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Dynamics of rumor spreading

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Abstract In this paper, we explain the rumor spreader model with a differential equation system and analyses and consider this system in dynamical system view. The model which we consider in a society contains ignoring, spreading, stifle and controlling factors. In this work, we study on a new rumor spreading model, Ignorant-Spreader-Stifler-Controller (ISRC) model, is developed. The model extends the classical Ignorant-Spreader-Stifler (ISR) rumor spreading model by adding a new kind of people that spread a new rumor against previous rumor to control and reduce the maximum rumor influence. The model is an extension of SIR model which has studied before. In this research, we give a dynamical system which explains SIRC dynamical factors. Moreover, we consider the equilibrium conditions near the equilibrium point

Keywords Epidemic model; Rumor spreading; Asymptotic behavior; Numerical simulations

1. Introduction

The classical model for the spread of a rumor consists of one of whom, the spreader knows a rumor. Those individuals who do not know the rumor are called ignorant. On day on the spreader chooses one individual at random, which could be the spreader himself to whom to tells the rumor. As noted it may not spread the first day. If an ignorant choosen, the spreader tells the rumor and on the next day there are two spreaders. Each spreader chooses one person at random (which could be himself, the other spreader, or an ignorant) to whom to tell the rumor. This continues until all persons have heard the rumor.

Rumors are an important form of social communications, and their spreading plays a significant role in a variety of human affairs Zhang and Zhang (2009). The spread of rumors can shape the public opinion in a country Galam (2003), greatly impact financial markets Kimmel (2004) and Kosfeld (2005), social networks Kosmidis and Bunde

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(2007), and cause panic in a society during wars and epidemics outbreaks. Traditionally, rumors spread on social networks through relationships between different individuals which it can produce a small effect on society stability. Nowadays, With the emergence of social web and other media, rumors can propagate at a faster velocity and generate greater influence on people's lives. Therefore, understanding the transmission mechanisms of rumors and then devising effective measures to suppress their spreading are of considerable importance Wang *et al.* (2014), Thomas (2007) and Bhavnani *et al.* (2009). The mathematical modeling of rumor spreading is an alternate way of evaluating agent behaviors.

The study of the rumor models has had a long history. The spreading of rumor is in very similar to the spreading of epidemic infection. Thus many epidemic models have been used to describe the spread of information in the form of rumors. The most popular transmission model of rumors was introduced by Daley and Kendall (1964) and investigated based on mathematical theory by Maki and Thompson (1973). These models have been used extensively for quantitative studies of rumor spreading Pittel (1990), Lefevre and Picard (1994) and Gu *et al.* (2008). A more detailed consideration of the models about rumor spreading was given in the literature Huo *et al.* (2011) and Zhao *et al.* (2011). Up to now, several rumor propagation models on social networks have been well studied. A rumor spreading model with network medium in complex social networks was established by J. Wang and Y. Wang (2015) and Zhao and Wang (2013) and investigated its dynamic behaviors and numerical simulation. Zanette (2001) and Zanette (2002) established a rumor spreading model on small-world networks and provided a threshold of rumor spreading. Some scholars studied applications of the stochastic version of the classical model for the spread of rumor on scale-free networks and showed that the uniformity of the network has a major impact on the dynamic mechanism of rumor spreading Moreno *et al.* (2004a), Moreno *et al.* (2004b) and Moreno *et al.* (2002). Dietz provides a review of the recent mathematical contributions to the description of the spread of epidemics and rumors in the survey paper Dietz (1967) which is worth seeing in this connection.

This paper is organized as follows. We first give a novel ISRC rumor transmission model and derive the corresponding mean-field equations in Section 2. In Section 3, we present analytical results for the ISRC model. In Section 4, numerical simulation on the dynamics results of the ISRC model is investigated to analyze the impact factors under different parameters. Conclusions and discussions are given in Section 5.

2. Model Formulation

In this section, we introduce and analyses an appropriate dynamical model for rumor spreading with controller agent. The total population is partitioned into Ignorant, spreaders, stifles, and controllers, respectively, denoted by $I(t)$, $S(t)$, $R(t)$ and $C(t)$. The total population size at time t is denoted by $N(t)$. Our assumptions on the dynamical transmission of rumor among humans are demonstrated in the following flowchart:

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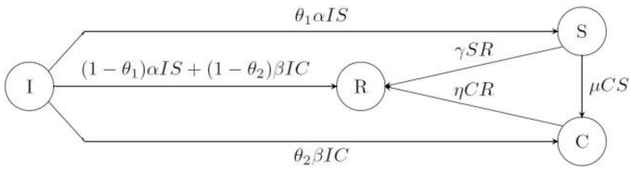

Figure 1. The transfer diagram of the model (2.1)

As shown in Figure 1, The ISRC rumor spreading rules can be summarized as follows

1. When an ignorant contacts a spreader, the ignorant becomes a spreader with probability $\theta_1\alpha$ or a stifler with probability $(1-\theta_1)\alpha$.
2. When an ignorant contacts a controller, the ignorant becomes a controller with probability $\theta_2\beta$ or a stifler with probability $(1-\theta_2)\beta$.
3. When a spreader contacts a stifler, the spreader becomes a stifler with probability γ
4. When a controller contacts a stifler, the controller becomes a stifler with probability η.
5. When a spreader contacts a controller, the spreader becomes a controller with probability μ. The model is described by the following system of differential equations:

$$\begin{cases} \dfrac{dI}{dt} = -\alpha I(t)S(t) - \beta I(t)C(t), \\ \dfrac{dS}{dt} = \theta_1 \alpha I(t)S(t) - \gamma S(t)R(t) - \mu C(t)S(t), \\ \dfrac{dR}{dt} = (1-\theta_1)\alpha I(t)S(t) + (1-\theta_2)\beta I(t)C(t) + \gamma S(t)R(t) - \eta C(t)R(t), \\ \dfrac{dC}{dt} = \theta_2 \beta I(t)C(t) - \eta C(t)R(t) + \mu C(t)S(t). \end{cases} \tag{2.1}$$

With the initial conditions

$$I(0) = N-2, S(0) = 1, R(0) = 0, C(0) = 1.$$

Table 1. Description of parameters in the model (2.1).

Parameters	Unit	Comments
α	Day^{-1}	$I(t)$ – to – $S(t)$ transmission rate
β	Day^{-1}	$I(t)$ – to – $C(t)$ transmission rate
θ_1	None	the rate of being $S(t)$ transmission
θ_2	None	the rate of being $C(t)$ transmission
μ	Day^{-1}	$S(t)$ – to – $C(t)$ transmission rate
γ	Day^{-1}	$S(t)$ – to – $R(t)$ transmission rate
η	Day^{-1}	$C(t)$ – to – $R(t)$ transmission rate
$I(0)$	Individual	The initial number of the susceptible individual
$S(0)$	Individual	The initial number of the spreader
$R(0)$	0	The initial number of the stifler
$C(0)$	Individual	The initial number of the controller

3. Model analysis

Lemma 3.1. The solutions $I(t)$, $S(t)$, $R(t)$ and $C(t)$ of system (2.1) with initial values $I(0) > 0$, $S(0) > 0$, $R(0) > 0$ and $C(0) > 0$ are positive for all $t > 0$

Proof. According to Sharomi *et al.* (2011), from the first equation of system (2.1), we have

$$\frac{d}{dt}(I(t)\exp\left\{\int_0^t(\alpha S(t)+\beta C(t))d\tau\right\})=0. \tag{3.1}$$

Hence

$$I(t_1)\exp\left\{\int_0^t(\alpha S(t)+\beta C(t))d\tau\right\}-I(0)=0. \tag{3.2}$$

Therefore,

$$I(t_1)=I(0)\exp\left\{\int_0^t(\alpha S(t)+\beta C(t))d\tau\right\}>0. \tag{3.3}$$

Similarly, it can be shown that

$$S(t_1) \;=\; S(0)\;\exp\left\{-\int_0^{t_1}(-\theta_1\alpha I(\tau)+\gamma R(\tau)+\mu C(\tau))d\tau\right\}, \tag{3.4}$$

And

$$C(t_2) \;=\; C(0)\;\exp\left\{-\int_0^{t_2}(-\theta_2\beta I(\tau)+\eta R(\tau)+\mu S(\tau))d\tau\right\}, \tag{3.5}$$

Furthermore, from the third equation of system (2.1), we have

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$$\frac{d}{dt}(R(t)\ \left\{\exp\int_0^t(-\gamma S(\tau)-\eta C(\tau))d\tau\right\})=\varphi(t)\ \exp\left\{\int_0^t(-\gamma S(\tau)-\eta C(\tau))d\tau\right\}, \qquad (3.6)$$

Where

$$\varphi(t)\ =\ (1-\theta_1)\alpha I(t)S(t)\ +\ (1-\theta_2)\beta I(t)C(t).$$

Hence,

$$R(t_1)\ exp\left\{\int_0^{t_1}(-\gamma S(\tau)-\eta C(\tau))d\tau\right\}-R(0)=\int_0^{t_1}\varphi(\zeta)\ exp\left\{\int_0^{\zeta}\zeta(-\gamma S(\tau)-\eta C(\tau))d\tau\}d\zeta\right\}. \qquad (3.7)$$

Therefore,

$$\begin{aligned} R(t_1)\ &= R(0)\ exp\left\{\int_0^{t_1}(\gamma S(\tau)\ +\eta C(\tau))d\tau\right\} \\ &+ exp\left\{\int_0^{t_1}(\gamma S(\tau)\ +\eta C(\tau))d\tau\right\}\times\int_0^{t_1}\varphi(\zeta)\ exp\left\{\int_0^{\zeta}(-\gamma S(\tau)-\eta C(\tau))d\tau\}d\zeta\right\}>0. \end{aligned} \qquad (3.8)$$

Thus, the solutions $I(t)$, $S(t)$, $R(t)$ and $C(t)$ of system (2.1) remain positive for all $t > 0$

In the whole process of rumor spreading, the number of spreaders and controllers first increase, then decrease and reach zero when the rumors dies out. At that time, the system reaches an equilibrium state and has only ignorant and stiflers. Since the total population $N(t)$ is independent of time and is a constant, denoted by N^*. We can obtain the equilibrium of the system (2.1) as follows

$E^* = (I^*. 0. R^*. 0)$

Where $I^* + R^* = N^*$, that is, the rumor and controller must disappear with time and all I^* and R^* represent stable situations. Next, we have a look at the stability of the equilibrium point $E^* = (I^*. 0. R^*. 0)$.

Theorem 3.1. If

$$I^* < min\left\{\frac{\gamma}{\alpha\theta_1+\gamma},\frac{\eta}{\beta\theta_2+\eta}\right\}$$

then the equilibrium E^* is locally asymptotically stable.

Proof. In order to verify the stability of equilibrium point E^* , we construct the Jacobian matrix of the system (2.1) at a point $E^* = (I^*. 0. R^*. 0)$ as follows:

$$J(E^*) = \begin{bmatrix} 0 & -\alpha I^* & 0 & -\beta I^* \\ 0 & \theta_1 \alpha I^* - \gamma R^* & 0 & 0 \\ 0 & (1-\theta_1)\alpha I^* + \gamma R^* & 0 & (1-\theta_2)\beta I^* + \eta R^* \\ 0 & 0 & 0 & \theta_{\partial} \beta I^* - \eta R^* \end{bmatrix},$$

With regard to the above matrix, we know that one zero eigenvalue corresponds to the fact that the order of the dynamical system is two. The other zero eigenvalue is related to the stable center manifold that is the straight line $I^* + R^* = N^*$ on the $(I^*. R^*)$ plane. Thus the stability of the equilibrium point depends on the signal of the reminder eigenvalues. The corresponding characteristic polynomial can be written as

$$p(\lambda) := \lambda_2 - (J_{22} + J_{44})\lambda + J_{22}J_{44} = 0, \tag{3.9}$$

Where

$$J_{22} = \theta_1 \alpha I^* - \gamma R^*$$

And

$$J_{44} = \theta_2 \beta I^* - \eta R^* .$$

From Routh-Hurwitz criterion, we can get when

$$0 < I^* < min\left\{ \frac{\gamma}{\alpha\theta_1 + \gamma}, \frac{\eta}{\beta\theta_2 + \eta} \right\}$$

is locally asymptotically stable.

4. Numerical Simulation

In this section, we use the Runge-Kutta method to solve the differential equations (2.1) and analyze the effects on the rumor spreading process by the new factors. Furthermore, this simulation allows us to find the key factors to affect the rumor diffusing and the ways of control rumor diffusing. In the following simulation we assume $N = (5 \times 10^6) + 2$. Thus $I(0) = 5 \times 10^6 . S(0) = 1$, $C(0) = 1$ and $R(0) = 0$.

Figure 2 shows the trends of ISRC model without the controller agent. The blue line represents the densities of controller agent which is zero, i.e. control mechanism is not considered. From the following simulation we can find there is a sharp increase in the number of spreaders as spreaders begin to propagate a rumor. With further spreading of the rumor, the number of spreaders reaches a peak and thereafter declines. Finally, the number of spreaders is zero and this leads to the termination of rumor spreading. In this whole process, the number of ignorant always reduces while the number of stiflers always increases until they reach the balance, respectively. As we see in Figure 2, the spreader density attains its maximum at time $t = 1.11$. In fact, the peak value of spreader density max $\{S(t)\}$ is 3.161573×10^6 , i.e., the highest densities of people are spreading the rumor is 3.161573×10^6.

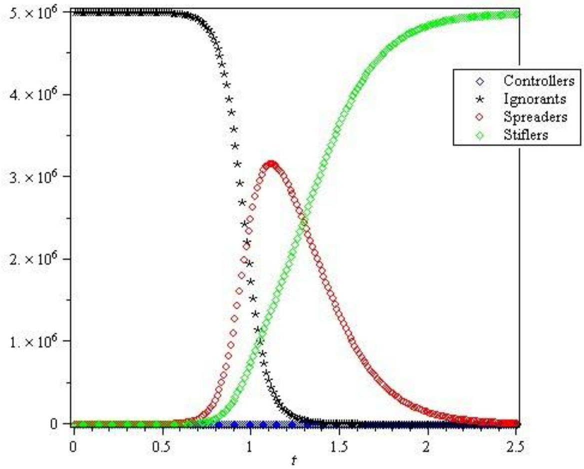

Figure 2. Densities of ISR over time with $\alpha = 4\gamma = 0.000004$, $\beta = \mu = \eta = 0$, $\theta_1 = 0.8$.

Figure 3 shows the general trends of the four kinds of agents in the ISRC rumor spreading model. This simulation illustrates the effect of a controller agent on the rate of spreader agent. From the following simulation we can find that the variation trend of the number of controllers is similar to that of the spreaders, which increases at first and then decreases to zero. Furthermore, this simulation shows that if, for example, we suppose the controller agent as a new rumor against the previous rumor with transmission rate β equal with transmission rate α , and then we can control and reduce the maximum rumor influence of the previous rumor. As we see in Figure 3, the spreader density is less than the controller density. This occurs because of the presence of the parameter μ. Furthermore, the spreader density attains its maximum at times $t = 0.69$ with $S(0.69) = 1.354440 \times 10^6$.

In addition, the peak value of controller density is 2.151224×10^6 which occurs at time $t = 0.78$.

Figure 4 shows how the densities of controllers change over time for different transmission rate between controllers and ignorant. This simulation represents the effect of transmission rate between controllers and ignorant on ISRC model. The blue asterisk line represents the scenario that trans- mission rate between controllers and ignorant is lower than transmission rate between spreaders and

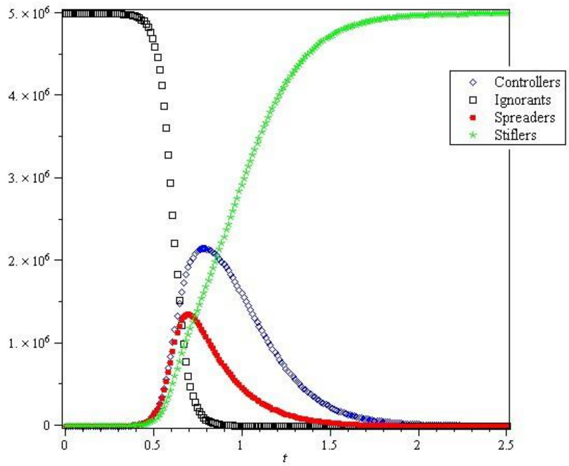

Figure 3. Densities of ISRC over time with $\alpha = \beta = 0.000006$, $\gamma = \mu = \eta = 0.000001$, $\theta_1 = \theta_2 = 0.8$.

Ignorant, i.e. $\beta < \alpha$. The other lines represent the scenario that $\beta \geq \alpha$. Furthermore, this simulation attempts to illustrate that one of the ways to control spreading rumor is spread a new rumor with high impact compared to the previous rumor.

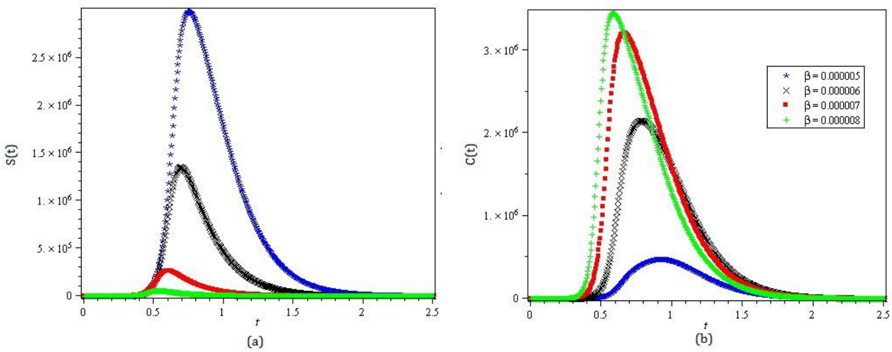

Figure 4. Density of spreaders and controllers over time under different transmission rate β with $\alpha = \beta = 0.000006$, $\gamma = \mu = \eta = 0.000001$, $\theta_1 = \theta_2 = 0.8$.

Figure 5 describes how the ratio between spreaders and controllers change with different transmission rate μ over time. The blue asterisk line represents the scenario that transmission rate between spreaders and controllers is zero, i.e. $\mu = 0$. In this case, the number of spreaders and controllers is equal. The other lines represent the scenario that $\mu > 0$. As we see in Figure 5, when $\mu > 0$, the ratio $\frac{S(t)}{C(t)}$ decreases with increasing time. Furthermore, this ratio decreases relatively faster with increasing μ. For instance, the number of controllers at time $t = 0.7$ will be about 1.46, 2.14 and 3.13 times the number of spreaders with respect to $\mu = 0.00001$, $\mu = 0.00002$ and $\mu = 0.00003$ respectively.

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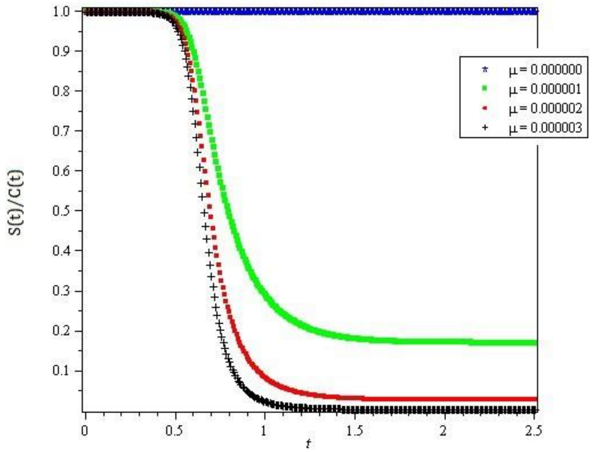

Figure 5. The ratio between spreaders and controllers over time under different transmission rate μ with $\alpha = \beta = 0.000006$, $\gamma = \eta = 0.000001$, $\theta_1 = \theta_2 = 0.8$.

5. Conclusion

In this paper, we studied the new pattern of rumor spreading corresponding to the appearance of controller agent. We showed that the model established in this paper possesses non-negative solutions, as desired in any rumor spreading dynamics. By using stability analysis, we obtained a sufficient condition on the parameters for the local asymptotical stability of the equilibrium point. Numerical simulations are also conducted to support our analytic results.

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