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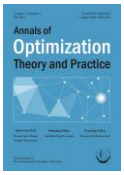

Countries credit ranking: a simple weighted non-linear programming model

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Abstract In this study a non-linear weighted sum model is proposed to rank countries based on economic factors. This ranking problem could be new and useful as most of previous researches rated countries not rank them. The countries are ranked from the best to the worst one by their score obtained by the model from credit point of view. As an advantage of the model, it is solvable by an analytical solution method manually instead of using optimization software. The analytical solution is useful for managers and decision makers to apply the model easily. The obtained ranking is compared with Moody's rating to discuss the efficiency of the model.

Keywords Country credit ranking; Country credit rating; Weighted additive model; Non-linear model

1. Introduction

Understanding the contribution of economic and political factors on the evaluation of countries credit rating has crucial policy implications. Up to now there are many different methodologies were suggested by researchers to calculate country credit rating (sovereign rating), however some of them were valuable. Most of these approaches and techniques are based on mathematical models Zopunidis and Doumpos (2000), (2002) and Hirth (2014) and some the others based on probabilistic and stochastic methods Hu *et al.* (2002) and Pantelous (2008) and Gonzalez and Hinojosa (2010). Famous rating agencies such as Standard & Poor's, Moody's, Fitch Rating, etc. announce a rating list monthly, quarterly, semiannually and annually which rate counties by considering several factors (economic and political). They employ different methodologies and techniques in addition to idea of expertise to rate the countries. In fact, it is impossible to create a unique model to rate countries, since there are many different quantitative and qualitative factors directly or indirectly affecting the outcome.

Credit rating announcement that is provided by the rating agencies affect the financial market in different ways. Recently, information provided by rating agencies is important for market contributors and regulators much more than past as they are under pressure after they fail to predict world financial crises 2007-2008 Abad and Robles (2014). The latest crisis demonstrated that the country credit rating is an important issue in the international market for both develop and developing countries Maltritz and Molchanov (2014). For that reason, nowadays credit rating agencies are not the only provider for rating information as econometrician, operation researcher, financial investigator and statisticians, etc. propose valuable approaches to rate countries Mirzaei and Vizvari (2011).

Credit rating methods mostly consider some predetermined levels for countries and each country is assigned to an appropriate level. The assignment of each country is done just by evaluation of the performance of that country in the economic factors considered for the rating. On the other hand credit ranking prepare a complete ranking of countries from the best to the worst one according to the economic factors. So, the performances of countries in the economic factors are compared to rank the countries.

In this study a mathematical model is proposed to rank (see Moreira *et al.* (2015)) the countries and it is subjected to selected economic factors. The suggested model is used to rank the countries (from best to worst) rather than rating them from credit point of view as the most of researches have focused on countries credit rating. The model calculate counties' score and the countries are ranked based on their obtained score. In other word, the result is subjective to the factors and number of countries which are selected. Therefore, if new counters or factors are selected and added to the dataset, then the ranking result may change accordingly.

As the rest of the paper, Section 2 reviews the existing literature of methodologies and techniques which were utilized in country credit rating. Section 3 proposes a non-linear data envelopment analysis model for countries credit ranking. Data collection and factor selection are discussed in Section 4. Empirical result and comparison of result with Moody's rating is covered by Section 5. Concluding remarks are discussed in the last section.

2. Literature review

Sovereign rating also known as country rating, is one of the import topics in the global financial market and is affected mostly by economic factors. Up to now, researchers proposed different methodologies which utilize mathematical, statistical, probabilistic and/or stochastic models to rate or classify countries. For instances, a model that is proposed in 2001 was based on the multi-criteria decision aid (MCDA) Kovacs *et al.* (2002) and Ballı and Korukoğlu (2014) and Chen and Lu (2015) and Izadikhah and Farzipoor Saen (2015) and multi-group hierarchical discrimination (M.H.DIS) Surma (2015), which use different factors to classify a number of countries in to specified classes Doumpos and Zopunidis (2001). The model was revised many times during next 2 years starting from 2000 to 2002 Zopunidis and Doumpos (2000), (2002). Later on, the

proposed model have modified and improved by Mirzaei and Vizvari (2011) and used to reconstruct the World Bank classification. Hu *et al.* (2002) constructed rating transition matrices for countries as an input of rating-based credit portfolio model. Hammer *et al.* (2007) applied reverse engineering by utilizing Logical Analysis of Data in the case of financial risk rating and the results were compared with Standard & Poor's rating result to prove the model accuracy.

One of the important issues in country rating is factors' selection. Although, economic and political factors both have essential effect on the country credit rating result, many economic theories emphasis in importance of macroeconomic factors for credit and default rating of countries Cifter *et al.* (2009). It seems instinctively obvious that macroeconomic conditions of a country should effect on the credit rating of that country Figlewski *et al.* (2012). The relations between macroeconomic factors and risk default were examined by several researchers such as Wilson (1997) and Nickell *et al.* (2000) and Allen and Saunders (2003) and Koopman and Lucas (2005) and Pesaran *et al.* (2006) and so on. The macroeconomic factors which appear to be significant for country credit rating are GDP, GDP per capita and GDP growth Afonso (2010) and Hischer and Nosbusch (2010) and Xu and Zhang (2014) and Eyssell *et al.* (2013), current account balance and public Amira (2004) and debt budget balance Baldacci and Kumar (2010). Later, Afonso (2003) mentioned that six factors appear to be important in deciding about country credit rating. Those six factors are GDP per capita, external debt, level of economic development, default history, real growth rate and inflation. Özatay *et al.* (2009) investigated the impact of worldwide financial conditions, domestic fundamentals and U.S. macroeconomic factor (news) on the emerging market bond index spread based on daily data. They found out that in the long run evolution of emerging market bond index spread is affected by global financial conditions, crises and domestic fundamentals which depend on sovereign rating. Schumacher (2014) investigated on the interactions between sovereign ratings and macroeconomic factors using a Panel Vector Autoregressive (PVAR) approach. He used annual data for European countries from 1996-2013. The results of the study proved that there is a significant tow-way interaction between macroeconomic factors and sovereign rating. Lei *et al.* (2014) studied the impact of tax convexity on the decisions taken for investing purposes.

3. The proposed model

In this section a mathematical model is introduced to rank some given countries according to some criteria (economic factors) in a way that the countries with higher ranking provide better environment for making investment by investors. The notations used in the model of the problem is introduced by Table 1.

Table 1. Notations used in formulation of the problem.

Notation	Type	Definition
I	Parameter	number of countries to be ranked
J	Parameter	number of criteria (economic factors) that effect ranking obtained for the countries

Table 1. Continued

Notation	Type	Definition
i	Index	index used to show each country
j	Index	index used to show each economic factor
w_j	Variable	relative importance obtained for the j-th economic factor
S_i	Variable	maximal possible score obtained for country i
x_{ij}	Parameter	performance of country i in economic factor j
r_{ij}	Parameter	normalized performance of country i in economic factor j

As the economic factors are divided to two types of positive (factors that higher performance of them is favored e.g. income) and negative (factors that lower performance of them is preferred e.g. inflation rate) factors, the normalization of x_{ij} is obtained by,

$$\begin{cases} r_{ij} = \dfrac{x_{ij} - \min\limits_{i=1,2,\dots,I}\{x_{ij}\}}{\max\limits_{i=1,2,\dots,I}\{x_{ij}\} - \min\limits_{i=1,2,\dots,I}\{x_{ij}\}} & \text{if } \ j \in POS \\ r_{ij} = \dfrac{\max\limits_{i=1,2,\dots,I}\{x_{ij}\} - x_{ij}}{\max\limits_{i=1,2,\dots,I}\{x_{ij}\} - \min\limits_{i=1,2,\dots,I}\{x_{ij}\}} & \text{if } \ j \in NEG \end{cases} \tag{1}$$

where POS and NEG are the sets of positive and negative economic factors, respectively.

Assuming that the factors are allowed to have positive relative importance such that $w_1, w_2, \dots, w_J \geq 0$, the aim is to aggregate multiple performance scores of each country obtained from different factors into a single score for the credit ranking problem. Therefore, a country with the highest score is ranked at the first place and so on.

To achieve this purpose, the following weighted sum model is proposed,

$$\max \ S_i = \sum_{j=1}^{J} r_{ij} w_j, \tag{2}$$

subject to

$$\sum_{j=1}^{J} w_j^2 = 1, \tag{3}$$

$$w_j \geq 0, \qquad j \in \{1, 2, \dots, J\} \tag{4}$$

The model (2)-(4) is applied to each country separately. The objective function calculated by Eq. (2) is a simple weighted sum of the normalized performances of each country in all the factors which maximizes the possible score obtained for the country. In constraint (3), the Euclidean norm of the relative importance of the factors is equal to 1. Applying this constraint, the model is an endogenous type model. Meaning that the relative importance of each factor is determined by the data of the model (normalized performances) endogenously instead of being determined by a decision maker. Constraint set (4) guarantee a positive weight for each factor.

Using Euclidean norm in the constraint (3) is an advantage for the model which gives a possibility to the model to be solved analytically. Thus, any financial manager and decision maker can apply and solve the model manually without any optimization solver. The analytical optimal solution of the model can be obtained by the method of Lagrange multipliers.

A simple explanation of Lagrange multipliers method can be found in the following steps,

Step 1: A model to be solved: $\max f(x) \;\; \text{s.t} \;\; g(x) = c$

Step 2: Construct an auxiliary function: $\Lambda(x, \lambda) = f(x) + \lambda(g(x) - c)$

Step 3: Solve equation $\nabla_{x,\lambda}\Lambda(x, \lambda) = 0$ to obtain optimal value of x.

To apply the Lagrange multipliers method to the model (6)-(8), first the Lagrange function Λ is defined by,

$$\Lambda(w_1, w_2, \dots, w_J, \lambda) = \sum_{j=1}^{J} r_{ij} w_j + \lambda\left(\sum_{j=1}^{J} w_j^2 - 1\right) \tag{5}$$

Then, $\nabla_{w_1,\dots,w_J,\lambda}\Lambda(w_1, \dots, w_J, \lambda) = 0$ implies that,

$$\frac{\partial \Lambda}{\partial w_j} = 0 \qquad j \in \{1, 2, \dots, J\} \tag{6}$$

so,

$$w_j = \frac{-r_{ij}}{2\lambda} \qquad j \in \{1, 2, \dots, J\} \tag{7}$$

And

$$\frac{\partial \Lambda}{\partial \lambda} = 0 \tag{8}$$

so,

$$\lambda = \pm\frac{\sqrt{\sum_{j=1}^{J} r_{ij}^2}}{2} \tag{9}$$

Replacing (14) in (12) results in the multiple optimal solution of the model (6)-(8) by,

$$w_j^* = \begin{cases} \dfrac{r_{ij}}{\sqrt{\sum_{j=1}^{J} r_{ij}^2}} \\ \dfrac{-r_{ij}}{\sqrt{\sum_{j=1}^{J} r_{ij}^2}} \end{cases} \qquad j \in \{1, 2, \dots, J\} \qquad (10)$$

Considering constraint (8), the optimal analytical solution for the model (6)-(8) for each country is obtained by the following equation for each country.

$$w_j^* = \frac{r_{ij}}{\sqrt{\sum_{j=1}^{J} r_{ij}^2}} \qquad j \in \{1, 2, \dots, J\} \qquad (11)$$

Finally, applying (16) in (6), the optimal (maximum) score for each country is obtained by the following equation.

$$S_i^* = \sum_{j=1}^{J} r_{ij} w_j^* = \sum_{j=1}^{J} \frac{r_{ij}^2}{\sqrt{\sum_{j=1}^{J} r_{ij}^2}} \qquad i \in \{1, 2, \dots, I\} \qquad (12)$$

Table 2. Economic factors selected from IMF database.

Factor Number	Factor Name and Type
1	Gross domestic product, constant prices (positive)
2	Gross domestic product, current prices (positive)
3	Gross domestic product based on purchasing-power-parity (PPP) valuation of country GDP (positive)
4	Gross domestic product based on purchasing-power-parity (PPP) per capita GDP (positive)
5	Gross domestic product per capita, current prices (positive)
6	Gross domestic product based on purchasing-power-parity (PPP) share of world total (positive)
7	Total investment (positive)
8	Gross national savings (positive)
9	Volume of exports of goods and services (positive)
10	General government revenue (positive)
11	General government total expenditure (positive)
12	General government net lending/borrowing
13	Current account balance ($)(positive)
14	Current account balance (percentage of GDP) (positive)
15	Volume of exports of goods (positive)
16	General government primary net lending/borrowing (positive)
17	Inflation, average consumer prices (index) (negative)
18	Inflation, average consumer prices (percentage change) (negative)
19	Inflation, end of period consumer prices (index) (negative)
20	Inflation, end of period consumer prices (percentage change) (negative)
21	Volume of imports of goods and services (negative)
22	Volume of Imports of goods (negative)

Table 2. Continued

Factor Number	Factor Name and Type
23	Unemployment rate (negative)
24	General government gross debt (negative)
25	Gross domestic product, deflator (negative)

4. Computational experiments on a case study

In this section the efficiency of the proposed model (2)-(4) and its analytical optimal solution obtained by (11) and (12) is measured. Some economic factors and countries are considered to be raked accordingly. Finally some economical comments on the obtained ranking are discussed.

4.1 Data collection and modification

All data in this study were collected from International Monetary Fund (IMF) database. Totally 25 factors (economical) and 53 countries are selected to elaborate the approximate ranking method. The number of selected countries is depended to availability of information in IMF database 2014. In addition, information related to the local currency rating of Moody's investor service (government bond rating for August 2014) is used to compare with the result of the proposed model. The 25 economic factors are listed in Table 2.

The performance of each country in each economic factor (x_{ij}) also is obtained from IMF database. The x_{ij} values of all countries in all 25 selected factors are normalized (r_{ij}) by the method mentioned in equation (1). The obtained r values of two factors (for instance) of Table 2 for all countries are depicted in Tables 3 and 4.

Table 3. The selected countries and original and normalized values of their performances in a positive factor for instance.

Country	Factor 1 (Positive Factor) Gross domestic product, constant prices		Country	Factor 1 (Positive Factor) Gross domestic product, constant prices	
	x_{ij}	r_{ij}		x_{ij}	r_{ij}
Albania	2.1	0.213252315	Kazakhstan	5.672	0.730034722
Australia	2.623	0.288917824	Korea	3.709	0.44603588
Austria	1.693	0.154369213	Kuwait	2.559	0.279658565
The Bahamas	2.297	0.241753472	Latvia	3.772	0.455150463
Belgium	1.22	0.0859375	Malaysia	5.2	0.661747685
Belize	2.5	0.271122685	Morocco	3.908	0.474826389
Bulgaria	1.6	0.140914352	Netherlands	0.832	0.029803241
Canada	2.299	0.242042824	New Zealand	3.254	0.380208333
Chile	3.634	0.435185185	Norway	1.791	0.168547454
China	7.538	1	Pakistan	3.102	0.358217593
Colombia	4.488	0.558738426	Panama	7.201	0.951244213
Costa Rica	3.8	0.459201389	Peru	5.519	0.707899306

Table 3. Continued

Country	Factor 1 (Positive Factor) Gross domestic product, constant prices		Country	Factor 1 (Positive Factor) Gross domestic product, constant prices	
	x_{ij}	r_{ij}		x_{ij}	r_{ij}
Denmark	1.481	0.123697917	Philippines	6.468	0.845196759
Egypt	2.256	0.235821759	Poland	3.088	0.35619213
Estonia	2.361	0.251012731	Portugal	1.166	0.078125
France	1.03	0.058449074	Romania	2.243	0.233940972
Germany	1.709	0.156684028	Russia	1.327	0.101417824
Honduras	3	0.343460648	Singapore	3.625	0.433883102
Hong Kong SAR	3.747	0.451533565	Slovak Republic	2.299	0.242042824
Hungary	1.984	0.196469907	South Africa	2.344	0.248553241
Iceland	2.682	0.297453704	Sweden	2.769	0.310040509
Ireland	1.699	0.155237269	Thailand	2.495	0.270399306
Israel	3.235	0.377459491	Tunisia	3	0.343460648
Italy	0.626	0	Turkey	2.267	0.237413194
Jamaica	1.275	0.093894676	United Kingdom	2.878	0.325810185
Japan	1.351	0.104890046	United States	2.768	0.309895833
Jordan	3.5	0.415798611	Uruguay	2.786	0.3125

4.2 Credit ranking of the countries

After normalizing the performances of each country in all factors, the model (2)-(4) is solved separately for each country using its analytical solution. The countries are sorted by decreasing order of their optimal scores. The ranking obtained for the countries is illustrated by Table 5.

Table 4. The selected countries and original and normalized values of their performances in a negative factor for instance.

Country	Factor 25 (Negative Factor) Gross domestic product, deflator		Country	Factor 25 (Negative Factor) Gross domestic product, deflator	
	x_{ij}	r_{ij}		x_{ij}	r_{ij}
Albania	204.797	0.913024625	Kazakhstan	215.747	0.904545909
Australia	101.95	0.992660296	Korea	118.852	0.979572874
Austria	117.278	0.980791641	Kuwait	243.694	0.88290621
The Bahamas	107.017	0.988736857	Latvia	112.598	0.984415422
Belgium	104.406	0.990758586	Malaysia	130.816	0.970309005
Belize	121.377	0.977617736	Morocco	126.58	0.973588991
Bulgaria	144.92	0.959388108	Netherlands	111.796	0.98503642
Canada	112.79	0.984266754	New Zealand	153.197	0.952979127
Chile	124.58	0.975137615	Norway	108.568	0.987535899
China	325.875	0.819272472	Pakistan	252.684	0.875945145
Colombia	148.576	0.956557223	Panama	150.013	0.955444536
Costa Rica	1,047.64	0.26040114	Peru	227.205	0.895673841
Denmark	120.123	0.978588723	Philippines	178.292	0.933547766

Table 4. Continued

Country	Factor 25 (Negative Factor) Gross domestic product, deflator		Country	Factor 25 (Negative Factor) Gross domestic product, deflator	
	x_{ij}	r_{ij}		x_{ij}	r_{ij}
Egypt	318.813	0.824740664	Poland	123.604	0.975893343
Estonia	149.611	0.95575581	Portugal	108.683	0.987446853
France	115.045	0.98252068	Romania	185.687	0.927821729
Germany	111.958	0.984910981	Russia	165.187	0.943695125
Honduras	227.674	0.895310689	Singapore	105	0.990298644
Hong Kong SAR	108.606	0.987506475	Slovak Republic	110.846	0.985772016
Hungary	135.014	0.967058443	South Africa	179.978	0.932242276
Iceland	202.286	0.914968923	Sweden	101.45	0.993047452
Ireland	101.638	0.992901882	Thailand	240.79	0.885154812
Israel	110.766	0.985833961	Tunisia	148.64	0.956507667
Italy	115.471	0.982190823	Turkey	1,383.94	0
Jamaica	214.183	0.905756933	United Kingdom	108.226	0.987800714
Japan	92.471	1	United States	108.216	0.987808457
Jordan	230.443	0.893166619	Uruguay	191.055	0.923665222

4.3 More discussion on the obtained ranking

It appears that country rating remain an important determinant of agencies credit rating. Although this study used a non-linear ranking model to rank the countries, it can be even used as an approach to rate countries. The model try to compare all countries by each other base on all economic factor levels, and rank the countries from the highest score to lowest score. Such a measure is suited for any country that have available information regarding to those economic factors.

The results that are depicted in Table 5 compare 54 countries, and rank them from the highest score to the lowest one. As it is expected, developed countries in Europe, China and US are listed among the top 10 countries. This is a valuable result and reflect the accuracy of the model in ranking the countries. If we compare our results with Moody's rating which is illustrated in Table 6, it can be concluded that the ranking model was successful in application. Also it is important to remind that the model which is utilized in this study is used to rank the countries not rating them. Since all of the rating agencies rate the countries and none of them rank them, there is no other source to compare the result of this study with them. Although, there are some miss ranking or error in ranking the model, overall result satisfy and show validity of the model. The reason for miss ranking may occur due to some political factors which were not the interest of this research. Since all of the factors which employed in this study are economic factors and are quantitative, another reason that may be cause of the miss ranking is lake of availability of some economic or other quantitative factors. In addition, as another reason of the miss ranking, some rating agencies are not willing to downgrade some countries since some of them are their clients and many other reasons that are not interest of this research.

One important note that was also mentioned earlier is that the model rank the countries not rate them. So, some time there is a small difference between two or more different ranks. For instance, Sweden and Singapore are ranked in 6th (with score 3.239604525) and 7th (with score 3.234437298) respectively in Table 5. Clearly, there is a minor difference between the scores of two countries, but with a small difference Sweden is preferred to Singapore.

Table 5. Complete credit ranking of the countries obtained by the model (2)-(4).

Country	Score (S_i^*)	Obtained Rank	Country	Score (S_i^*)	Obtained Rank
Kuwait	3.691875294	1	New Zealand	2.886315808	28
Norway	3.480280683	2	Iceland	2.873243364	29
China	3.437855554	3	Portugal	2.858175795	30
United States	3.30366353	4	Chile	2.85028914	31
Australia	3.267500871	5	Slovak Republic	2.846360342	32
Sweden	3.239604525	6	Colombia	2.844780097	33
Singapore	3.234437298	7	Estonia	2.84170649	34
Germany	3.200642842	8	Panama	2.829547343	35
Denmark	3.179865668	9	Poland	2.824456333	36
Netherlands	3.160279787	10	Romania	2.806000468	37
Austria	3.113361836	11	Philippines	2.797381073	38
Belgium	3.086349456	12	Morocco	2.764841354	39
France	3.06213882	13	Uruguay	2.748532632	40
Canada	3.036992008	14	The Bahamas	2.720146396	41
Italy	3.03352462	15	Belize	2.718025531	42
Korea	3.031142269	16	Kazakhstan	2.710471842	43
Malaysia	3.022203108	17	Turkey	2.689606545	44
Ireland	2.988980148	18	Albania	2.689481653	45
Hong Kong SAR	2.987219531	19	Jordan	2.668579912	46
Israel	2.9813513	20	Tunisia	2.572606402	47
Bulgaria	2.941056949	21	Costa Rica	2.458448512	48
United Kingdom	2.935241441	22	Pakistan	2.449036743	49
Thailand	2.916347181	23	South Africa	2.439498125	50
Hungary	2.903752619	24	Honduras	2.409879802	51
Peru	2.899935046	25	Russia	2.303064757	52
Latvia	2.896582265	26	Jamaica	2.177198077	53
Japan	2.895213607	27	Egypt	2.082532847	54

Table 6. Moody's rating for 54 countries

Country	Moody Scale	Country	Moody Scale	Country	Moody Scale
Kuwait	Aa2	Hong Kong SAR	Aa1	Romania	Baa3
Norway	Aaa	Israel	A1	Philippines	Baa3
China	Aa3	Bulgaria	Baa2	Morocco	Ba1
United States	Aaa	United Kingdom	Aa1	Uruguay	Baa2
Australia	Aaa	Thailand	Baa1	The Bahamas	Baa1
Sweden	Aaa	Hungary	Ba1	Belize	Caa2
Singapore	Aaa	Peru	A3	Kazakhstan	Baa2
Germany	Aaa	Latvia	Baa1	Turkey	Baa3
Denmark	Aaa	Japan	Aa3	Albania	B1
Netherlands	Aaa	New Zealand	Aaa	Jordan	B1
Austria	Aaa	Iceland	Baa3	Tunisia	Ba3
Belgium	Aa3	Portugal	Ba1	Costa Rica	Baa3
France	Aa1	Chile	Aa3	Pakistan	Caa1
Canada	Aaa	Slovak Republic	A2	South Africa	Baa1
Italy	Baa2	Colombia	Baa2	Honduras	B3
Korea	Aa3	Estonia	A1	Russia	Baa1
Malaysia	A3	Panama	Baa2	Jamaica	Caa3
Ireland	Baa1	Poland	A2	Egypt	Caa1

5. Conclusion

Country credit rating changes have influences on investment and every sector of the related countries. Most of researches in the literature focus on rating changes of countries and those rating affected mostly by macroeconomic outcomes. Numerous downgraded of European countries in past years have shown how vital it is to examine the issue.

This study starts for the first time to rank a set of countries based on several important macroeconomic factors instead of rating them. A weighted sum model was proposed and solved analytically. The analytical optimal solution was obtained by the Lagrange Multipliers method easily. This easy solution method is an advantage of the model that helps the managers to apply the model easily without any optimization software. The results was compared to Moody's rating (in 2013) to show accuracy of the model. This study is a worthy empirical analysis for comparing several specified countries for investment. In particular, if there is a set of specified countries which are interested for investment, it is possible to compare them and find the best candidate. The presented model can be an alternative to country risk rating of agencies, since there is a broad question agency variation in credit quality assessment in the country perspective.

Future studies on the country credit ranking problem may focus on using linear data envelopment analysis models to rank countries. On the other hand, simultaneous countries credit rating-ranking problem may be an interesting study.

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