ \langle[0.1,0.3],0.2\rangle,\\ \langle[0.3,0.5],0.2\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.1,0.3],0.3\rangle,\\ \langle[0.1,0.5],0.6\rangle,\\ \langle[0.1,0.4],0.3\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.3,0.6],0.2\rangle,\\ \langle[0.1,0.3],0.7\rangle,\\ \langle[0.2,0.3],0.2\rangle\end{pmatrix}$
a_3	$\begin{pmatrix}\langle[0.2,0.4],0.3\rangle,\\ \langle[0.2,0.5],0.6\rangle,\\ \langle[0.1,0.3],0.2\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.4,0.5],0.4\rangle,\\ \langle[0.7,0.3],0.3\rangle,\\ \langle[0.1,0.3],0.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.2,0.4],0.2\rangle,\\ \langle[0.2,0.5],0.6\rangle,\\ \langle[0.1,0.3],0.4\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.1,0.4],0.2\rangle,\\ \langle[0.2,0.3],0.6\rangle,\\ \langle[0.1,0.5],0.6\rangle\end{pmatrix}$
a_4	$\begin{pmatrix}\langle[0.2,0.3],0.4\rangle,\\ \langle[0.5,0.6],0.5\rangle,\\ \langle[0.4,0.2],0.3\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.7,0.4],0.6\rangle,\\ \langle[0.1,0.2],0.1\rangle,\\ \langle[0.3,0.2],0.4\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.1,0.2],0.3\rangle,\\ \langle[0.2,0.4],0.6\rangle,\\ \langle[0.3,0.2],0.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.3,0.1],0.2\rangle,\\ \langle[0.1,0.3],0.2\rangle,\\ \langle[0.5,0.4],0.3\rangle\end{pmatrix}$

Table 3. Spherical cubic fuzzy information of 3rd decision maker

	h_1	h_2	h_3	h_4
a_1	$\begin{pmatrix}\langle[0.1,0.4],0.1\rangle,\\ \langle[0.1,0.2],0.4\rangle,\\ \langle[0.2,0.3],0.4\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.1,0.3],0.2\rangle,\\ \langle[0.2,0.4],0.3\rangle,\\ \langle[0.3,0.4],0.2\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.1,0.6],0.7\rangle,\\ \langle[0.2,0.3],0.2\rangle,\\ \langle[0.2,0.4],0.2\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.1,0.3],0.6\rangle,\\ \langle[0.2,0.2],0.3\rangle,\\ \langle[0.4,0.7],0.2\rangle\end{pmatrix}$
a_2	$\begin{pmatrix}\langle[0.1,0.3],0.2\rangle,\\ \langle[0.1,0.2],0.4\rangle,\\ \langle[0.3,0.2],0.3\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.1,0.4],0.3\rangle,\\ \langle[0.1,0.5],0.5\rangle,\\ \langle[0.5,0.4],0.3\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.2,0.3],0.2\rangle,\\ \langle[0.1,0.2],0.1\rangle,\\ \langle[0.5,0.5],0.4\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.1,0.2],0.3\rangle,\\ \langle[0.1,0.5],0.6\rangle,\\ \langle[0.1,0.2],0.3\rangle\end{pmatrix}$
a_3	$\begin{pmatrix}\langle[0.3,0.7],0.1\rangle,\\ \langle[0.2,0.2],0.2\rangle,\\ \langle[0.1,0.3],0.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.2,0.4],0.1\rangle,\\ \langle[0.3,0.1],0.6\rangle,\\ \langle[0.2,0.4],0.2\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.2,0.4],0.2\rangle,\\ \langle[0.1,0.3],0.2\rangle,\\ \langle[0.2,0.7],0.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.1,0.3],0.2\rangle,\\ \langle[0.4,0.3],0.2\rangle,\\ \langle[0.1,0.6],0.4\rangle\end{pmatrix}$
a_4	$\begin{pmatrix}\langle[0.1,0.2],0.2\rangle,\\ \langle[0.5,0.6],0.2\rangle,\\ \langle[0.3,0.1],0.4\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.2,0.3],0.5\rangle,\\ \langle[0.6,0.8],0.1\rangle,\\ \langle[0.6,0.3],0.2\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.2,0.3],0.3\rangle,\\ \langle[0.2,0.4],0.1\rangle,\\ \langle[0.1,0.2],0.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.2,0.2],0.4\rangle,\\ \langle[0.1,0.2],0.4\rangle,\\ \langle[0.1,0.2],0.4\rangle\end{pmatrix}$

By using the SCFWA operator and $\overline{\omega} = (0.35,0.4,0.25)^{T}$, is the weight vector of decision maker.

The aggregated results discussed in the following Table 4.

Table 4. Spherical cubic fuzzy weighted average aggregation information of decision makers

	h_1	h_2	h_3	h_4
a_1	$\begin{pmatrix}\langle[0.1,0.33],0.46\rangle,\\ \langle[0.25,0.34],0.39\rangle,\\ \langle[0.2,0.23],0.28\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.14,0.38],0.32\rangle,\\ \langle[0.16,0.28],0.26\rangle,\\ \langle[0.17,0.4],0.2\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.28,0.6],0.67\rangle,\\ \langle[0.2,0.17],0.29\rangle,\\ \langle[0.2,0.4],0.16\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.14,0.39],0.57\rangle,\\ \langle[0.21,0.37],0.4\rangle,\\ \langle[0.3,0.58],0.28\rangle\end{pmatrix}$
a_2	$\begin{pmatrix}\langle[0.13,0.3],0.4\rangle,\\ \langle[0.13,0.2],0.4\rangle,\\ \langle[0.17,0.23],0.44\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.28,0.4],0.44\rangle,\\ \langle[0.1,0.3],0.29\rangle,\\ \langle[0.23,0.47],0.26\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.13,0.3],0.22\rangle,\\ \langle[0.13,0.37],0.3\rangle,\\ \langle[0.19,0.42],0.36\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.31,0.43],0.42\rangle,\\ \langle[0.1,0.41],0.55\rangle,\\ \langle[0.17,0.3],0.31\rangle\end{pmatrix}$
a_3	$\begin{pmatrix}\langle[0.21,0.49],0.23\rangle,\\ \langle[0.2,0],0.4\rangle,\\ \langle[0.16,0.3],0.39\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.33,0.52],0.29\rangle,\\ \langle[0.5,0.29],0.3\rangle,\\ \langle[0.17,0.36],0.33\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.24,0.4],0.2\rangle,\\ \langle[0.17,0.44],0.36\rangle,\\ \langle[0.17,0.41],0.54\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.1,0.32],0.2\rangle,\\ \langle[0.24,0.3],0.43\rangle,\\ \langle[0.13,0.52],0.51\rangle\end{pmatrix}$
a_4	$\begin{pmatrix}\langle[0.15,0.28],0.36\rangle,\\ \langle[0.28,0.52],0.37\rangle,\\ \langle[0.29,0.19],0.32\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.5,0.32],0.52\rangle,\\ \langle[0.23,0.39],0.18\rangle,\\ \langle[0.33,0.28],0.3\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.21,0.42],0.69\rangle,\\ \langle[0.2,0.31],0.3\rangle,\\ \langle[0.16,0.2],0.28\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.25,0.21],0.25\rangle,\\ \langle[0.1,0.24],0.24\rangle,\\ \langle[0.24,0.21],0.43\rangle\end{pmatrix}$

We utilized SCFWA operator, comprising $\overline{\omega} = (0.1,0.2,0.25,0.45)^{T}$ as the criteria vector, we have the accumulated SCFNs for alternatives a_i $(i = 1,2,3,4)$.

Table 5. Row wise spherical cubic fuzzy aggregated information of decision makers

a_1	$([0.18,0.45],0.56)$	$([0.2,0.28],0.34)$	$([0.23,0.45],0.22)$
a_2	$([0.26,0.38],0.38)$	$([0.11,0.36],0.4)$	$([0.19,0.35],0.32)$
a_3	$([0.21,0.41],0.22)$	$([0.25,0.33],0.38)$	$([0.15,0.43],0.46)$
a_4	$([0.31,0.31],0.48)$	$([0.16,0.3],0.25)$	$([0.24,0.22],0.35)$

Step 3:

Row wise aggregation information given in Table 5. By Definition [8], we will find the scores $\overline{S}(a_i)$ of all a_i $(i = 1,2,3,4)$ as shown below :

$\overline{S}(a_1) = 0.32,\quad \overline{S}(a_2) = 0.28,\quad \overline{S}(a_3) = 0.3,\quad \overline{S}(a_4) = 0.26.$

Step 4:

First of all, we arrange the SCFNs in descending order to choose the best alternatives a s follows:

$a_1 > a_3 > a_2 > a_4.$

Hence, a_1 is best one.

For Spherical Cubic Fuzzy Ordered Weighted Averaging (SCFOWA) Operator

Step 1:

The collected data provided by three decision makers is based on the different significance of all the decision-makers in Table 4.

Step 2:

By using the SCFOWA operator and $\overline{\omega} = (0.35,0.4,0.25)^T$, is the weight vector of decision maker. The aggregated results discussed in the following table 6.

Table 6. Spherical cubic fuzzy ordered weighted average aggregation information of decision makers

	h_1	h_2	h_3	h_4
a_1	$\begin{pmatrix}\langle[0.26,0.51],0.62\rangle,\\ \langle[0.3,0.38],0.46\rangle,\\ \langle[0.2,0.44],0.16\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.15,0.46],0.42\rangle,\\ \langle[0.41,0.32],0.31\rangle,\\ \langle[0.23,0.33],0.2\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.14,0.44],0.54\rangle,\\ \langle[0.17,0.32],0.27\rangle,\\ \langle[0.2,0.37],0.24\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.14,0.42],0.43\rangle,\\ \langle[0.17,0.37],0.26\rangle,\\ \langle[0.32,0.6],0.29\rangle\end{pmatrix}$
a_2	$\begin{pmatrix}\langle[0.31,0.49],0.33\rangle,\\ \langle[0.1,0.34],0.5\rangle,\\ \langle[0.15,0.37],0.26\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.26,0.3],0.38\rangle,\\ \langle[0.1,0.5],0.5\rangle,\\ \langle[0.29,0.4],0.36\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.22,0.38],0.24\rangle,\\ \langle[0.14,0.28],0.27\rangle,\\ \langle[0.27,0.35],0.28\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.13,0.27],0.21\rangle,\\ \langle[0.14,0.36],0.28\rangle,\\ \langle[0.29,0.47],0.47\rangle\end{pmatrix}$
a_3	$\begin{pmatrix}\langle[0.24,0.49],0.2\rangle,\\ \langle[0.36,0.49],0.47\rangle,\\ \langle[0.21,0.36],0.2\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.3,0.51],0.22\rangle,\\ \langle[0.5,0.28],0.28\rangle,\\ \langle[0.14,0.39],0.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.15,0.34],0.2\rangle,\\ \langle[0.27,0.43],0.4\rangle,\\ \langle[0.26,0.41],0.46\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.21,0.4],0.2\rangle,\\ \langle[0.18,0.39],0.36\rangle,\\ \langle[0.21,0.54],0.63\rangle\end{pmatrix}$
a_4	$\begin{pmatrix}\langle[0.24,0.44],0.56\rangle,\\ \langle[0.46,0.6],0.28\rangle,\\ \langle[0.3,0.23],0.18\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.49,0.3],0.4\rangle,\\ \langle[0.32,0.45],0.21\rangle,\\ \langle[0.34,0.28],0.36\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.13,0.24],0.36\rangle,\\ \langle[0.15,0.36],0.47\rangle,\\ \langle[0.23,0.24],0.4\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.25,0.24],0.17\rangle,\\ \langle[0.13,0.3],0.21\rangle,\\ \langle[0.35,0.33],0.46\rangle\end{pmatrix}$

We utilized SCFOWA operator, comprising $\overline{\omega} = (0.1,0.2,0.25,0.45)^T$ as the criteria vector, we have the accumulated SCFNs for alternatives a_i $(i = 1,2,3,4)$.

Table 7. Row wise spherical cubic fuzzy aggregated information of decision makers

a_1	$([0.16,0.44],0.47)$	$([0.25,0.35],0.28)$	$([0.27,0.49],0.24)$
a_2	$([0.21,0.33],0.25)$	$([0.13,0.38],0.33)$	$([0.28,0.42],0.37)$
a_3	$([0.23,0.42],0.2)$	$([0.32,0.39],0.36)$	$([0.21,0.47],0.52)$
a_4	$([0.3,0.28],0.28)$	$([0.24,0.39],0.27)$	$([0.32,0.29],0.38)$

Step 3:

Row wise aggregation information given in Table 7. By Definition [8], we will find the scores $\overline{S}(a_i)$ of all a_i $(i = 1,2,3,4)$ as shown below :

$$\overline{S}(a_1) = 0.1, \qquad \overline{S}(a_2) = 0.05, \qquad \overline{S}(a_3) = 0.06, \qquad \overline{S}(a_4) = 0.02.$$

Step 4:

First of all, we arrange the SCFNs in descending order to choose the best alternatives a s follows:

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$a_1 > a_3 > a_2 > a_4$.

Hence, a_1 is best one.

For Spherical Cubic Fuzzy Hybrid Averaging (SCFHA) Operator

Step 1:

The collected data provided by three decision makers is based on the different significance of all the decision-makers in Table 4.

Step 2:

Using the formula $\tilde{S}_p = q\bar{\omega}_p S_p = \left(\langle[\tilde{a}_p^-, \tilde{a}_p^+], \tilde{\mu}_p\rangle, \langle[\tilde{n}_p^-, \tilde{n}_p^+], \tilde{\delta}_p\rangle, \langle[\tilde{b}_p^-, \tilde{b}_i^+], \tilde{\vartheta}_p\rangle\right)$, $(p = 1,2, \dots, n)$ to the data given in table 4, having weight $\bar{\omega} = (0.1,0.2,0.25,0.45)^T$ of a_i, the outcome is provided in table 8, as listed below:

Table 8. Spherical cubic fuzzy hybrid weighted average aggregation information of decision makers

	h_1	h_2	h_3	h_4
a_1	$\begin{pmatrix}\langle[0.26,0.38], 0.46\rangle,\\ \langle[0.04,0.19], 0.06\rangle,\\ \langle[0.05,0.10], 0.22\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.24,0.42], 0.39\rangle,\\ \langle[0.05,0.22], 0.04\rangle,\\ \langle[0.03,0.15], 0.13\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.38,0.63], 0.68\rangle,\\ \langle[0.06,0.18], 0.03\rangle,\\ \langle[0.05,0.11], 0.11\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.68,0.25], 0.49\rangle,\\ \langle[0.13,0.28], 0.19\rangle,\\ \langle[0.10,0.32], 0.18\rangle\end{pmatrix}$
a_2	$\begin{pmatrix}\langle[0.31,0.52], 0.30\rangle,\\ \langle[0.03,0.21], 0.06\rangle,\\ \langle[0.05,0.14], 0.12\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.37,0.64], 0.53\rangle,\\ \langle[0.03,0.15], 0.09\rangle,\\ \langle[0.18,0.36], 0.05\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.19,0.40], 0.28\rangle,\\ \langle[0.01,0.13], 0.06\rangle,\\ \langle[0.04,0.13], 0.07\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.19,0.45], 0.32\rangle,\\ \langle[0.07,0.17], 0.36\rangle,\\ \langle[0.04,0.19], 0.09\rangle\end{pmatrix}$
a_3	$\begin{pmatrix}\langle[0.27,0.49], 0.29\rangle,\\ \langle[0.03,0.09], 0.12\rangle,\\ \langle[0.03,0.38], 0.13\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.37,0.63], 0.52\rangle,\\ \langle[0.02,0.11], 0.06\rangle,\\ \langle[0.12,0.36], 0.04\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.20,0.41], 0.29\rangle,\\ \langle[0.03,0.11], 0.06\rangle,\\ \langle[0.03,0.24], 0.08\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.38,0.57], 0.53\rangle,\\ \langle[0.07,0.17], 0.36\rangle,\\ \langle[0.04,0.19], 0.09\rangle\end{pmatrix}$
a_4	$\begin{pmatrix}\langle[0.25,0.37], 0.41\rangle,\\ \langle[0.22,0.41], 0.19\rangle,\\ \langle[0.16,0.17], 0.11\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.43,0.56], 0.52\rangle,\\ \langle[0.04,0.11], 0.02\rangle,\\ \langle[0.10,0.17], 0.06\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.43,0.63], 0.74\rangle,\\ \langle[0.06,0.15], 0.22\rangle,\\ \langle[0.05,0.12], 0.18\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[0.29,0.42], 0.31\rangle,\\ \langle[0.02,0.09], 0.21\rangle,\\ \langle[0.08,0.33], 0.19\rangle\end{pmatrix}$

Step 3:

By definition of score function [8], we will find the scores $\bar{S}(a_i)$ of all a_i $(i = 1,2,3,4)$ as shown below :

$\bar{S}(a_1) = 0.11,\qquad \bar{S}(a_2) = 0.05,\qquad \bar{S}(a_3) = 0.08,\qquad \bar{S}(a_4) = 0.04.$

Step 4:

First of all, we arrange the SCFNs in descending order to choose the best alternatives a s follows:

$a_1 > a_3 > a_2 > a_4$.

Hence, a_1 is best one.

Ranking of Alternatives

The following table 9 represents the ranking of different alternatives.

Table 9. Ranking of Alternatives

Operator	$\bar{S}(a_1)$	$\bar{S}(a_2)$	$\bar{S}(a_3)$	$\bar{S}(a_4)$	Ranking
SCFWA	0.32	0.28	0.3	0.26	$a_1 > a_3 > a_2 > a_4$
SCFOWA	0.1	0.05	0.06	0.02	$a_1 > a_3 > a_2 > a_4$
SCFHWA	0.11	0.05	0.08	0.04	$a_1 > a_3 > a_2 > a_4$

The following Figure 1 represents the comparative study of the supplier selection.

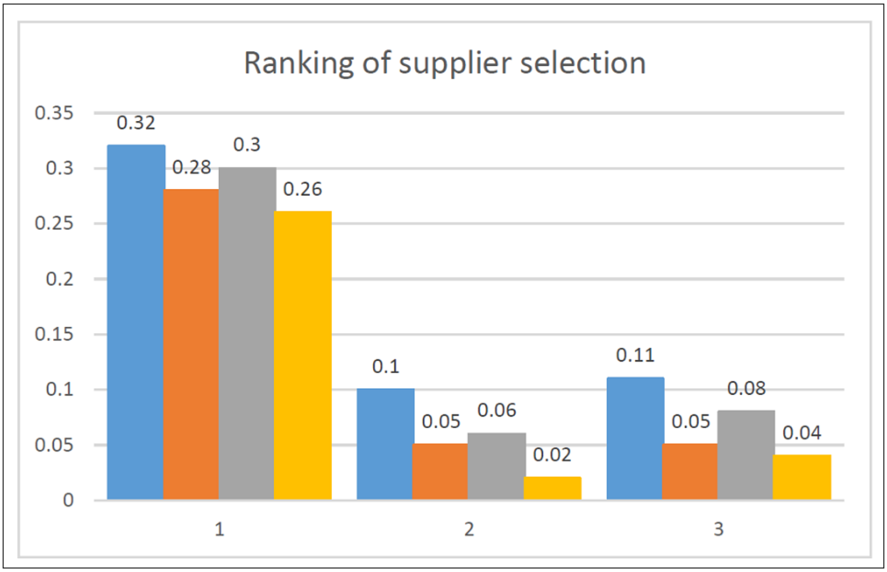

Figure 1. Ranking of supplier selection using spherical cubic aggregation operators

4.2 Verification by VIKOR Method

By VIKOR method, we prove the result of SCFWA operators. The estimated information of each of the decision-makers under the spherical cubic fuzzy weighted averaging operator indicated in table 4. By using the weight vector $\overline{\omega} = (0.1,0.2,0.25,0.45)^T$ as criteria weight, we apply the VIKOR method on the information given in table 4. Following are the steps to verify the example by the VIKOR method.

Step 1: First of all we will normalize the decision matrix given in Table 4.

Step 2 : Determine PIS R^+ and NIS R^- which are defined as $R^- = (\varrho_1^-, \varrho_2^-, \varrho_3^-, \varrho_4^-)$, $R^+ = (\varrho_1^+, \varrho_2^+, \varrho_3^+, \varrho_4^+)$ and further $\varrho_j^+ = \max\{\varrho_{ij} \mid 1 < j < 4\}$ and $\varrho_j^- = \min\{\varrho_{ij} \mid 1 <$

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$j < 4\}$these are computed by the score function $\bar{S}(\tilde{S}_c) = \frac{(\tilde{a}^- + \tilde{a}^+ + \tilde{\mu})^2 + (\tilde{n}^- + \tilde{n}^+ + \tilde{\delta})^2 - (\tilde{b}^- + \tilde{b}^+ + \tilde{\vartheta})^2}{9}$.

Step 3 : Compute $\widetilde{P}_i$, $\widetilde{Q}_i$, $\widetilde{R}_i$ with the help of following formulas.

$$\widetilde{P}_i = \sum_{j=1}^{m} \frac{\bar{\omega}_j d(\varrho_{ij}, \varrho_j^+)}{d(\varrho_j^+, \varrho_j^-)}$$

$$\widetilde{Q}_i = \max_{1 \le j \le m} \frac{\bar{\omega}_j d(\varrho_{ij}, \varrho_j^+)}{d(\varrho_j^+, \varrho_j^-)}$$

$$\widetilde{R}_i = \frac{x(\widetilde{P}_i - P^*)}{P^- - P^*} + \frac{(1-x)(\widetilde{Q}_i - Q^*)}{Q^- - Q^*}$$

suppose that $x = 0.4$, the computed values shown in table 10.

Step 4 : After ranking the values of $\widetilde{R}_i$, we get the following order

$\widetilde{R}_4 > \widetilde{R}_2 > \widetilde{R}_3 > \widetilde{R}_1$

Step 5 : From the ranking outcomes we get that $\widetilde{R}_1$ is minimum so a_1 is the best supplier compared to all.

Table 10. Verification by VIKOR Method

i	$\widetilde{P}_i$	$\widetilde{Q}_i$	$\widetilde{R}_i$
1	0.56	0.12	0.01
2	0.4	0.3	0.60
3	0.92	0.42	0.50
4	0.82	0.2	0.91

5. Comparison

In the section, we compared and conclude our function with our advanced fuzzy aggregation operators with our predefined fuzzy aggregation operators. Although SFS theory is important in many areas, there are some problems that have not been deal with by SFSs. Every element of SFSs is defined as an ordered triplet, differentiated as membership, neutral and non-membership. For SCFS, any element has a membership, a neutral and a non-membership function of which membership functions, neutral and non-member, all are cubic, fuzzy numbers. The numerical problem solved by SCFS in section 4 is a revolutionary concept. Due to the restricted method of previous aggregation operators, the problem described in this article cannot be resolved. But we can solve it easily by SCFNs. SCFA operators are also more authentic in addressing the unpredictable problems.

We did not use the definition of the score function in the numerical issue stated. We used the notion of cubic fuzzy by following membership, neutral and non-membership, as cubic fuzzy number i.e. here, SCFN $\left(\langle[\tilde{a}^-,\tilde{a}^+],\tilde{\mu}\rangle,\langle[\tilde{n}^-,\tilde{n}^+],\tilde{\delta}\rangle,\langle[\tilde{b}_1^-,\tilde{b}^+],\tilde{\vartheta}\rangle\right)$, as a collection of three cubic numbers $S_1=\langle[\tilde{a}^-,\tilde{a}^+],\tilde{\mu}\rangle$, $S_2=\langle[\tilde{n}^-,\tilde{n}^+],\tilde{\delta}\rangle$ and $S_3=\langle[\tilde{b}_1^-,\tilde{b}^+],\tilde{\vartheta}\rangle$, $\frac{1}{3}\left(\bar{S}(S_1)+\bar{S}(S_2)+\bar{S}(S_3)\right)=\frac{1}{3}\left(\frac{(\tilde{a}^-+\tilde{a}^++\tilde{\mu})^2+(\tilde{n}^-+\tilde{n}^++\tilde{\delta})^2-(\tilde{b}^-+\tilde{b}^++\tilde{\vartheta})^2}{9}\right)$. Table 9, shows the ranking outcomes of the alternatives and compare these results by using SCF score function. Finally, the findings show that a_1 is the better of all options as shown in Figure 2.

With intuitionistic cubic fuzzy set and Pythagorean cubic fuzzy, aggregation operators have some problems with MADM and we do not have some ambiguous conditions to handle the problems. While there are no such limitations on SCF aggregation operators, so we get more precise results. The ranking of the four possible alternatives obtained by the proposed aggregation operator can easily be seen from Table 9 to be relatively close to each other. The best alternative that these operators have obtained is the same, namely, a_1. These approaches are ideal for addressing the circumstances in which the input and opinions collaborate, which might consider the interaction between the experts and standards that are more appropriate to deal with these types of issues.

The following is Figure 2 which indicates the analysis of the comparison.

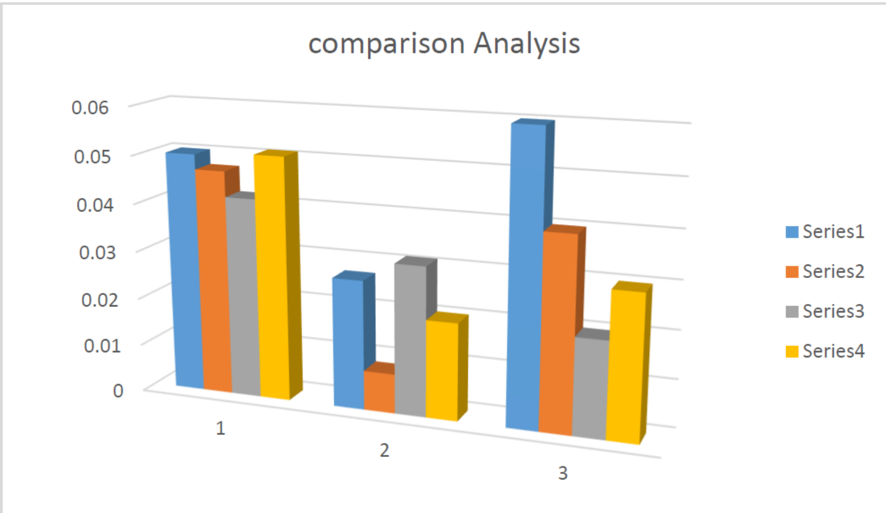

Figure 2. Comparison analysis of supplier selection using intuitionistic cubic aggregation operators

6. Conclusion

The definition of the spherical cubic fuzzy set, which is the generalization of the interval valued spherical fuzzy set, was introduced in this article. Several spherical cubic fuzzy

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operational laws were developed. For the comparison of spherical cubic fuzzy numbers, we have established a score and accuracy degree. The spherical cubic fuzzy distance between spherical cubic fuzzy numbers was also described. Aggregating the spherical cubic fuzzy information, we proposed (SCFWA) operator, (SCFOWA) and (SCFHWA) operator, we also addressed some of its properties such as Idempotency, Boundary, Monotonicity and showed a relationship between these developed operators. We, moreover, in order to demonstrate the strength and efficiency of the existing operators, they suggested a multi-attribute decision-making (MADM) approach. In addition, to clarify the decision-making issues, we have applied the existing aggregation operators. A numerical illustration has been suggested that indicates that the proposed operators have an alternate way to more effectively solve the decision-making process. Finally, in order to illustrate the validity, practicality, and effectiveness of the novel approach, we have given some comparison with the existing operators.

In future work, we will set up more aggregating operators for SCF data such as, spherical cubic Hamacher aggregation operator (SCFHA), spherical cubic fuzzy Dombi aggregation (SCFDA), spherical cubic TOPSIS, VIKOR and GRA Process. SCFDA aggregation operator. Many would be implemented to solve the MCGDM problems under unclear situations. Also, we can extend the work to (Ullah *et al.*,2020), (Garg *et al.*,2019), (Garg,2021) and (Garg,2020).

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