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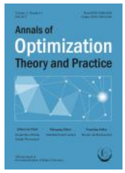

Simulation of the game of chaos and fractals

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Abstract In this paper, the role of the iteration function system in the geometric modeling of two-dimensional and three-dimensional fractal objects is presented. We also study chaos game display technology, which is a very important and efficient tool in the biochemistry and military fields. This method provides a good template for amplified and generated sequences such as the DNA sequence. See Reference 5. The game of chaos provides many interesting patterns and is therefore a very important tool for examining the organization of units. In this paper, we investigate the game of chaos and chaotic systems, and by giving an example, we have obtained the speed of creating a system disorder by Liapanov's view.

Keywords Fractal; Absorbent; Chaos games; Random algorithm; Lyapunov view

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1. Introduction

Dynamic systems have recently received much attention, as they have played a significant role in natural issues, geography, economics, physics, biology, etc. Dynamic systems have been used for many modeling Mitra and Privileggi (2005). Because in many cases the fractal nature of these systems is seen, and especially in chaotic systems, including the irregular appearance, discussions of fractal geometry usually involve the sweet talk of dynamic systems.

In response to the question, "What are dynamic systems? First, let's take a simple example.

"The emission rate of a pollutant is 2 cubic centimeters per second"

The system shows how the emission or concentration of the pollutant changes over time. Suppose x_n is the concentration of this pollutant at time t = n or that is, its initial concentration in the medium and x_{n+1} is its concentration at the next time. Therefore, the dynamic system in question can be represented in the form of an equation, as follows:

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$$x_{n+1} = x_n + 2 \tag{1}$$

If the initial concentration of the pollutant is 2, by entering this number into the above equation, the secondary concentration in the medium is equal to 14.

$$x_{n+1} = x_n + 2 = 12 + 2 = 14$$

Each time we go from right to left in the equation, it is called a repetition. We can repeat the equation again and add new pollutant concentration to the equation $x_{n+2} = 16$. Repeat the equation three times to obtain the concentration of pollutant $x_{n+3} = 18$ after 1 second. It is a deterministic dynamical system, if we want to convert it to a non-random system, we can say "the contaminant concentration increases by about 5 cm3 per second" and the equation follows.:

$$x_{n+1} = x_n + 2 + p \tag{2}$$

Where p is the error sentence (which is very small compared to 0) and indicates the existence and impact of various probabilities in the system.

We are now defining the dynamic system.

Suppose D is a subset of R^n, which in some cases can be R^n itself. Also suppose $f: D \to D$ is a continuous mapping. In the usual way, we use f^k to represent the k-th repetition, that is

$$f^2(x) = f(f(x), f^1(x) = f(x), f^0(x) = x, ... \tag{3}$$

Note that if x is a point on D, $f^k(x)$ is on D for every k. Typically $f^2(x)$,$f(x)$,x and so on are values of a quantity at times 2,1,0 and .... Therefore, the quantity at time k + 1 is given by the function f in terms of the quantity at time k. The population growth system, the amount of investment under taxation, and the location of a fluid particle in a uniform flow are all examples of this type of system.

A duplicate scheme $\{f^k(x)\}$ is called a discrete dynamic system Jeffrey (1990). We would like to investigate the sequence behavior of the repetitions, or circuits, $\{f^k(x)\}_{k=1}^{\infty}$ for various $x \in D$ starting points, especially for large k. For example, we know that the curvature of a curve is inversely proportional to its radius of curvature, so if the curvature of a curve is at x, then its curvature radius is $f(x) = \frac{1}{x}$. Then $f^k(x)$ For every element x if $k \to 1$,it tends to zero. Sometimes the distribution of duplicates seems almost random. $\{f^k(x)\}_{k=1}^{\infty}$ can refer to a fixed point ω (which in this example is a zero point), that is, a point of D such as ω where $f(\omega) = \omega$. More generally, $\{f^k(x)\}_{k=1}^{\infty}$ can favor a circuit with periodic points T set $\{\omega, f(\omega), \dots, f^{T-1}(\omega)\}$ which That T is the smallest positive integer that $f^T(\omega) = \omega$, since if we have $k \to \infty$: $\left|f^k(x) - f^i(w)\right| \to 0$. Although we may see a random motion of $f^k(x)$ around, it always remains close to a given set, which may be a fractal. A fractal is similar in its so-called object or phenomenon. The Mandelbrot Bionite was the first to express the fractal word.

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The term fractal Latin means fractured to produce irregular pieces. Fractals produced by dynamical systems, such as the Mandelbrot set and the Julia set, are called algebraic fractals Garg *et al.* (2014). In the field of computer graphics activities, researchers are always trying to find new ways to geometrically model objects and objects. Polynomials are one of the tools needed to find mathematical models. In this article, we first start with a simple statement of the game of chaos and, by giving an example, we try to implement the concept of chaos and examine the application of Lyapunov's view. We also give an example of their use in military systems by expressing the nature of fractals.

2. Chaos Game

The science of chaos began with a great deal in the 1980s, and even some scientists considered it a parallel to two major revolutions in physical theory, namely, relativity and quantum mechanics Aswathy (2015). When these theories were opposed to Newton's theory of dynamical systems, chaos challenged traditional beliefs in the Newtonian framework.

The growth of chaos reveals the stunning dependency between order and disorder that results from successive instability. Over the past 5 years, successful innovations in computer graphics have been one of the most important factors that have enabled scientists and mathematicians to advance the nonlinear systems from which chaos results.

Now, through computer simulations, scientists can observe gradual changes in dynamical systems and complex chaotic effects caused by motion and dynamic equations. (see Garg *et al.*, 2014). Sun has helped a great deal in understanding many of the phenomena in the various branches of science. These include biology, mathematics, physics, economics, simulation and military science.

The phrase game chaos in mathematics was first introduced by Michael Burnsley. The game of chaos is a way of creating fractals by polynomials First we analyze the word chaos.

Perhaps in the first place, the word chaos is a system of chaos and disorder. It looks like that. Figure 1 shows a picture of a riot.

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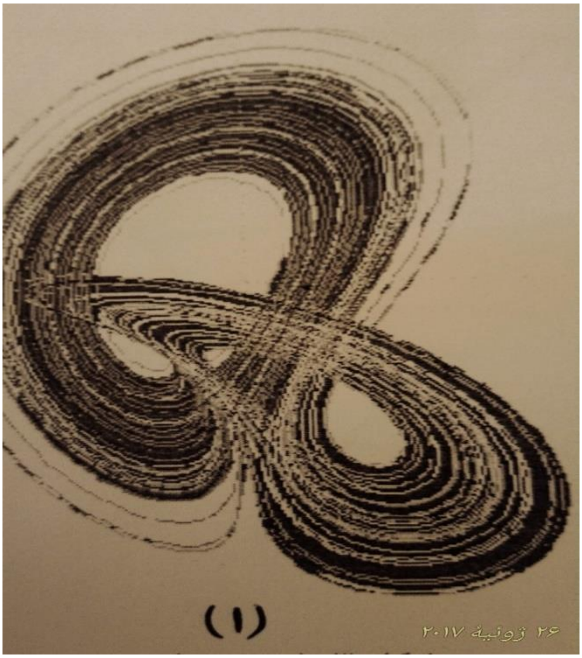

Figure 1.

The curves show how a dynamic system (especially this particular system known as the Lorenz system) changes with time in 3D space. Notice how the curves rotate without interrupting. These curves are called system circuits. Also note that the system rotates around the two main zones. The range from which the system is moving in a certain direction is called the "absorption domain" and is called the "absorber" where the system is moving. More precisely, in a dynamic system mapped by $f: D \to D$ the subset F of D is called an attractor for f if a set is closed under f (ie $f(F) = F$) such that for any x belonging to an open set V containing F interval $f^k(x)$ tends to zero until k is infinitely long, in this case the set V is called the "absorption domain" F.

Now let's give a precise definition of chaos.

A chaotic system is a system that is first of all non-linear, meaning that the outputs of the system are not proportional to the inputs (proportional to being multiplied by a fixed number or subtracted or subtracted by a fixed number); eventually, they continue in cycles called strange adsorbents.

The sensitivity phenomenon of chaotic systems was first discovered by a meteorologist named Lorenz in Year 2. As he rounded up the numerical values of the weather forecasting equations from 2 decimal places to 5 decimal places (because his printer was only able to print numbers up to 5 decimal places) he suddenly realized that all the sequence numbers he obtained, is far from reality (although its initial values only differed from the fourth decimal place, the difference between the initial values being less than 0.001). This suggests that long-term weather forecasts (as a chaotic system) are

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impossible. No matter how accurate the calculations are, a very small difference, even for example 0.000000001 and smaller, can make a lot of difference in subsequent results. Researchers in this field have an example to express the sensitivity of chaotic systems:

"Waving a butterfly in Shiraz can cause a storm in Tehran! »

Chaotic systems are highly sensitive to initial values, which means that if the initial values are very small, there will be a lot of difference after a few steps due to system replication. Consider the following simple system known as the logistics system:

$$x_{n+1} = 4x_n[1 - x_n] \quad (4)$$

In this system, the input variable is x_n and the output is x_{n+1}. This system is nonlinear, so the output is not proportional to the input. Now let's take a look at the system. Suppose x_n = 0.75 In this case the output is: x_{n+1} = 4 (0.75) [1-0.75] = 0.75

If this equation indicates the concentration of a type of toxic gas in the environment, the concentration of 0.75 will be a base number and the concentration of the environment will be the same at a later time. The value of 0.75 is called the equation constant. But suppose the initial ambient concentration is x_0 = 0.7499, then the output is x_1 = 4 (0.7499) [1-0.7499] = 0.7502 and so if we consider this value as the next step input:

x_2 = 4 (0.7502) [1-0.7502] = 0.7496

And by repeating this system we have:

x_{99} = 0.966007, x_{100} = 0.131350

It is observed that only the first few iterations of the outputs are approximately the same, but from the next step, the outputs of the equation diverge widely and there is a great divergence between the circuits.

3. Lyapunov's facade

The question that arises here is how fast do these circuits go away? The answer to this question is obtained by examining Lyapunov's perspective as presented below.

Suppose l represents the error in our initial investigation, or the difference between the two initial values. In the previous example, this value is between 0.75 and 0.7499. Suppose R is a distance (positive or negative) around a reference circuit. Now our question is, how fast (after a few iterations) does another circuit - which has an initial error l - fall into the radius R of the reference circuit? Or, alternatively, after n times the equation is repeated, what circuit (R) exceeds the reference circuit by the initial error l? The answer is a function of the repetitions, n and the Lyapunov view of γ.

$$R = l.e^{\gamma n} \quad (5)$$

In this function, e = 0.7182818 is the base of the natural logarithm.

The Lyapunov γ view is obtained from the following relation:

$$\gamma = \sum (1/n) \log |df / dx_n| \quad (6)$$

For example, we compute the Lyapunov view of the logistic equation of the previous example.

Derivative to the right of the logistic equation

$$x_{n+1} = 4x_n[1 - x_n] = 4x_n - 4x_n^{\ 2} \quad (7)$$

Is equal to

$$df/dx_n = 4 - 8x_n \quad (8)$$

So for the first iteration of the second circuit we have x_n= 0.7502:

$\left| df/dx_n \right|$ = | 4 [1-2 (0.7502)] | = 2.0016

And we have log (2.0016) = 0.6936. If we add and average this last value and the rest of the values obtained in the same way, we get Lyapunov's view. The true value of Lyapunov's view is 0.693147, where even this first sentence is very close to the true value. If we start with x_0 = 0.1, we get slower to the true value of Lyapunov's view.

Now that we obtain Lyapunov's view of the logistic equation, we use the following example to calculate the speed of circuits of this equation from a reference circuit.

Example. We want to know how long after starting from the initial value of 0.7499 we exit the neighborhood of the reference circuit (circuit with the initial value of 0.75) with a radius of 0.01.

In this case R = 0.01. For a circuit with an initial value of 0.7499, the difference in the initial value is l = 0.75-0.7499 = 0.0001. So by placing Lyapunov's view (0.693147) and the rest of the data in relation to $R = l.e^{\gamma n}$ we have:

$$0.01 = 0.0001e^{0.693147n}$$

Solving this equation yields n based on: n = 6.64. Looking at the table below, we see that for n = 7 (seventh iteration) the output is x_7 = 762688 and this is outside the neighborhood of radius 0.01 of the reference circuit, outside the range (0.74, 0.76). Take it.

period	x_n	$log\left|{}^{df}/_{dx_n}\right|$
1	0.360000	0.113329
2	0.921600	1.215743
3	0.289014	0.523479
4	0.821939	0.946049
5	0.585421	-0.380727
6	0.970813	1.326148
7	0.113339	1.129234
8	0.401974	-0.243079
9	0.961563	1.306306
10	0.147837	1.035782
average		0.697226

Similarly, for the third circuit with an initial value of 0.74999, the difference in the initial values is l = 0.00001. So by putting the data in the equation we will have:

$$R = l.e^{\gamma n}$$

From which n = 9.96 is obtained. In this example, the system is divergent, because the Lyapunov view is positive. If the Lyapunov view is negative, the value of $l.e^{\gamma n}$ decreases with each step. So the condition for the system to be chaotic is to have a positive Lyapunov view.

Also note that the logistic equation is a simple equation with only one variable x_n . Therefore, this equation is the only data of a Lyapunov view. In general, an equation with m variable may have m Lyapunov view. In this case, it is a chaotic system if at least one of Lyapunov's views is positive.

Fractal drawing using chaos game and its application

Chaos game is an algorithm that presents the structure of fractal formation. The image produced by the game of chaos is known as a catcher.

One of the methods of fractal drawing is the Function Repeating System (IFS) method. In this way, simply start with a set of X_0 and apply some functions $f_1, f_2, \ldots, f_n$. We define the set x_1 as $X_1 = f_1(X_0) \cup f_2(X_0) \cup \ldots \cup f_n(X_0)$ and so on, $X_2 = f_1(X_1) \cup f_2(X_1) \cup \ldots \cup f_n(X_1)$ and so on. Repeating this to a finite number is the form that we will eventually achieve.

To create a fractal by IFS we follow the following algorithm:

1. Build a set of transformations or mappings.

2. Draw a basic pattern on the page.

3. Apply the modifications to the initial templates and consider this step as the first step.

4. Apply the transforms again to the image obtained in the previous step, ie we combine the transforms with the initial transform.

5. Repeat step 4 again and again.

In most cases, the choice of conversion or starting point is random, so set $\{X; f_1, f_2, \dots, f_n, p_1, p_2, \dots, p_n\}$, where p_i are the probabilities of choosing each of the f_i, as a system of repetitive functions and thereby the desired fractal we draw.

We provide the following example to understand fractal drawing as well as its application.

Example. Suppose an area is besieged by an enemy on three sides. Here we consider points A, B and C as the enemy force. We may at any moment be accidentally threatened or attacked by any of these three points. We want to model these attacks and how we respond and create a safe zone within our region that has the highest security factor. To this end, consider the following pattern.

Points A, B, C have three points on the plate, forming a triangle. We consider an arbitrary point as the starting point, which is the location of our own forces. Because we are threatened on three sides, we throw a three-sided dice that has the letters A, B, C written on each side. If for example A appears, then the point moves halfway to point A toward that point, that is, half way up to point A, advancing toward the enemy. Consider the obtained point as the reference point. Throw the dice again and act as before. That is to say, in mathematical terms, we convert the following transformations to the area.

$$f(x,y) = (\tfrac{1}{2}(\alpha + x_0), \tfrac{1}{2}(\beta + y_0)) \tag{9}$$

Where (x_0, y_0) are the starting point coordinates and (α, β) the coordinates of the threat location. The process of drawing this fractal is shown in Figure 2.

By several steps in the process, in other words, the combination of these transformations is repeated several times, form (3) the desired fractal. This fractal is known as the Serpinski triangle. It is observed that in the middle region of this triangle there are no geometrical lines or objects, and in our view this area is represented as the least threatened security zone.

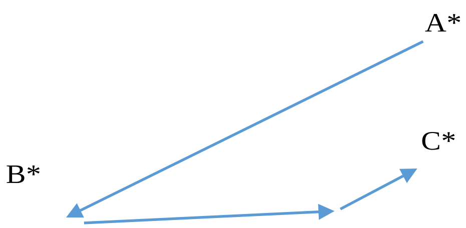

Figure 2.

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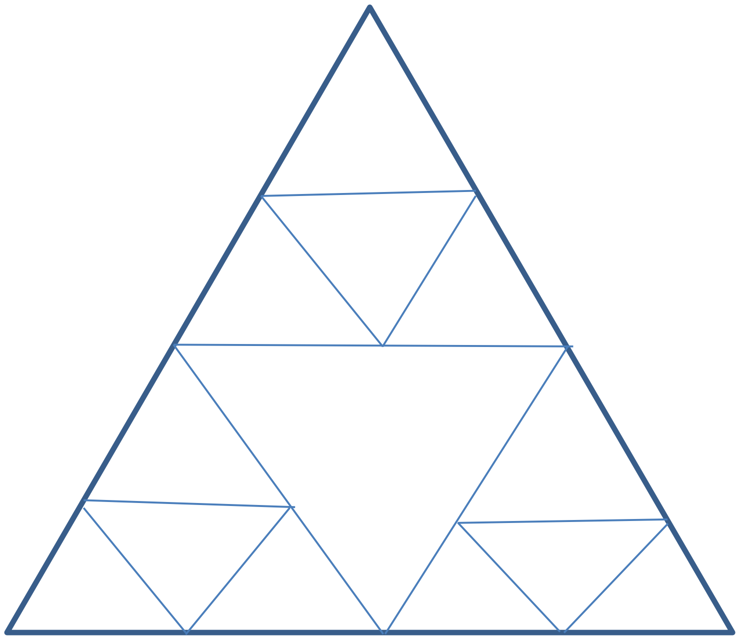

Figure 3.

4. Results

Investigating the structure of computer images of complex objects is a difficult task and a complicated process. Things like trees, plants, mountains and clouds. Conventional geometry is not enough to describe these objects. Mathematicians seek to discover various techniques for modeling and analyzing complex and complex objects and objects. The algorithms presented in this paper are deterministic algorithms and stochastic hybrid algorithms that are introduced on the iteration system. The fundamental feature of the device is the repetitive function of the image produced by its mappings is a fractal. We call this image absorbent. A set of offsets and associated probabilities sets up a duplicate function system.

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