



Performance analysis of multi computer system consisting of active parallel homogeneous clients

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Abstract Reliability is among the performance factors applied to multi computer systems consisting of active parallel hosts (clients) and a central server. For reliability evaluation and system performance, this study analyzed a multi computer system consisting of a three hosts (clients) connected to a central server. The system is configured as series-parallel system consisting of two subsystems A and B. Subsystem A consist of three clients working in parallel with each other while subsystem B consist of a central server. Both clients and server failure and repair time are to be exponentially distributed. The system is analyzed using first order differential difference equations to derive the expressions for availability, mean time to failure, probability of busy period of repairman due partial or complete failure. Reliability characteristics such as availability, MTTF, profit function as well as sensitivity analysis have been discussed. Some particular cases have also been derived and examined to see the practical effect of the model. The computed results are demonstrated by tables and graphs. Impact of both clients and server failure and repair rates on availability, MTTF and profit is determine and presented in tables and graphs. Maximum, median and minimum level of availability, MTTF and profit is also determine through Box and Whiskers plot.

Keywords Multi computer; Client; Server; Reliability analysis; Industrial systems; Availability

1. Introduction

During the past two decades, it is hard to find any society in which computer systems do not play dominant role. Computer systems are often used in most critical areas such as communication, banks, education institutes and defense. Transmission of data or information within the aforementioned examples require the use of multi computer. A multi computer system or simply computer network is defined as a collection of computers interconnected by a single technology. Examples of multi computer systems include client-server architecture, peer to peer, etc. A Client-Server architecture has one or more client and server, where the client can send a requests to any one of the server while the server process the request and respond to the request. Reliability analysis of such multi computer systems is a very meaningful measure, and achieving required level of reliability and availability is an essential requisite. Reliability is an important factor in equipment maintenance because lower equipment reliability means higher maintenance. Maintaining the availability and quality of assets at a required performance level solely depends on this essential measure (reliability). Reliability can be seen as the ability of a system to perform its intended function under stipulated conditions for a specified period of time. In many communication industries, majority of their assets are repairable systems. The nature of these repairable systems can be described in terms of reliability, availability and maintainability. The success of these repairable systems will directly determine the quality of product, the production costs, the service to the customers and the expected profit. Many of these assets include telecommunication networks, thermal power plants and electric generators. Designing a suitable system together with a scientific maintenance planning has make the jobs of maintenance managers and system designers more challenging. For this reason, many researchers have studied reliability problem of redundant systems under different operational situations and circumstances in assessing their reliability characteristics. For example, [Kumar et al. \(2020\)](#) have recently studied the reliability analysis of a redundant system with 'FCFS' repair policy subject to weather conditions. [Chauhan and Malik \(2017\)](#) have introduced the evaluation of reliability and MTSF of a parallel system with Weibull failure laws. [Ibrahim et al. \(2017\)](#) have studied the reliability assessment of complex system consisting two subsystems connected in series configuration using Gumbel-Hougaard family copula distribution. [Kakkar et al. \(2016\)](#) have discussed the reliability analysis of two dissimilar parallel unit repairable system with failure during preventive maintenance. [Niwas and Garg \(2018\)](#) have presented an approach for analyzing the reliability and profit of an industrial system based on the cost-free warranty policy. [Kakkar et al. \(2017\)](#) have examined availability analysis of two parallel unit system under the provision of maintenance. [Garg \(2014\)](#) studied the reliability, availability and maintainability analysis of industrial systems using PSO and fuzzy methodology. [Garg et al. \(2014a\)](#) dealt with bi-objective optimization of the reliability-redundancy allocation problem for series-parallel system. [Garg et al. \(2014b\)](#) Intuitionistic fuzzy optimization technique for solving multi-objective reliability optimization problems in interval environment. [Yusuf et al. \(2018\)](#) have analyzed some reliability characteristics of a linear consecutive 2-out-of-4system connected to 2-out-of-4 supporting device for operation. [Zhao et al. \(2020\)](#) have discussed the reliability analysis of aero-engine

compressor rotor system considering cruise characteristics. [Yang et al. \(2019\)](#) have studied the reliability and availability analysis of standby systems with working vacations and retrieval of failed components. [Temraz \(2019\)](#) analyzed availability and reliability of a parallel system under imperfect repair and replacement: analysis and cost optimization. [Singh and Ayagi \(2017\)](#) have presented the study of reliability measures of system consisting of two subsystems in configuration using copula. [Kakkar et al. \(2015\)](#) have studied the reliability analysis of two-unit parallel repairable industrial system.

To improve system reliability of multi computer systems, some researchers have proposed models to represent the performance of flow networks and proclaimed better performance by their operations. For instance, [Zhang \(2019\)](#) analysed the reliability of computer network based on intelligent cloud computing method. [Yusuf et al. \(2020\)](#) discussed the performance of multi-computer system consisting of three subsystems in series configuration using copula repair policy. [Yakubu et al. \(2017\)](#) presented a modified math client server application for e-learning. [Ahlawat and Anand \(2014\)](#) dealt with introduction of computer networking. [Fong and Hui \(1999\)](#) Presented an application of middleware in the three tier client/server database. [Garg \(2019\)](#) introduced an approach in resolving heterogeneity using RPC in client server systems. [Kirubanand and Palaniammal \(2010\)](#) dealt with performance modelling in client server network comparison of hub, switch and Bluetooth technology using Markov algorithm. [Kovalev et al. \(2015\)](#) dealt with modelling of reliability of the distributed computer system with architecture client-server. [Dilawar and Syed \(2014\)](#) presented Mathematical modelling and analysis of Network service failure in data centre. [Minkevičius and Kulvietis \(2011\)](#) Investigates the Reliability of multi-server computer networks. Mohan (2013) discussed the network, analysis and remote application control software based on client-server architecture. [Oluwatosin \(2014\)](#) dealt with the modelling of client-server. Pradeep and [Singh \(2010\)](#) studied software reliability growth model for three-tier client server system. [Potapov et al. \(2019\)](#) dealt with reliability modelling of an information system with client server architecture.

In this paper, we study the reliability analysis of repairable multi computer systems consisting of three clients in active parallel and a central server. The computer is considered as a system with two subsystems A and B in which subsystem A has three identical clients in parallel while subsystem B has only a central server. The primary contribution of the work is divided into three folds as follows:

- ✓ To developed the expressions for availability, MTTF, busy period probability due to partial and complete failure as well as profit function.
- ✓ To determine the impact of both clients and server failure and repair rates on availability, MTTF and profit.
- ✓ To determine the maximum, median and minimum value of availability, MTTF and profit through Box and Whiskers plot.

The rest of the paper is organized as follows. Section 2 gives the notations and assumptions used throughout the study. Section 3 presents the description and states of

the system. Section 4 deals with the models' formulation. The results of our numerical simulations and discussion are provided in section 5 and the paper is concluded in section 6.

2. Notations and Assumptions

Notations

μ_1 = Repair rate of clients

λ_1 = Failure rate of clients

μ_0 = Repair rate of server

λ_0 = Failure rate of server

$P(t)$ = Probability row

$p_j(t)$ = Probability that the system is in state j , for $j=0,1,2,3,4,5,6$

A_v = Availability of the system

B_{p1} = Busy period of repairman due to repair of partial failure

B_{p2} = Busy period of repairman due to repair of complete failure

P_f : Profit function

C_0 : Total revenue generated

C_1 : Cost due to partial failure

C_2 : Cost due to complete failure

Assumptions

System under study consist of three identical clients in active all connected to one server. At initially state, both clients and server are in perfect state. It is assumed that clients and server failures are repairable. It is also assumed that failure times and repair times of clients and server are assumed exponentially distributed. Each of the client failed independent from others and the server. Three or two clients cannot fail simultaneously.

3. Description and states of the system

The architecture of a multi computer system in this paper has two subsystems A and B connected in series-parallel arrangement where subsystem A has three active parallel clients while subsystem B has one server. The three clients are connected to the server. The clients send a request individually to the server and the server respond to the request by each clients. Each client fails on its own with exponential failure distribution of λ_1 parameter and repair rate of μ_1 . At the failure of one or two clients, the system works in reduced capacity while the remaining client continuing working. This called partial failure. System failure occur when the three clients have failed or at the failure of the server with exponential failure distribution of λ_0 parameter and repair rate of μ_0 . The Table 1 below presents the states status of the model.

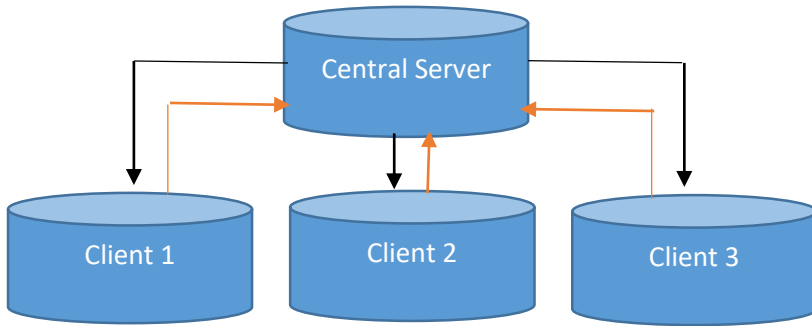


Figure 1. Reliability block diagram of the system.

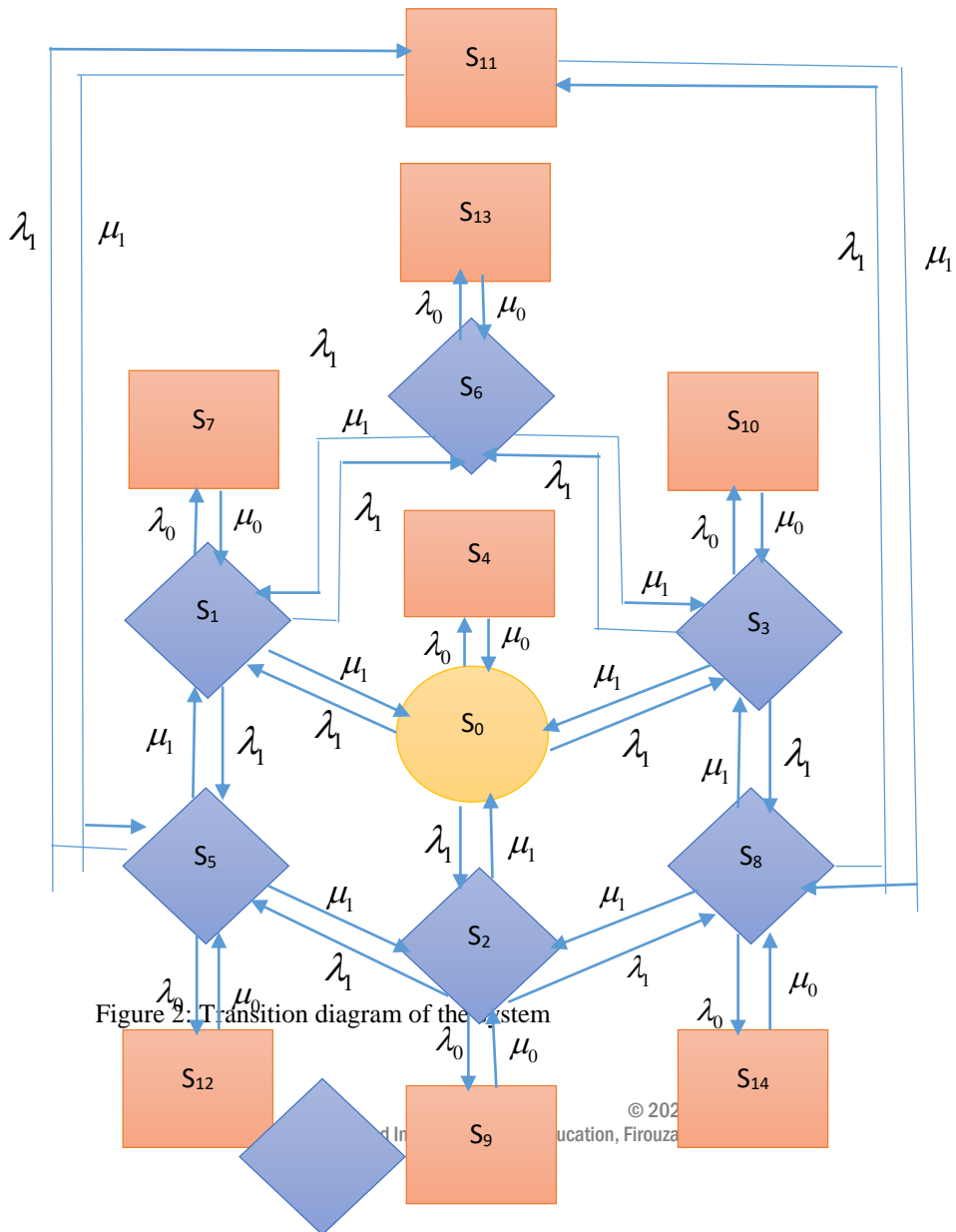


Figure 2: Transition diagram of the system

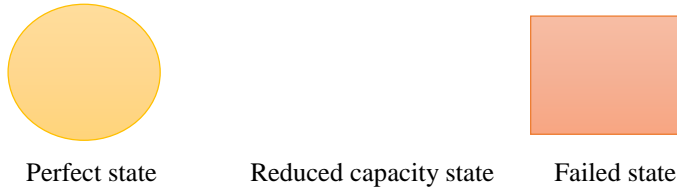


Figure 2. Transition diagram of the system.

Table 1. Description of the states of the system.

State	Description of the state
S ₀	Initial state, clients and servers are working. The system is operational.
S ₁	First client has failed and is under repair, the rest of the client and the server are working. The system is operational.
S ₂	Second client has failed and is under repair, the rest of the client and the server are working. The system is operational.
S ₃	Third client has failed and is under repair, the rest of the client and the server are working. The system is operational.
S ₄	The server has failed and is under repair. The system is down
S ₅	Client 1 and 2 have failed and are under repair, Client 3 and the server are working. The system is operational.
S ₆	Client 1 and 3 have failed and are under repair, Client 2 and the server are working. The system is operational.
S ₇	Client 1 and the server have failed and are under repair. The system is down.
S ₈	Client 2 and 3 have failed and are under repair, Client 1 and the server are working. The system is operational.
S ₉	Client 2 and the server have failed and are under repair. The system is down.
S ₁₀	Client 3 and the server have failed and are under repair. The system is down.
S ₁₁	Client 1, 2 and 3 have failed and are under repair. The system is down.
S ₁₂	Client 1, 2 and the server have failed and are under repair. The system is down.
S ₁₃	Client 1 and 3 and the server have failed and are under repair. The system is down.
S ₁₄	Client 2, 3 and the server have failed and are under repair. The system is down.

4. Model formulation

From Figure 2, the following set of differential difference equations are as follows:

$$\left(\frac{d}{dt} + y_0\right) p_0(t) = \mu_1 p_1(t) + \mu_1 p_2(t) + \mu_1 p_3(t) + \mu_0 p_4(t) \quad (1)$$

$$\left(\frac{d}{dt} + y_1\right) p_1(t) = \lambda_1 p_0(t) + \mu_1 p_5(t) + \mu_1 p_6(t) + \mu_0 p_7(t) \quad (2)$$

$$\left(\frac{d}{dt} + y_1\right) p_2(t) = \lambda_1 p_0(t) + \mu_1 p_5(t) + \mu_1 p_8(t) + \mu_0 p_9(t) \quad (3)$$

$$\left(\frac{d}{dt} + y_1\right) p_3(t) = \lambda_1 p_0(t) + \mu_1 p_6(t) + \mu_1 p_8(t) + \mu_0 p_{10}(t) \quad (4)$$

$$\left(\frac{d}{dt} + \mu_0\right) p_4(t) = \lambda_0 p_0(t) \tag{5}$$

$$\left(\frac{d}{dt} + y_2\right) p_5(t) = \lambda_1 p_1(t) + \mu_1 p_2(t) + \mu_1 p_{11}(t) + \mu_0 p_{12}(t) \tag{6}$$

$$\left(\frac{d}{dt} + y_2\right) p_6(t) = \lambda_1 p_1(t) + \mu_1 p_3(t) + \mu_1 p_{11}(t) + \mu_0 p_{13}(t) \tag{7}$$

$$\left(\frac{d}{dt} + \mu_0\right) p_7(t) = \lambda_0 p_1(t) \tag{8}$$

$$\left(\frac{d}{dt} + y_2\right) p_8(t) = \lambda_1 p_2(t) + \mu_1 p_3(t) + \mu_1 p_{11}(t) + \mu_0 p_{14}(t) \tag{9}$$

$$\left(\frac{d}{dt} + \mu_0\right) p_9(t) = \lambda_0 p_2(t) \tag{10}$$

$$\left(\frac{d}{dt} + \mu_0\right) p_{10}(t) = \lambda_0 p_3(t) \tag{11}$$

$$\left(\frac{d}{dt} + 3\mu_1\right) p_{11}(t) = \lambda_1 p_5(t) + \lambda_1 p_6(t) + \lambda_1 p_8(t) \tag{12}$$

$$\left(\frac{d}{dt} + \mu_0\right) p_{12}(t) = \lambda_0 p_5(t) \tag{13}$$

$$\left(\frac{d}{dt} + \mu_0\right) p_{13}(t) = \lambda_0 p_6(t) \tag{14}$$

$$\left(\frac{d}{dt} + \mu_0\right) p_{14}(t) = \lambda_0 p_8(t) \tag{15}$$

(1) to (15) are expressed below in the format $\frac{d}{dt} Q(t) = TP(t)$ (16)

where $y_0 = (\lambda_0 + 3\lambda_1)$, $y_1 = (\lambda_0 + 2\lambda_1 + \mu_1)$, $y_2 = (\lambda_0 + \lambda_1 + 2\mu_1)$ and

$$T = \begin{pmatrix} -y_0 & \mu_1 & \mu_1 & \mu_1 & \mu_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_1 & -y_1 & 0 & 0 & 0 & \mu_1 & \mu_1 & \mu_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_1 & 0 & -y_1 & 0 & 0 & \mu_1 & 0 & 0 & \mu_1 & \mu_0 & 0 & 0 & 0 & 0 \\ \lambda_1 & 0 & 0 & -y_1 & 0 & 0 & \mu_1 & 0 & \mu_1 & 0 & \mu_0 & 0 & 0 & 0 \\ \lambda_0 & 0 & 0 & 0 & -\mu_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_1 & \lambda_1 & 0 & 0 & -y_2 & 0 & 0 & 0 & 0 & \mu_1 & \mu_0 & 0 & 0 \\ 0 & \lambda_1 & 0 & \lambda_1 & 0 & 0 & -y_2 & 0 & 0 & 0 & \mu_1 & 0 & \mu_0 & 0 \\ 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_1 & \lambda_1 & 0 & 0 & 0 & 0 & -y_2 & 0 & \mu_1 & 0 & 0 & \mu_0 \\ 0 & 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_1 & \lambda_1 & 0 & \lambda_1 & 0 & 0 & -3\mu_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_0 & 0 & 0 & 0 & 0 & -\mu_0 \end{pmatrix}$$

The MTTF is computed by taking the transpose matrix of T and delete the rows and columns for the failed state and designation the new matrix by M and applying

$$MTTF = P(0)(-M^{-1})[1,1,1,1,1,1]^T = \frac{\lambda_0^2 + 6\lambda_0\lambda_1 + 3\lambda_0\mu_1 + 11\lambda_1^2 + 7\lambda_1\mu_1 + 2\mu_1^2}{\lambda_0^3 + 6\lambda_0^2\lambda_1 + 3\lambda_0^2\mu_1 + 11\lambda_0\lambda_1^2 + 7\lambda_0\lambda_1\mu_1 + 2\lambda_0\mu_1^2 + 6\lambda_1^3} \quad (17)$$

where

$$M = \begin{pmatrix} -y_0 & \lambda_1 & \lambda_1 & \lambda_1 & 0 & 0 & 0 \\ \mu_1 & -y_1 & 0 & 0 & \lambda_1 & \lambda_1 & 0 \\ \mu_1 & 0 & -y_1 & 0 & \lambda_1 & 0 & \lambda_1 \\ \mu_1 & 0 & 0 & -y_1 & 0 & \lambda_1 & \lambda_1 \\ 0 & \mu_1 & \mu_1 & 0 & -y_2 & 0 & 0 \\ 0 & \mu_1 & 0 & \mu_1 & 0 & -y_2 & 0 \\ 0 & 0 & \mu_1 & \mu_1 & 0 & 0 & -y_2 \end{pmatrix}$$

(16) is now

$$\begin{pmatrix} p'_0 \\ p'_1 \\ p'_2 \\ p'_3 \\ p'_4 \\ p'_5 \\ p'_6 \\ p'_7 \\ p'_8 \\ p'_9 \\ p'_{10} \\ p'_{11} \\ p'_{12} \\ p'_{13} \\ p'_{14} \end{pmatrix} = \begin{pmatrix} -y_0 & \mu_1 & \mu_1 & \mu_1 & \mu_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_1 & -y_1 & 0 & 0 & 0 & \mu_1 & \mu_1 & \mu_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_1 & 0 & -y_1 & 0 & 0 & \mu_1 & 0 & 0 & \mu_1 & \mu_0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_1 & 0 & 0 & -y_1 & 0 & 0 & \mu_1 & 0 & \mu_1 & 0 & \mu_0 & 0 & 0 & 0 & 0 \\ \lambda_0 & 0 & 0 & 0 & -\mu_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_1 & \lambda_1 & 0 & 0 & -y_2 & 0 & 0 & 0 & 0 & \mu_1 & \mu_0 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & \lambda_1 & 0 & 0 & -y_2 & 0 & 0 & 0 & \mu_1 & 0 & \mu_0 & 0 & 0 \\ 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_1 & \lambda_1 & 0 & 0 & 0 & 0 & -y_2 & 0 & \mu_1 & 0 & 0 & \mu_0 & 0 \\ 0 & 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_1 & \lambda_1 & 0 & \lambda_1 & 0 & 0 & -3\mu_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 \end{pmatrix} \begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \\ p_9 \\ p_{10} \\ p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \end{pmatrix} \quad (18)$$

Setting the (18) to zero in steady state to give:

$$\begin{pmatrix}
 -y_0 & \mu_1 & \mu_1 & \mu_1 & \mu_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 \lambda_1 & -y_1 & 0 & 0 & 0 & \mu_1 & \mu_1 & \mu_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 \lambda_1 & 0 & -y_1 & 0 & 0 & \mu_1 & 0 & 0 & \mu_1 & \mu_0 & 0 & 0 & 0 & 0 & 0 \\
 \lambda_1 & 0 & 0 & -y_1 & 0 & 0 & \mu_1 & 0 & \mu_1 & 0 & \mu_0 & 0 & 0 & 0 & 0 \\
 \lambda_0 & 0 & 0 & 0 & -\mu_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & \lambda_1 & \lambda_1 & 0 & 0 & -y_2 & 0 & 0 & 0 & 0 & \mu_1 & \mu_0 & 0 & 0 & 0 \\
 0 & \lambda_1 & 0 & \lambda_1 & 0 & 0 & -y_2 & 0 & 0 & 0 & \mu_1 & 0 & \mu_0 & 0 & 0 \\
 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & \lambda_1 & \lambda_1 & 0 & 0 & 0 & 0 & -y_2 & 0 & \mu_1 & 0 & 0 & \mu_0 & 0 \\
 0 & 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & \lambda_1 & \lambda_1 & 0 & \lambda_1 & 0 & 0 & -3\mu_1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & -\mu_0
 \end{pmatrix}
 \begin{pmatrix}
 p_0(\infty) \\
 p_1(\infty) \\
 p_2(\infty) \\
 p_3(\infty) \\
 p_4(\infty) \\
 p_5(\infty) \\
 p_6(\infty) \\
 p_7(\infty) \\
 p_8(\infty) \\
 p_9(\infty) \\
 p_{10}(\infty) \\
 p_{11}(\infty) \\
 p_{12}(\infty) \\
 p_{13}(\infty) \\
 p_{14}(\infty)
 \end{pmatrix}
 = 0 \quad (19)$$

The steady-state availability, busy period probability due repair of partial and complete failure are given by

$$A_v(\infty) = p_0(\infty) + p_1(\infty) + p_2(\infty) + p_3(\infty) + p_5(\infty) + p_6(\infty) + p_8(\infty) \quad (20)$$

$$B_{P1}(\infty) = p_1(\infty) + p_2(\infty) + p_3(\infty) + p_5(\infty) + p_6(\infty) + p_8(\infty) \quad (21)$$

$$B_{P2}(\infty) = p_4(\infty) + p_7(\infty) + p_9(\infty) + p_{10}(\infty) + p_{11}(\infty) + p_{12}(\infty) + p_{13}(\infty) + p_{14}(\infty) \quad (22)$$

Combining (19) and normalizing condition

$$\sum_{j=0}^{14} p_j(\infty) = 1 \quad (23)$$

To produce

$$\begin{pmatrix}
 -y_0 & \mu_1 & \mu_1 & \mu_1 & \mu_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 \lambda_1 & -y_1 & 0 & 0 & 0 & \mu_1 & \mu_1 & \mu_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 \lambda_1 & 0 & -y_1 & 0 & 0 & \mu_1 & 0 & 0 & \mu_1 & \mu_0 & 0 & 0 & 0 & 0 & 0 \\
 \lambda_1 & 0 & 0 & -y_1 & 0 & 0 & \mu_1 & 0 & \mu_1 & 0 & \mu_0 & 0 & 0 & 0 & 0 \\
 \lambda_0 & 0 & 0 & 0 & -\mu_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & \lambda_1 & \lambda_1 & 0 & 0 & -y_2 & 0 & 0 & 0 & 0 & 0 & \mu_1 & \mu_0 & 0 & 0 \\
 0 & \lambda_1 & 0 & \lambda_1 & 0 & 0 & -y_2 & 0 & 0 & 0 & 0 & \mu_1 & 0 & \mu_0 & 0 \\
 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & \lambda_1 & \lambda_1 & 0 & 0 & 0 & 0 & -y_2 & 0 & 0 & \mu_1 & 0 & 0 & \mu_0 \\
 0 & 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & \lambda_1 & \lambda_1 & 0 & \lambda_1 & 0 & 0 & -3\mu_1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 & 0 \\
 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
 \end{pmatrix}
 \begin{pmatrix}
 p_0(\infty) \\
 p_1(\infty) \\
 p_2(\infty) \\
 p_3(\infty) \\
 p_4(\infty) \\
 p_5(\infty) \\
 p_6(\infty) \\
 p_7(\infty) \\
 p_8(\infty) \\
 p_9(\infty) \\
 p_{10}(\infty) \\
 p_{11}(\infty) \\
 p_{12}(\infty) \\
 p_{13}(\infty) \\
 p_{14}(\infty)
 \end{pmatrix}
 = 0 \quad (24)$$

To obtain the states probabilities $p_j(t)$, (24) is solved using MATLAB package

Explicit expressions for (20) to (22) are as follows:

$$A_V(\infty) = \frac{\mu_0\mu_1^3 + 3\mu_0\mu_1^2 + 3\mu_0\mu_1\lambda_1^2}{(\lambda_0 + \mu_0)(\mu_1^3 + 3\mu_1^2\lambda_1 + 3\mu_1\lambda_1^2) + \mu_0\lambda_1^3} \quad (25)$$

$$B_{P1}(\infty) = \frac{3\lambda_1\mu_0\mu_1^2 + 3\lambda_1^2\mu_0\mu_1 + 3\lambda_0\lambda_1\mu_1^2 + \lambda_1^3\mu_0 + 3\lambda_0\lambda_1^2\mu_1}{(\lambda_0 + \mu_0)(\mu_1^3 + 3\mu_1^2\lambda_1 + 3\mu_1\lambda_1^2) + \mu_0\lambda_1^3} \quad (26)$$

$$B_{P2}(\infty) = \frac{\lambda_0\mu_1^3 + 3\lambda_0\lambda_1\mu_1^2 + \lambda_1^3\mu_0 + 3\lambda_0\lambda_1^2\mu_1}{(\lambda_0 + \mu_0)(\mu_1^3 + 3\mu_1^2\lambda_1 + 3\mu_1\lambda_1^2) + \mu_0\lambda_1^3} \quad (27)$$

and the profit generated is given by

$$P_F(\infty) = C_0A_V(\infty) - C_1B_{P1}(\infty) - C_2B_{P2}(\infty) \quad (28)$$

5. Results and Discussion

In this section, numerical examples are presented to validate the expressions using MATLAB package. The following parameter values are used in the simulations:

$$\mu_0 = 0.8, \mu_1 = 0.8, \lambda_0 = 0.1 \text{ and } \lambda_1 = 0.1$$

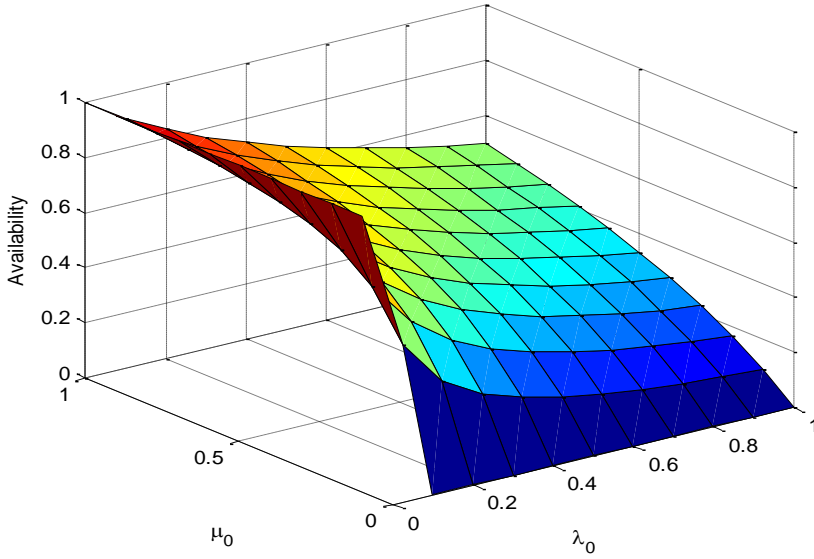


Figure 3a. Availability against μ_0 and λ_0 .

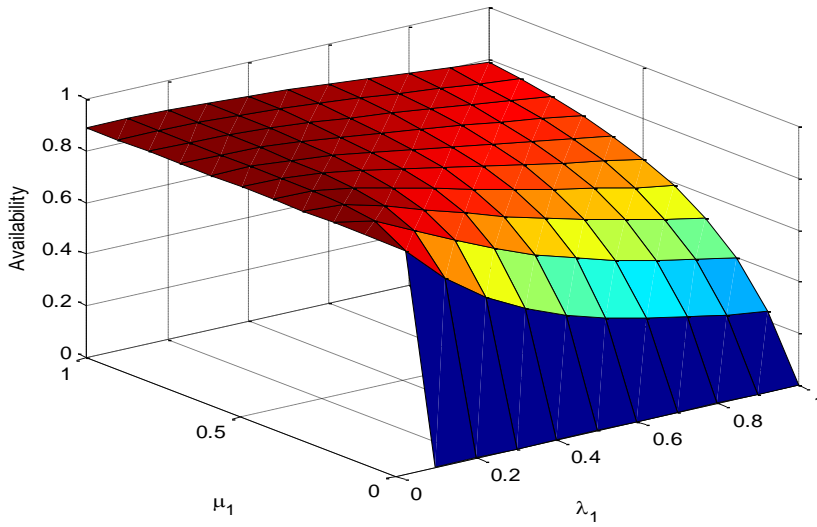
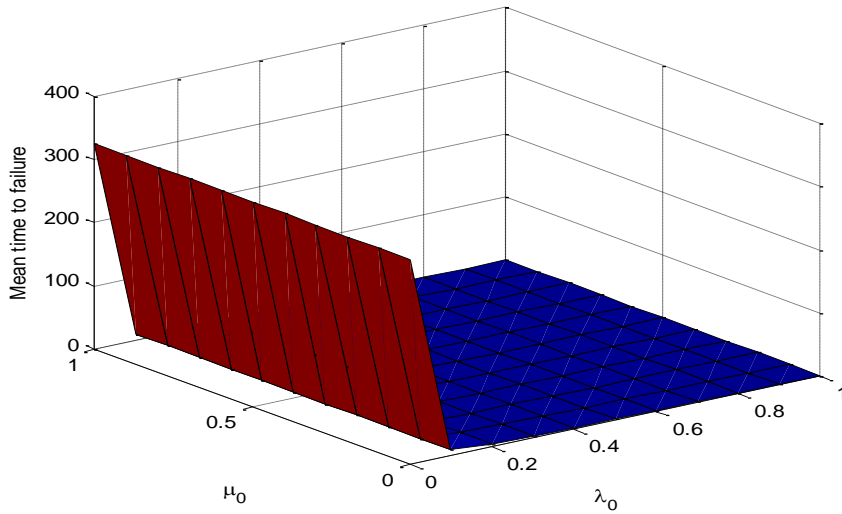
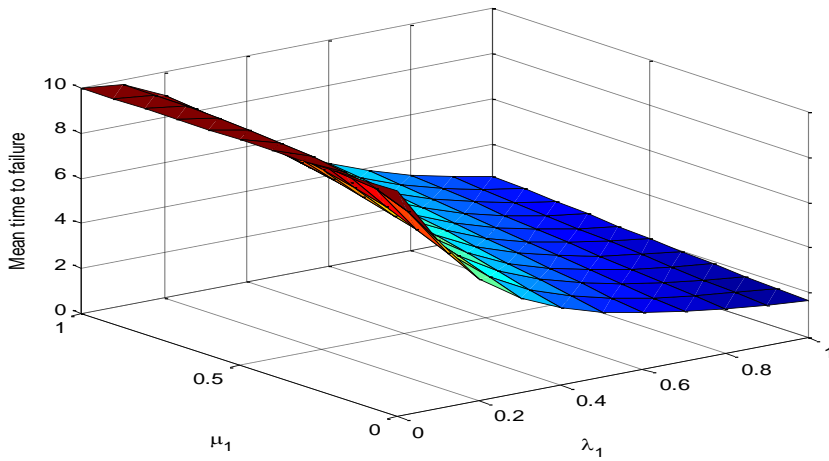
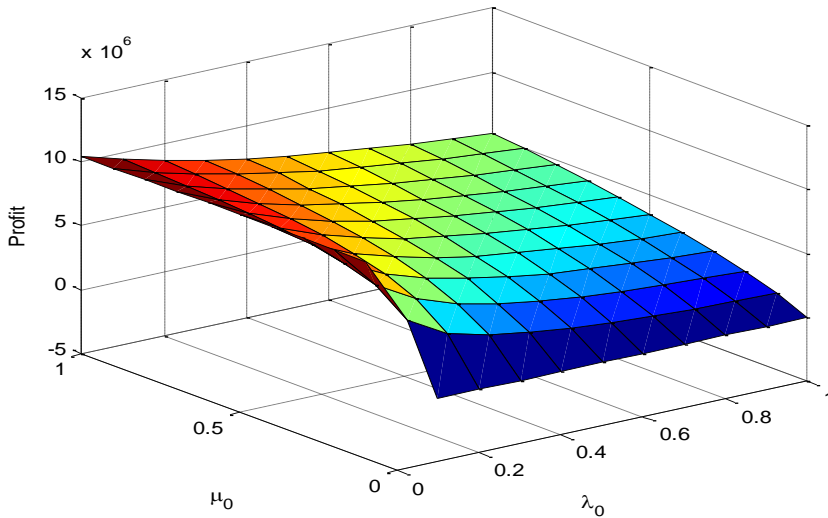
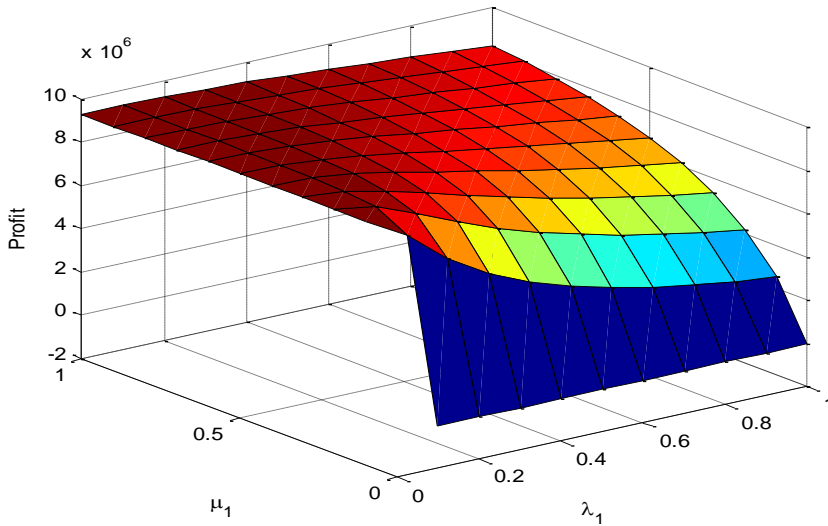


Figure 3b. Availability against μ_1 and λ_1 .

Figure 4a. Mean time to failure against μ_0 and λ_0 .Figure 4b. Mean time to failure against μ_1 and λ_1 .

Figure 5a. Profit against μ_0 and λ_0 .Figure 5b. Profit against μ_1 and λ_1 .

From the surface plots depicted in Figure 3a, 4a and 5a, by fixing μ_1 and λ_1 , availability, mean time to failure and profit decreases as the failure rate λ_0 increases whereas availability, mean time to failure and profit increases as repair rate μ_0 increases. From these figures, it is clear that availability, mean time to failure and profit of the system is higher for higher values of μ_0 and lower values of λ_0 . From these plots it can be observe

that reducing the occurrence of server failure will lead to higher values of availability, profit as well as the expected life time of the system. On the other hand, simulations depicted in Figures 3b, 4b and 5b have shown that availability, mean time to failure and profit increases as repair rate μ_1 is increasing from 0 to 1 and decreases as λ_1 increases from 0 to 1. This indicates that adequate maintenance action to clients such as inspection, perfect repair or replacement should be practice to enhance the availability, mean time to failure and profit.

In this section, sensitivity analyses are presented in the tables below. The following parameter values are used for Tables 2 and 3:

$\lambda_1 = 0.1, \mu_1 = 0.8, C_0 = 10,500,000, C_1 = 1150, C_2 = 1950$ and $\lambda_0 = 0.1, \mu_0 = 0.8, C_0 = 10,500,000, C_1 = 1150, C_2 = 1950$ for Tables 4 and 5.

Table 2. Sensitivity analysis with respect to λ_0 for of $\mu_0 \in [0.3:0.3:0.9]$.

λ_0	Availability			MTTF			Profit*10 ⁷		
	μ_0			μ_0			μ_0		
	0.3	0.6	0.9	0.3	0.6	0.9	0.3	0.6	0.9
0	0.9986	0.9986	0.9986	325	325	325	1.0458	1.0458	1.0458
0.1	0.7492	0.8561	0.8989	9.7414	9.7414	9.7414	0.7839	0.8961	0.9410
0.2	0.5995	0.7492	0.8173	4.9427	4.4927	4.4927	0.6266	0.7839	0.8553
0.3	0.4997	0.6661	0.7492	3.3108	3.3108	3.3108	0.5218	0.6965	0.7839
0.4	0.4283	0.5995	0.6916	2.4887	2.4887	2.4887	0.4469	0.6266	0.7234
0.5	0.3748	0.5450	0.6423	1.9935	1.9935	1.9935	0.3907	0.5694	0.6716
0.6	0.3332	0.4997	0.5995	1.6626	1.6626	1.6626	0.3469	0.5218	0.6266
0.7	0.2999	0.4612	0.5621	1.4259	1.4259	1.4259	0.3120	0.4814	0.5873
0.8	0.2726	0.4283	0.5290	1.2481	1.2481	1.2481	0.2834	0.4469	0.5526
0.9	0.2499	0.3998	0.4997	1.1098	1.1098	1.1098	0.2595	0.4169	0.5218

Table 3. Sensitivity analysis with respect to μ_0 for $\lambda_0 \in [0.1:0.3:0.7]$

μ_0	Availability			MTTF			Profit*10 ⁶		
	λ_0			λ_0			λ_0		
	0.1	0.4	0.7	0.1	0.4	0.7	0.1	0.4	0.7
0	0	0	0	9.7414	2.4887	1.4259	-0.0296	-0.0296	-0.0296
0.1	0.4997	0.1999	0.1250	9.7414	2.4887	1.4259	5.2178	2.0702	1.2829
0.2	0.6661	0.3332	0.2222	9.7414	2.4887	1.4259	6.9653	3.4695	2.3035
0.3	0.7492	0.4283	0.2999	9.7414	2.4887	1.4259	7.8388	4.4686	3.1197
0.4	0.7991	0.4997	0.3635	9.7414	2.4887	1.4259	8.3627	5.2178	3.7874
0.5	0.8324	0.5551	0.4164	9.7414	2.4887	1.4259	8.7120	5.8004	4.3437
0.6	0.8561	0.5995	0.4612	9.7414	2.4887	1.4259	8.9615	6.2664	4.8144
0.7	0.8739	0.6358	0.4997	9.7414	2.4887	1.4259	9.1486	6.6476	5.2178
0.8	0.9978	0.6661	0.5329	9.7414	2.4887	1.4259	9.2941	6.9653	5.5674
0.9	0.8989	0.6916	0.5621	9.7414	2.4887	1.4259	9.4105	7.2342	5.8732

Table 4. Sensitivity analysis with respect to λ_1 for $\mu_1 \in [0.2:0.3:0.8]$.

λ_1	Availability			MTTF			Profit*10 ⁶		
	μ_1			μ_1			μ_1		
	0.2	0.5	0.8	0.2	0.5	0.8	0.2	0.5	0.8
0	0.8889	0.8889	0.8889	10.0000	10.0000	10.0000	9.3058	9.3058	9.3058
0.1	0.8595	0.8852	0.8878	8.8462	9.5161	9.7414	8.9964	9.2669	9.2941
0.2	0.7887	0.8704	0.8826	6.7347	8.0000	8.6992	8.2530	9.1111	9.2388
0.3	0.7140	0.8470	0.8728	5.1786	6.4000	7.3000	7.4684	8.8648	9.1364
0.4	0.6468	0.8188	0.8595	4.1374	5.1392	6.0042	6.7624	8.5692	8.9964
0.5	0.5888	0.7887	0.8436	3.4211	4.2130	4.9597	6.1531	8.2530	8.8298
0.6	0.5392	0.7583	0.8261	2.9064	3.5329	4.1543	5.6319	7.9335	8.6460
0.7	0.4966	0.7285	0.8077	2.5218	3.0237	3.5364	5.1862	7.6206	8.4519
0.8	0.4599	0.6999	0.7887	2.2248	2.6331	3.0576	4.7999	7.3196	8.2530
0.9	0.4281	0.6726	0.7697	1.9890	2.3263	2.6807	4.4654	7.0332	8.0529

Table 5. Sensitivity analysis with respect to μ_1 for of $\lambda_1 \in [0.3:0.2:0.7]$.

μ_1	Availability			MTTF			Profit*10 ⁶		
	λ_1			λ_1			λ_1		
	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
0	0	0	0	4.2143	2.8977	2.2045	-0.0304	-0.0304	-0.0304
0.1	0.5392	0.4002	0.3170	4.7059	3.1569	2.3608	5.6319	4.1727	3.2987
0.2	0.7140	0.5888	0.4966	5.1786	3.4211	2.5218	7.4684	6.1531	5.1852
0.3	0.7887	0.6906	0.6071	5.6216	3.6869	2.6866	8.2530	7.2224	6.3458
0.4	0.8261	0.7508	0.6793	6.0294	3.9516	2.8542	8.6460	7.8544	7.1034
0.5	0.8470	0.7887	0.7285	6.4000	4.2130	3.0237	8.6448	8.2530	7.6206
0.6	0.8595	0.8139	0.7634	6.7339	4.4690	3.1944	8.9964	8.5174	7.9865
0.7	0.8675	0.8313	0.7887	7.0330	4.7183	3.3656	9.0804	8.6998	8.2530
0.8	0.8728	0.8436	0.8077	7.3000	4.9597	3.5364	9.1364	8.8298	8.4519
0.9	0.8765	0.8527	0.8221	7.5380	5.1923	3.7064	9.1753	8.9251	8.6035

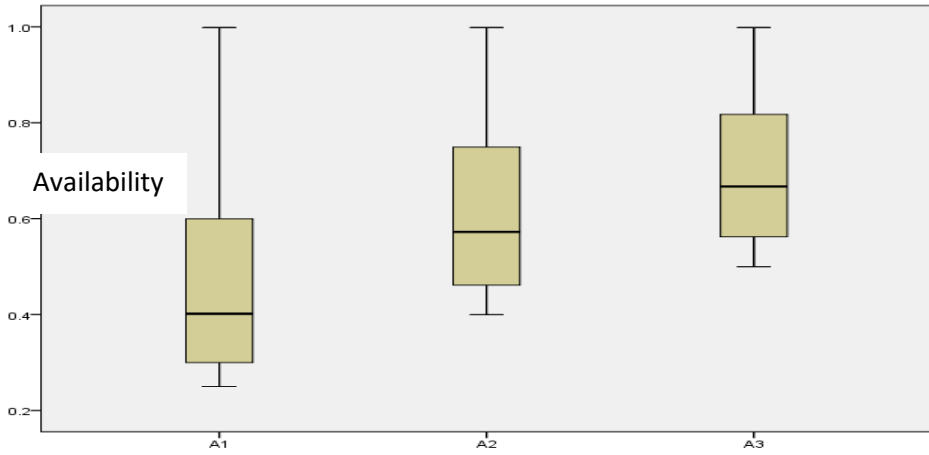


Figure 6a. Box plot for availability when $\mu_0 \in [0.3:0.3:0.9]$.

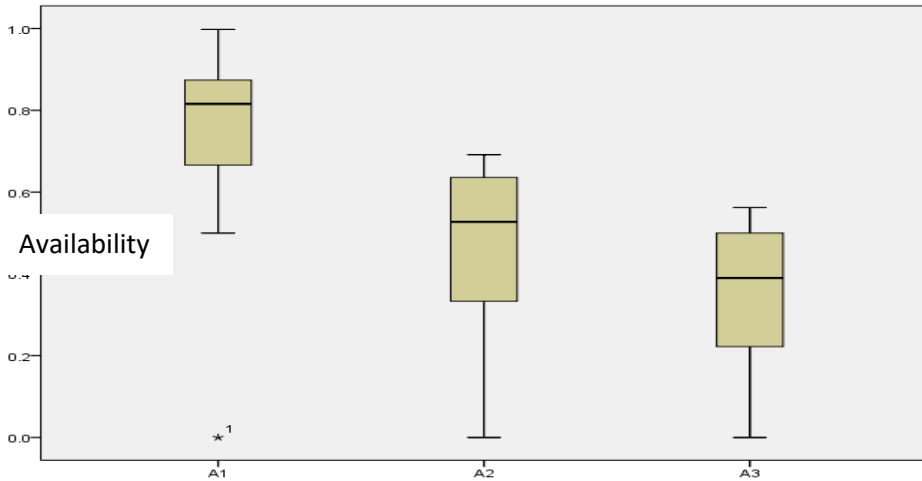


Figure 6b. Box plot for availability for $\lambda_0 \in [0.1:0.3:0.7]$.

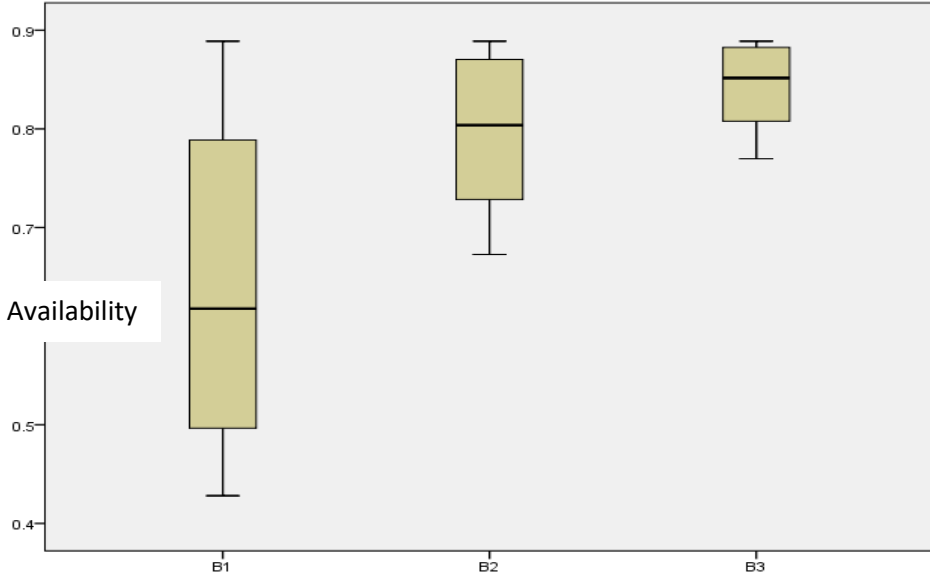


Figure 6c. Box plot for Availability for $\mu_1 \in [0.2:0.3:0.8]$.

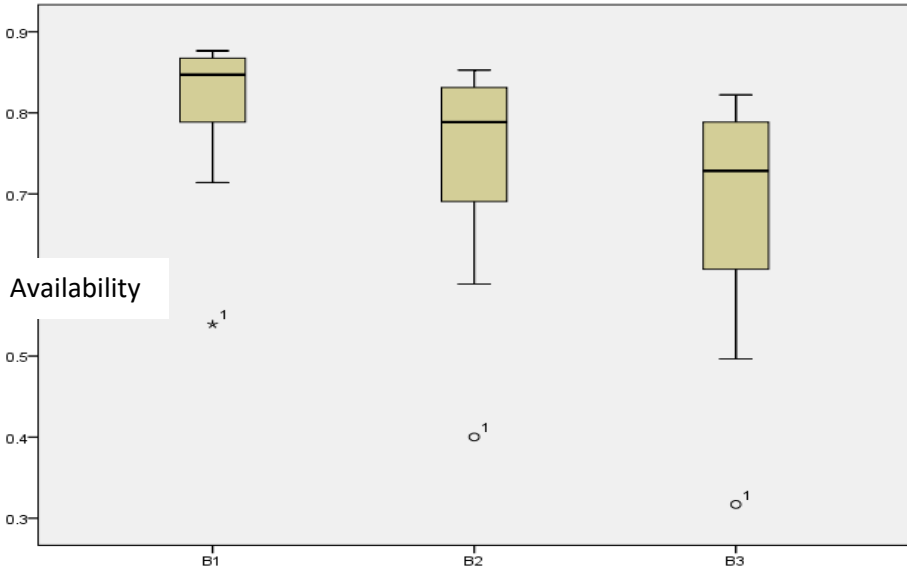


Figure 6d. Box plot for Availability for $\lambda_1 \in [0.3:0.2:0.7]$.

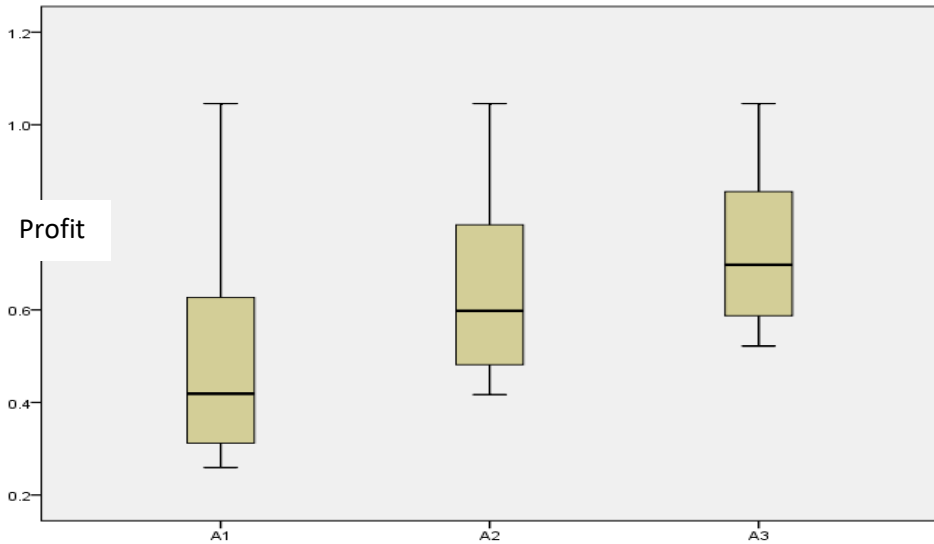


Figure 7a. Box plot for Profit when $\mu_0 \in [0.3:0.3:0.9]$.

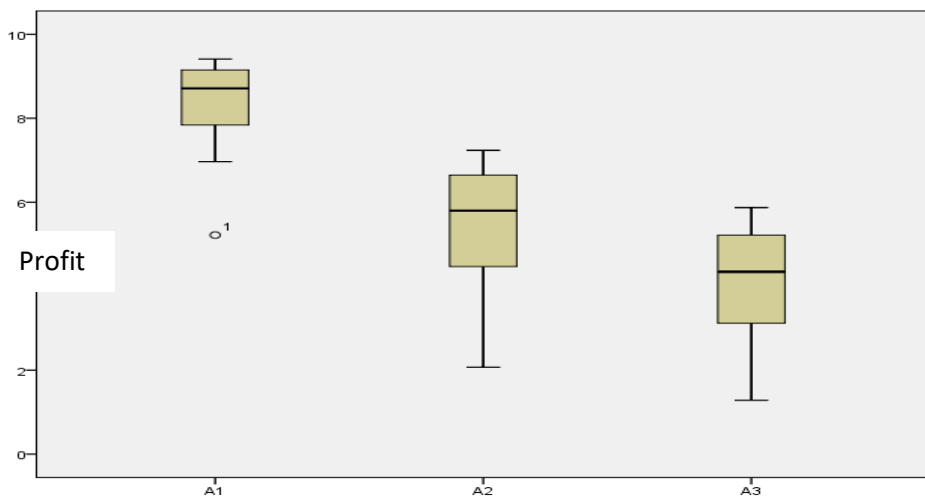


Figure 7b. Box plot for Profit for $\lambda_0 \in [0.1:0.3:0.7]$.

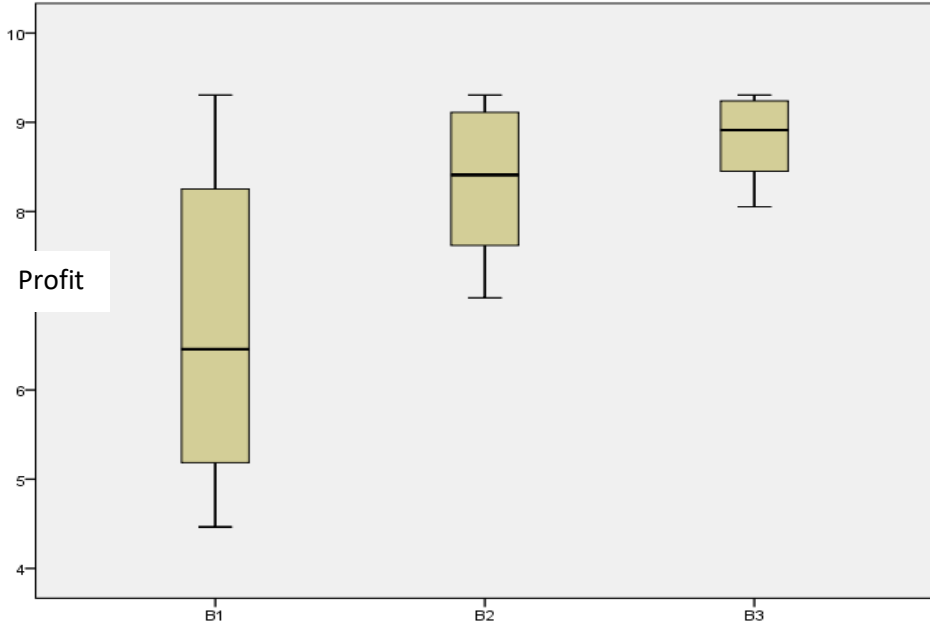


Figure 7c. Box plot for Profit for $\mu_1 \in [0.2:0.3:0.8]$.

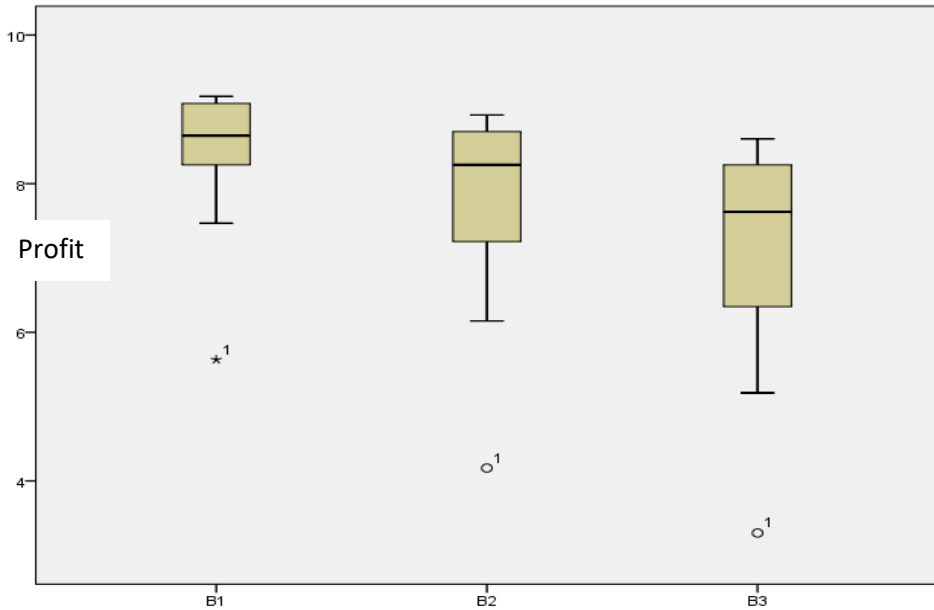


Figure 7d. Box plot for Profit μ_1 for $\lambda_1 \in [0.3:0.2:0.7]$.

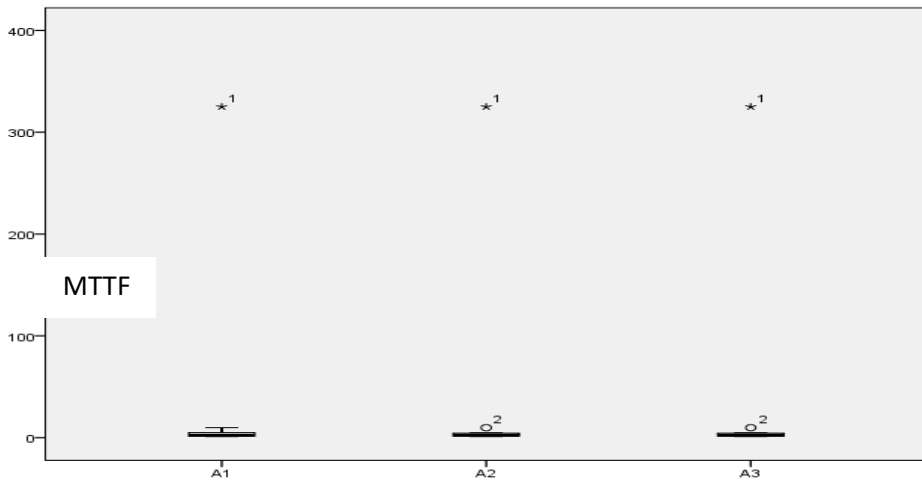


Figure 8a. Box plot for MTTF for $\mu_0 \in [0.3:0.3:0.9]$.

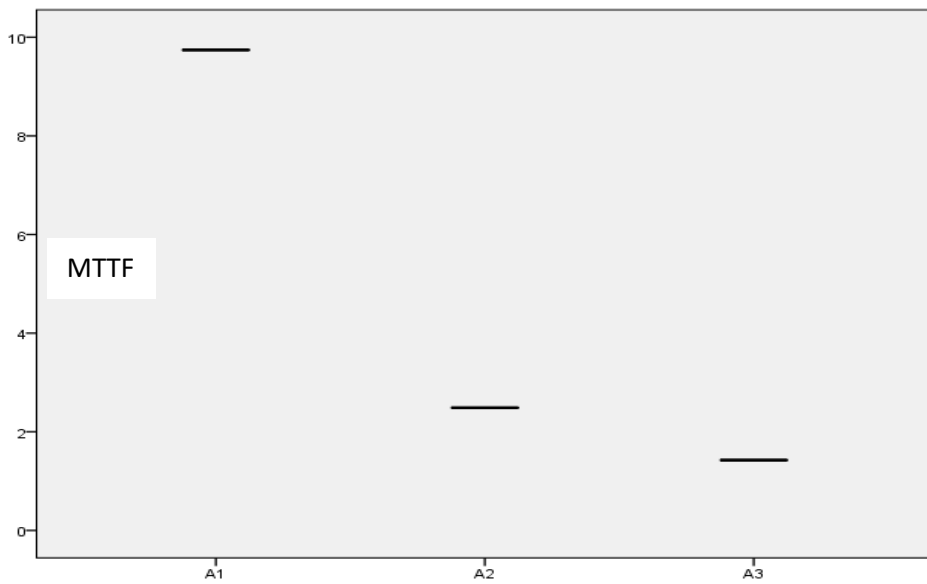
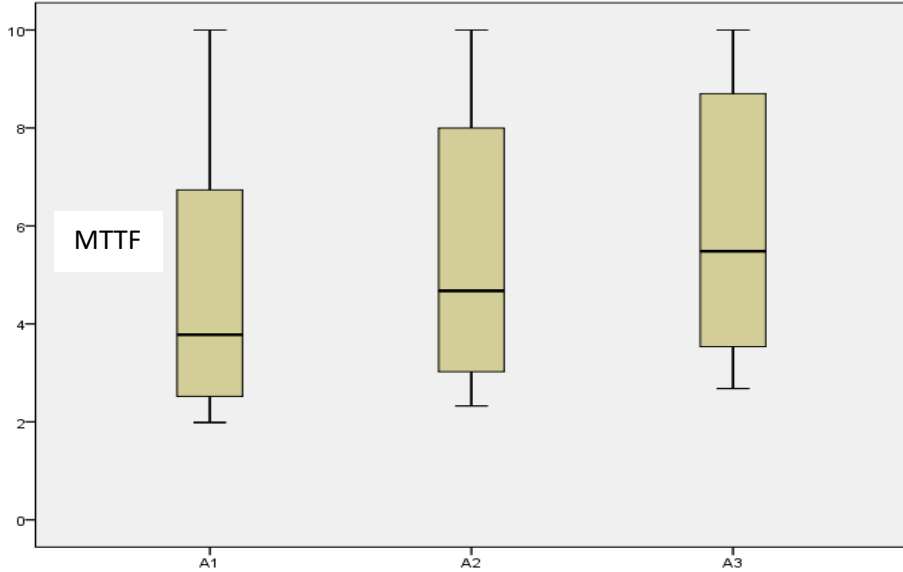
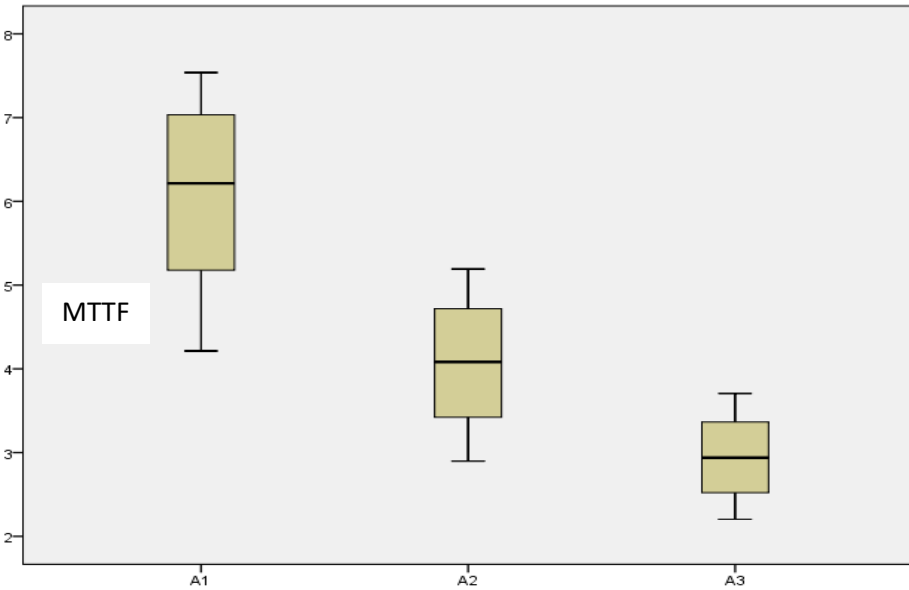


Figure 8b. Box plot for MTTF for $\lambda_0 \in [0.1:0.3:0.7]$.

Figure 8c. Box plot for MTTF for $\mu_1 \in [0.2:0.3:0.8]$.Figure 8d. Box plot for MTTF for $\lambda_1 \in [0.3:0.2:0.7]$.

Sensitivity analysis presented in Tables 2 to 5 and Figures 6a to 8d depict the variation of availability, mean time to failure and profit with respect to λ_j and $\mu_j, j = 0, 1$. It is clear from Tables 2 and 4 that availability, mean time to failure and profit decreases as λ_0 and λ_1 increases from 0 to 0.9 for different values of μ_0 and μ_1 . On the other hand,

Box plot in Figures 6a, 7a and 8a and Figures 6c,7c and 8c, availability, mean time to failure and profit increases at the points A_1 , A_2 and A_3 and B_1 , B_2 and B_3 and reaches their highest medium and maximum of the minimum value at A_3 and B_3 compared to A_1 and A_2 and B_1 and B_2 . Increasing in the values of μ_0 and μ_1 increases the availability, mean time to failure and profit significantly. Availability, mean time to failure and profit tend to be higher at A_3 and B_3 whenever $\mu_0, \mu_1 \geq 0/9$. It is evident from Table 3 and 5 that availability, mean time to failure and profit increases as μ_0 and μ_1 increases from 0 to 0.9 for different values of λ_0 and λ_1 and decreases in Figures 6b, 7b and 8b and Figures 6d, 7d and 8d. Availability, mean time to failure and profit reaches their highest medium and maximum of the minimum value at A_1 and B_1 compared to A_2 and A_3 and B_2 and B_3 . Decreasing in the values of λ_0 and λ_1 decrease the values of availability, mean time to failure and profit. Availability, mean time to failure and profit tend to be higher at A_1 and B_1 whenever $0 \leq \lambda_0, \lambda_1 \leq 0/1$. This sensitivity analyses implies that maintenance action such as inspection, preventive maintenance, etc. should be invoked to reduce the occurrence of failure in order to attain maximum value of availability, mean time to failure and profit.

6. Conclusion

In this paper, a multi computer system consisting two serial subsystems A and B is considered. Subsystem A has three identical active parallel clients while subsystem B has one server. Explicit expression for the steady-state availability function, mean time to failure, busy period probabilities of repairman as well as profit are. Impact of various failure and repair on system performance measures are studied and presented in graphs and Tables using MATLAB and SPSS packages. These are the main contribution of the paper. On the basis of the tables and figures, it is evident that the availability, mean time to failure and profit can be improve through higher values repair rates together with zero or lower values of failure rates. Thus, higher system availability, mean time to failure and profit can be achieved through perfect repair of any of clients or server or both, regular inspection, condition monitoring, and used of fault tolerant clients and server, multiple clients, server replication. It is evident from this study that availability can be worthwhile by adding more servers on standby or replication. This will prevent loss of data, enhancing availability, less maintenance cost, as well as increase in the output. The present work can be extended further for a system containing multi-clients with multi servers and solve using fourth order Runge Kutta techniques, optimization of reliability by considering intuitionistic fuzzy programming technique as well as optimization of reliability by considering nonlinear resource constraints using the penalty guided based biogeography based optimization.

References

1. Ahlawat, S., Anand, A. (2014). An Introduction to Computer Networking, *International Journal of Computer Science and Information Technology Research*, Vol. 2, Issue 2, pp 373-377.

2. Chauhan, S. K., Malik, S. C. (2017). Evaluation of reliability and MTSF of a parallel system with Weibull failure laws. *Journal of Reliability and Statistical Studies*, 137-148.
3. UsmanDilawar, M., Syed, F. (2014). Mathematical modeling and analysis of network service failure in datacentre. *Int. J. Mod. Educ. Comput. Sci.(IJMECS)*, 6(6), 30-36.
4. Fong, J., Hui, R. (1999). Application of middleware in the three tier client/server database design methodology. *Journal of the Brazilian Computer Society*, 6(1), 50-64.
5. Garg, S. (2019). An approach to resolve heterogeneity using RPC in client server systems, *International Journal of Engineering Applied Sciences and Technology*, Vol. 4, Issue 4, 301-305.
6. Garg, H., Rani, M., Sharma, S. P., Vishwakarma, Y. (2014). Bi-objective optimization of the reliability-redundancy allocation problem for series-parallel system. *Journal of Manufacturing Systems*, 33(3), 335-347.
7. Garg, H., Rani, M., Sharma, S. P., Vishwakarma, Y. (2014). Intuitionistic fuzzy optimization technique for solving multi-objective reliability optimization problems in interval environment. *Expert Systems with Applications*, 41(7), 3157-3167.
8. Garg, H., Sharma, S. P. (2012). Behavior analysis of synthesis unit in fertilizer plant. *International Journal of Quality & Reliability Management*.
9. Ibrahim, K. H., Singh, V. V., Lado, A. K. (2017). Reliability assessment of complex system consisting two subsystems connected in series configuration using Gumbel-Hougaard family copula distribution. *Journal of Applied Mathematics and Bioinformatics*, 7(2), 1-27.
10. Kakkar, M., Chitkara, A., Bhatti, J. (2016). Reliability analysis of two dissimilar parallel unit repairable system with failure during preventive maintenance. *Management Science Letters*, 6(4), 285-296.
11. Kakkar, M.K., Bhardwaj, N., Sharma, N., Bhatti, J. (2017). Availability analysis of two parallel unit system under the provision of maintenance. *International Journal of Recent Research Aspects*, 4(2), 155-160.
12. Kakkar, M., Chitkara, A., Bhatti, J. (2015). Reliability analysis of two unit parallel repairable industrial system. *Decision Science Letters*, 4(4), 525-536.
13. Kirubanand, V.B., Palaniammal, S. (2010). Performance modelling in client server network comparison of hub, switch and bluetooth technology using Markov algorithm and queuing petri nets with the security of steganography, *Int. J. Of Advanced Networking and Applications*, Volume: 01, Issue: 05, Pages: 331-336.
14. Kovalev, I. V., Zelenkov, P. V., Karaseva, M. V., Tsarev, M. Y., Tsarev, R. Y. (2015, January). Model of the reliability analysis of the distributed computer systems with architecture" client-server. In *IOP Conference Series: Materials Science and Engineering* (Vol. 70, No. 1, p. 012009).
15. Kumar, P., Singh, Y. (2010). A software reliability growth model for three-tier client server system. *International Journal of Computer Applications*, 975, 8887.
16. Minkevičius, S., Kulvietis, G. (2011, June). Investigation of the reliability of multiserver computer networks. In *International Conference on Analytical and*

- Stochastic Modeling Techniques and Applications* (pp. 249-256). Springer, Berlin, Heidelberg.
17. Niwas, R., Garg, H. (2018). An approach for analyzing the reliability and profit of an industrial system based on the cost free warranty policy. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, 40(5), 265.
 18. Mohan, R. (2013). Network Analysis and Remote Application Control Software based on Client-Server Architecture, *International Journal of Computer Applications*, 68 (12), 0975 – 8887.
 19. Oluwatosin, H. S. (2014). Client-server model. *IOSR J Comput Eng (IOSR-JCE)*, 16(1), 67.
 20. Potapov, V. I., Shafeeva, O. P., Gritsay, A. S., Makarov, V. V., Kuznetsova, O. P., Kondratukova, L. K. (2019, August). Reliability in the model of an information system with client-server architecture. In *Journal of Physics: Conference Series* (Vol. 1260, No. 2, p. 022007). IOP Publishing.
 21. Singh, V. V., Ayagi, H. I. (2017). Study of reliability measures of system consisting of two subsystems in series configuration using copula. *Palest J Math*, 6(1), 1-10.
 22. Temraz, N. S. Y. (2019). Availability and reliability of a parallel system under imperfect repair and replacement: analysis and cost optimization. *International Journal of System Assurance Engineering and Management*, 10(5), 1002-1009.
 23. Xin, J., Guo, L., Huang, N., Li, R. (2013). Network service reliability analysis model. *Chemical Engineering Transactions*, 33, 511-516.
 24. Yang, D. Y., Tsao, C. L. (2019). Reliability and availability analysis of standby systems with working vacations and retrieval of failed components. *Reliability Engineering & System Safety*, 182, 46-55.
 25. Yakubu, M., Ngene, C. U., Gambo, Y. (2017). Modified Math Client-Server Application for E-Learning. *International Journal of Computer Applications*, 170(4), 26-29.
 26. Yusuf, I., Ismail, A. L., Singh, V. V., Ali, U. A., Sufi, N. A. (2020). Performance Analysis of Multi-Computer System Consisting of Three Subsystems in Series Configuration Using Copula Repair Policy. *SN Computer Science*, 1(5), 1-11.
 27. Zhang, F. (2019). Research on reliability analysis of computer network based on intelligent cloud computing method. *International Journal of Computers and Applications*, 41:4, 283-288.
 28. Yusuf, I., Yusuf, B., Babagana, M., Sani, B., Lawan, M. A. (2018). Some reliability characteristics of a linear consecutive 2-out-of-4 system connected to 2-out-of-4 supporting device for operation. *International Journal of Engineering & Technology*, 7(1), 135-139.