Some induced generalized Einstein aggregating operators and their application to group decision-making problem using intuitionistic fuzzy numbers

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**Abstract** The paper aims to develop an idea of some inducing operators, namely induced intuitionistic fuzzy Einstein hybrid averaging operator, induced intuitionistic fuzzy Einstein hybrid geometric operator, induced generalized intuitionistic fuzzy Einstein hybrid averaging operator and induced generalized intuitionistic fuzzy Einstein hybrid geometric operator along with their wanted structure properties such as, monotonicity, idempotency and boundedness. The proposed operators are competent and able to reflect the complex attitudinal character of the decision maker by using order inducing variables and deliver more information to experts for decision-making. To show the legitimacy, practicality and effectiveness of the new operators, the proposed operators have been applied to decision making problems.

**Keywords** I-IFEHA operator; I-IFEHG operator; I-GIFEHA operator; I-GIFEHG operator; MAGDM problem
1. **Introduction**

Multi-attribute decision-making play an importance acts and role in much area our daily such as management, medical, engineering, business and economics. Commonly, it has been recognized and assumed that the data and information which touch the alternatives in form of criteria and weight are articulated in real numbers. However due to the complication of the system time-by-time, it is not easy to an expert to make a correct decision, as most of the model value during the process of decision-making filled with uncertainty. Thus in decision-making process, the decision and verdict information and data is mostly and often incomplete, indeterminate and inconsistent information. Therefore, Zadeh (1965) firstly developed and familiarize the concept fuzzy set (FS) which is more powerful and controlling to process fuzzy information in our daily life problems. However, there are some problems in this approach and tactic, because it has only element and member called membership function. To control and cover the above limitations, Atanassov (1986) and Atanassov (1989) familiarized the idea of intuitionistic fuzzy set (IFS) by adding and including a new element called non-membership function. Intuitionistic fuzzy set is the more suitable method and ways for the controlling and managing of those problems and difficulties which occur in the form of fuzzy set. After the development of intuitionistic fuzzy set several researchers, such as Garg (2020) introduced the notion of some averaging and geometric operators. Garg and Arora (2019) generalized intuitionistic fuzzy soft power aggregation operators. Garg and Kumar (2020) presented the possibility measure of interval-valued intuitionistic fuzzy set. Xu (2007) developed models for MAGDM problem. Yu et al. (2012) introduced the idea of prioritized operators for IVIFNs. Bustince and Burillo (1995) introduced the idea of interval-valued intuitionistic fuzzy numbers. Xu and Xia (2011) introduced the idea of induced aggregation operators. Tan (2011) introduced the notion of some generalized intuitionistic fuzzy geometric operators. Xia and Xu (2010) introduced the notion of similarity measures for intuitionistic fuzzy values. Li (2011) introduced the notion of some generalized ordered weighted averaging operators. Wei (2008) and Wei (2010) and Wei (2011) developed the maximizing deviation method, GRA method and gray relational analysis method for IFSs. Wei and Zhao (2011) explore the notion of Minimum deviation models. Wei et al. (2011) developed the notion of correlation coefficient to IVIFNs. Grzegorzewski (2004) find distances between intuitionistic fuzzy sets. Yager (2009) developed some aspects of intuitionistic fuzzy sets. Ye (2010) and Ye (2011) introduced the concept of weighted correlation coefficient and fuzzy cross entropy for IVIFSs. Garg (2016) and Garg (2019) introduced the idea of generalized intuitionistic fuzzy interactive geometric interaction operators and Intuitionistic fuzzy Hamacher aggregation operators with entropy weight and their applications. Garg and Kumar (2018) and Garg and Kumar (2019) introduced the idea of power geometric aggregation operators and a novel correlation coefficient of intuitionistic fuzzy sets. Garg and Rani (2019) introduced the idea of some generalized geometric aggregation operators for complex intuitionistic fuzzy sets. Xu (2007) and Xu and Yager (2006) presented the idea of some geometric and arithmetic operators using algebraic rules, namely intuitionistic...
fuzzy hybrid averaging operator, intuitionistic fuzzy weighted averaging operator, intuitionistic fuzzy ordered weighted averaging operator, intuitionistic fuzzy hybrid geometric operator, intuitionistic fuzzy ordered weighted geometric operator, intuitionistic fuzzy weighted geometric operator, respectively along with their wanted properties and also applied these on multi-attributes group decision making problems in daily life to select the best alternative from all under consideration alternatives. Einstein product and sum are the better ways from the algebraic laws. Xu (2011) familiarized the notion of intuitionistic fuzzy power operators and their application. Wang and Liu (2011) and Wang and Liu (2012) presented the concept of Einstein operators namely, intuitionistic fuzzy Einstein weighted geometric operator, intuitionistic fuzzy Einstein weighted averaging operator, intuitionistic fuzzy Einstein ordered weighted averaging operator and utilized these operators to MAGDM problems. After the developing of Einstein operators, Zhao and Wei (2013) generalized them and presented the idea of intuitionistic fuzzy Einstein hybrid averaging operator, intuitionistic fuzzy Einstein hybrid geometric operator. Zhao et al. (2010) familiarized the idea of generalized intuitionistic fuzzy weighted averaging operator, generalized intuitionistic fuzzy ordered weighted averaging operator, generalized intuitionistic fuzzy hybrid averaging operator, generalized interval-valued intuitionistic fuzzy weighted averaging operator, generalized interval-valued intuitionistic fuzzy ordered weighted averaging operator, generalized interval-valued intuitionistic fuzzy hybrid averaging operator. Wei (2010) and Yu and Xu (2013) and Su et al. (2011) and Xu et al. (2013) introduced the notion of some new type’s methods and utilized them to group decision making. Rahman et al. (2018) and Rahman et al. (2020) introduced some generalized intuitionistic fuzzy Einstein hybrid aggregation Operators and confidence levels and their application to group decision making problems. Jamil et al. (2020) introduced the notion of induced generalized interval-valued intuitionistic fuzzy Einstein operator and applied them on group decision making problem.

Motivated from (37), in which the authors used some algebraic operational laws and develop some algebraic operators for intuitionistic fuzzy values such as, induced intuitionistic fuzzy hybrid averaging operator, induced intuitionistic fuzzy hybrid geometric operator. But Einstein product and sum are the best ways from the algebraic operational laws. Therefore, in this paper we utilize the Einstein sum and product and propose some new methods such as, induced intuitionistic fuzzy Einstein hybrid averaging operator, induced intuitionistic fuzzy Einstein hybrid geometric operator, induced generalized intuitionistic fuzzy Einstein hybrid averaging operator, induced generalized intuitionistic fuzzy Einstein hybrid geometric operator. The new operators are competent to reflect the complex attitudinal character of the decision maker with the help of inducing variables and provide more information to decision maker. The proposed methods provide more general, more accurate and precise results are comparing to their existing methods. Therefore, these methods play a vital role in real world problems.
The rest of the paper is structured as follows. Section two, contains some basic results. Section three, contains new methods, such as induced intuitionistic fuzzy Einstein hybrid averaging operator, induced intuitionistic fuzzy Einstein hybrid geometric operator, induced generalized intuitionistic fuzzy Einstein hybrid averaging operator and induced generalized intuitionistic fuzzy Einstein hybrid geometric operator. Section four, contains multi-attributes group decision making problem. Section five, contains numerical example. Section six, contains conclusion.

2. Preliminaries

In this section, we present fuzzy set, intuitionistic fuzzy set, score function, accuracy function, some Einstein operational laws, induced intuitionistic fuzzy hybrid averaging operator and induced intuitionistic fuzzy hybrid geometric operator.

**Definition 1:** [1] Let \( Z \) be a general set, then FS can be defined as:

\[
F = \left\{ \{z, \xi_F(z)\} \mid z \in Z \right\}
\]

where \( \xi_F(z) : Z \rightarrow [0,1] \) called membership function.

**Definition 2:** [2, 3] Let \( Z \) be a general set, then intuitionistic fuzzy set can be defined as:

\[
I = \left\{ \{z, \xi_I(z), \kappa_I(z)\} \mid z \in Z \right\}
\]

where \( \xi_I(z) : Z \rightarrow [0,1] \), \( \kappa_I(z) : Z \rightarrow [0,1] \) with \( 0 \leq \xi_I(z) + \kappa_I(z) \leq 1, \forall z \in Z \). Let \( \pi_I(z) = 1 - \xi_I(z) - \kappa_I(z) \) then \( \pi_I(z) \) is called the degree of indeterminacy.

**Definition 3:** [31] Let \( \mathcal{R} = (\xi_R, \kappa_R) \) be an intuitionistic fuzzy value, then the score function and accuracy function of \( \mathcal{R} \) can be defined as: \( S(\mathcal{R}) = \xi_R - \kappa_R \) and \( H(\mathcal{R}) = \xi_R + \kappa_R \) respectively.

**Definition 4:** [31] Let \( \mathcal{R}_1 = (\xi_{R_1}, \kappa_{R_1}) \) and \( \mathcal{R}_2 = (\xi_{R_2}, \kappa_{R_2}) \) be the two IFVs, then

1. If \( S(\mathcal{R}_1) > S(\mathcal{R}_2) \), then \( \mathcal{R}_1 \prec \mathcal{R}_2 \)
2. If \( S(\mathcal{R}_2) = S(\mathcal{R}_1) \), then
3. If \( H(\mathcal{R}_2) = H(\mathcal{R}_1) \), then, \( \mathcal{R}_1 = \mathcal{R}_2 \)
2. If $H(\mathfrak{R}_2) < H(\mathfrak{R}_1)$, then $\mathfrak{R}_1 \succ \mathfrak{R}_2$

**Definition 5:** [31] Let $\langle u_j, \mathfrak{R}_j \rangle (j = 1, 2)$ be a family of 2-tuples and $\sigma > 0$, then

$$\mathfrak{R}_1 \otimes E \mathfrak{R}_2 = \left( \frac{\xi \mathfrak{R}_1 + \xi \mathfrak{R}_2}{1 + (1 - \xi \mathfrak{R}_1)(1 - \xi \mathfrak{R}_2)}, \frac{\xi \mathfrak{R}_1 \xi \mathfrak{R}_2}{1 + (1 - \xi \mathfrak{R}_1)(1 - \xi \mathfrak{R}_2)} \right)$$  \hspace{1cm} (3)

$$\mathfrak{R}_1 \otimes E \mathfrak{R}_2 = \left( \frac{\xi \mathfrak{R}_1 + \xi \mathfrak{R}_2}{1 + (1 - \xi \mathfrak{R}_1)(1 - \xi \mathfrak{R}_2)}, \frac{\xi \mathfrak{R}_1 \xi \mathfrak{R}_2}{1 + \xi \mathfrak{R}_1 \xi \mathfrak{R}_2} \right)$$  \hspace{1cm} (4)

$$\langle \mathfrak{R} \rangle^\sigma = \left( \frac{2(\xi \mathfrak{R})^\sigma}{(2 - \xi \mathfrak{R})^\sigma + (\xi \mathfrak{R})^\sigma}, \frac{(1 + \xi \mathfrak{R})^\sigma - (1 - \xi \mathfrak{R})^\sigma}{(1 + \xi \mathfrak{R})^\sigma + (1 - \xi \mathfrak{R})^\sigma} \right)$$  \hspace{1cm} (5)

$$\sigma(\mathfrak{R}) = \left( \frac{(1 + \xi \mathfrak{R})^\sigma - (1 - \xi \mathfrak{R})^\sigma}{(1 + \xi \mathfrak{R})^\sigma + (1 - \xi \mathfrak{R})^\sigma}, \frac{2(\xi \mathfrak{R})^\sigma}{(2 - \xi \mathfrak{R})^\sigma + (\xi \mathfrak{R})^\sigma} \right)$$  \hspace{1cm} (6)

**Definition 6:** [37] The induced intuitionistic fuzzy hybrid averaging operator can be defined as:

$$\text{I-IFHA}_{\xi, \varnothing} \left( \langle u_1, \mathfrak{R}_1 \rangle, \langle u_2, \mathfrak{R}_2 \rangle, \ldots, \langle u_n, \mathfrak{R}_n \rangle \right) = \left\{ 1 - \prod_{j=1}^{n} \left( 1 - \xi \mathfrak{R}_j \varnothing(j) \right)^{\varnothing_j}, \prod_{j=1}^{n} \left( \xi \mathfrak{R}_j \varnothing(j) \right)^{\varnothing_j} \right\}$$ \hspace{1cm} (7)

Where $\mathfrak{R}_j \varnothing(j)$ be the weighted intuitionistic fuzzy value $\mathfrak{R}_j (\mathfrak{R}_j = n \varnothing_j \mathfrak{R}_j, j = 1, \ldots, n)$ of the IFOWA pair $\langle u_j, \mathfrak{R}_j \rangle$ having the jth largest $u_j$ in $\langle u_j, \mathfrak{R}_j \rangle$. $\varnothing = (\varnothing_1, \ldots, \varnothing_n)^T$ be the associated vector of induced intuitionistic fuzzy hybrid averaging operator with $\varnothing_j \in [0,1]$ and $\sum_{j=1}^{n} \varnothing_j = 1$. $\varnothing = (\varnothing_1, \ldots, \varnothing_n)^T$ be the weighted vector with $\varnothing_j \in [0,1]$ and $\sum_{j=1}^{n} \varnothing_j = 1$.

**Definition 7:** [37] The induced intuitionistic fuzzy hybrid geometric operator can be defined as:

$$\text{I-FHG}_{\xi, \varnothing} \left( \langle u_1, \mathfrak{R}_1 \rangle, \langle u_2, \mathfrak{R}_2 \rangle, \ldots, \langle u_n, \mathfrak{R}_n \rangle \right) = \left\{ \prod_{j=1}^{n} \left( \xi \mathfrak{R}_j \varnothing(j) \right)^{\varnothing_j}, 1 - \prod_{j=1}^{n} \left( 1 - \xi \mathfrak{R}_j \varnothing(j) \right)^{\varnothing_j} \right\}$$ \hspace{1cm} (8)
where \( \mathcal{R}_j(j) \) is the weighted intuitionistic fuzzy value \( \mathcal{R}_j(\mathcal{R}_j = (\mathcal{R}_j)^n \delta(j), j = 1, 2, \ldots, n) \) of the intuitionistic fuzzy ordered weighted geometric pair \( \left\langle u_j, \mathcal{R}_j \right\rangle \) having the jth largest \( u_j \) in \( \left\langle u_j, \mathcal{R}_j \right\rangle \). \( \mathcal{Z} = (\mathcal{Z}_1, \ldots, \mathcal{Z}_n)^T \) be the associated vector of I-IFHG operator with \( \mathcal{Z}_j \in [0,1] \) and \( \sum_{j=1}^{n} \mathcal{Z}_j = 1 \). \( \mathcal{Z} \) be the weighted vector with \( \mathcal{Z}_j \in [0,1] \) and \( \sum_{j=1}^{n} \mathcal{Z}_j = 1 \).

3. **Some induced Einstein hybrid aggregation operators**

**Definition 8:** The induced intuitionistic fuzzy Einstein hybrid averaging operator can be stated as:

\[
I-IFEHA\left(\mathcal{R}_1, \ldots, \mathcal{R}_n; \mathcal{Z}\right) = \sum_{j=1}^{n} \left( \frac{1}{1+\varepsilon\mathcal{R}_j(\mathcal{R}_j)} \right)^{3_j} - \frac{1}{1-\varepsilon\mathcal{R}_j(\mathcal{R}_j)} \right)^{3_j} - \sum_{j=1}^{n} \left( \frac{1}{2-\kappa\mathcal{R}_j(\mathcal{R}_j)} \right)^{3_j} + \sum_{j=1}^{n} \left( \frac{1}{\kappa\mathcal{R}_j(\mathcal{R}_j)} \right)^{3_j}
\]

where \( \mathcal{R}_j(j) \) is the weighted intuitionistic fuzzy value \( \mathcal{R}_j(\mathcal{R}_j = n\mathcal{Z}(\mathcal{R}_j), j = 1, \ldots, n) \) of the intuitionistic fuzzy ordered weighted averaging (IFOWA) pair \( \left\langle u_j, \mathcal{R}_j \right\rangle \) having the jth largest \( u_j, u_j \) in \( \left\langle u_j, \mathcal{R}_j \right\rangle \) is the order inducing variable and \( \mathcal{R}_j \) is the intuitionistic fuzzy argument variable, \( \mathcal{Z} = (\mathcal{Z}_1, \ldots, \mathcal{Z}_n)^T \) be the associated vector of I-IFEHA operator with \( \mathcal{Z}_j \in [0,1] \) and \( \sum_{j=1}^{n} \mathcal{Z}_j = 1 \). \( \mathcal{Z} \) be the weighted vector with \( \mathcal{Z}_j \in [0,1] \) and \( \sum_{j=1}^{n} \mathcal{Z}_j = 1 \).

**Theorem 1:** Let \( \left\langle u_j, \mathcal{R}_j \right\rangle (j = 1, 2, 3) \) be a family of 2-tuples, and \( \sigma > 0 \), then

1. \( \mathcal{R}_1 \oplus \mathcal{R}_2 = \mathcal{R}_2 \oplus \mathcal{R}_1 \)
2. \( \mathcal{R}_1 \otimes \mathcal{R}_2 = \mathcal{R}_2 \otimes \mathcal{R}_1 \)
3. \( (\mathcal{R}_1 \oplus \mathcal{R}_2) \oplus \mathcal{R}_3 = \mathcal{R}_1 \oplus (\mathcal{R}_2 \oplus \mathcal{R}_3) \)

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4. \((R_1 \otimes R_2) \otimes R_3 = R_1 \otimes (R_2 \otimes R_3)\)

5. \(\sigma(R_1 \otimes R_2) = \sigma R_1 \otimes \sigma R_2\)

6. \((R_1 \otimes R_2)^\sigma = (R_1)^\sigma \otimes (R_2)^\sigma\)

**Theorem 2:** Let \(\{u_j, R_j\}_{j=1,2,3}\) be a family of 2-tuples, and \(\sigma\), then

1. \((R_1)^c \cup (R_2)^c = (R_1 \cup R_2)^c\)

2. \((R_1)^c \cap (R_2)^c = (R_1 \cap R_2)^c\)

3. \((R_1 \cup R_2) \cap R_2 = R_2\)

4. \((R_1 \cap R_2) \cup R_2 = R_2\)

5. \((R^c)^\sigma = (\sigma R)^c\)

6. \(\sigma(R^c) = (R^\sigma)^c\)

**Theorem 3:** Let \(\{u_j, R_j\}_{j=1,2,3}\) be a family of 2-tuples, then

1. \((R_1 \cup R_2) \oplus (R_1 \cap R_2) = R_1 \oplus R_2\)

2. \((R_1 \cup R_2) \otimes (R_1 \cap R_2) = R_1 \otimes R_2\)

3. \((R_1 \cup R_2) \cap R_3 = (R_1 \cap R_3) \cup (R_2 \cap R_3)\)

4. \((R_1 \cap R_2) \cup R_3 = (R_1 \cup R_3) \cap (R_2 \cup R_3)\)

5. \((R_1 \cup R_2) \oplus R_3 = (R_1 \oplus R_3) \cup (R_2 \oplus R_3)\)

6. \((R_1 \cap R_2) \oplus R_3 = (R_1 \oplus R_3) \cap (R_2 \oplus R_3)\)

**Theorem 4:** Let \(\{u_j, R_j\}_{j=1,2,\ldots,n}\) be a family of 2-tuples, then their aggregated value by using the induced intuitionistic fuzzy Einstein hybrid averaging (I-IFEHA) operator is also intuitionistic fuzzy value, and
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\[
I-I\text{FEHA}_{\delta,\mathcal{I}}(\langle u_1, \mathcal{R}_1 \rangle, ..., \langle u_n, \mathcal{R}_n \rangle) = \left( \prod_{j=1}^{n} \left( 1 + \frac{1}{\kappa_{\delta}(j)} \right)^{3j} \right) - \left( \prod_{j=1}^{n} \left( 1 - \frac{1}{\kappa_{\delta}(j)} \right)^{3j} \right) + 2 \left( \prod_{j=1}^{n} \left( \kappa_{\delta}(j) \right)^{3j} \right) \tag{10}
\]

**Proof:** By mathematical induction. For \( n=2 \)

\[
\mathcal{A}_1 \langle u_1, \mathcal{R}_1 \rangle = \left( \prod_{j=1}^{n} \left( 1 + \frac{1}{\kappa_{\delta}(j)} \right)^{3j} \right) - \left( \prod_{j=1}^{n} \left( 1 - \frac{1}{\kappa_{\delta}(j)} \right)^{3j} \right) + 2 \left( \prod_{j=1}^{n} \left( \kappa_{\delta}(j) \right)^{3j} \right)
\]

\[
\mathcal{A}_2 \langle u_2, \mathcal{R}_2 \rangle = \left( \prod_{j=1}^{n} \left( 1 + \frac{1}{\kappa_{\delta}(j)} \right)^{3j} \right) - \left( \prod_{j=1}^{n} \left( 1 - \frac{1}{\kappa_{\delta}(j)} \right)^{3j} \right) + 2 \left( \prod_{j=1}^{n} \left( \kappa_{\delta}(j) \right)^{3j} \right)
\]

Thus

\[
I-I\text{FEHA}_{\delta,\mathcal{I}}(\langle u_1, \mathcal{R}_1 \rangle, \langle u_2, \mathcal{R}_2 \rangle) = \left( \prod_{j=1}^{n} \left( 1 + \frac{1}{\kappa_{\delta}(j)} \right)^{3j} \right) - \left( \prod_{j=1}^{n} \left( 1 - \frac{1}{\kappa_{\delta}(j)} \right)^{3j} \right) + 2 \left( \prod_{j=1}^{n} \left( \kappa_{\delta}(j) \right)^{3j} \right)
\]

The given result is true for \( n=2 \), so it is true for \( n=k \).

\[
I-I\text{FEHA}_{\delta,\mathcal{I}}(\langle u_1, \mathcal{R}_1 \rangle, ..., \langle u_k, \mathcal{R}_k \rangle) = \left( \prod_{j=1}^{n} \left( 1 + \frac{1}{\kappa_{\delta}(j)} \right)^{3j} \right) - \left( \prod_{j=1}^{n} \left( 1 - \frac{1}{\kappa_{\delta}(j)} \right)^{3j} \right) + 2 \left( \prod_{j=1}^{n} \left( \kappa_{\delta}(j) \right)^{3j} \right)
\]

If the given result is true for \( n=k \), then it is true for \( n=k+1 \)

\[
I-I\text{FEHA}_{\delta,\mathcal{I}}(\langle u_1, \mathcal{R}_1 \rangle, ..., \langle u_{k+1}, \mathcal{R}_{k+1} \rangle) = \left( \prod_{j=1}^{n} \left( 1 + \frac{1}{\kappa_{\delta}(j)} \right)^{3j} \right) - \left( \prod_{j=1}^{n} \left( 1 - \frac{1}{\kappa_{\delta}(j)} \right)^{3j} \right) + 2 \left( \prod_{j=1}^{n} \left( \kappa_{\delta}(j) \right)^{3j} \right)
\]

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Let
\[ p_1 = \prod_{j=1}^{k} \left( 1 + \varepsilon_{j\theta(j)} \right)^{\alpha_j} - \prod_{j=1}^{k} \left( 1 - \varepsilon_{j\theta(j)} \right)^{\alpha_j}, \quad p_2 = \left( 1 + \varepsilon_{j\theta_{k+1}} \right)^{\alpha_{k+1}} - \left( 1 - \varepsilon_{j\theta_{k+1}} \right)^{\alpha_{k+1}}, \quad \eta = 2 \prod_{j=1}^{k} \left( \kappa_{j\theta(j)} \right)^{\alpha_j}, \]

\[ q_1 = \prod_{j=1}^{k} \left( 1 + \varepsilon_{j\theta(j)} \right)^{\alpha_j} + \prod_{j=1}^{k} \left( 1 - \varepsilon_{j\theta(j)} \right)^{\alpha_j}, \quad q_2 = \left( 1 + \varepsilon_{j\theta_{k+1}} \right)^{\alpha_{k+1}} + \left( 1 - \varepsilon_{j\theta_{k+1}} \right)^{\alpha_{k+1}}, \quad r_2 = 2 \left( \kappa_{j\theta_{k+1}} \right)^{\alpha_{k+1}} \]

and \[ s_1 = \prod_{j=1}^{k} \left( 2 - \kappa_{j\theta(j)} \right)^{\alpha_j} + \prod_{j=1}^{k} \left( \kappa_{j\theta(j)} \right)^{\alpha_j}, \quad s_2 = \left( 2 - \kappa_{j\theta_{k+1}} \right)^{\alpha_{k+1}} + \left( \kappa_{j\theta_{k+1}} \right)^{\alpha_{k+1}}. \]

Now putting these values in equation (11), we have
\[ \text{I-IFEHA}_{\theta, \alpha} \left( \langle u_1, \theta_1 \rangle, \ldots, \langle u_{k+1}, \theta_{k+1} \rangle \right) = \left( \frac{p_1}{q_1}, \frac{\eta}{s_1} \right) \otimes \left( \frac{p_2}{q_2}, \frac{r_2}{s_2} \right) \]

\[ = \left( \frac{\left( p_1 + p_2 \right)}{q_1 q_2} \frac{\eta r_2}{s_1 s_2} \frac{1}{1 + \left( \frac{p_1}{q_1} \right) \frac{p_2}{q_2} \frac{1/\eta}{1/\eta - 1/\eta - 1/\eta}} \right) \]

\[ = \left\{ \frac{p_1 q_2 + p_2 q_1}{q_1 q_2 + p_1 p_2} \frac{\eta r_2}{2 s_1 s_2 - s_1 s_2 - s_1 s_2 + \eta r_2} \right\} (12) \]

Putting the values of \[ p_1 q_2 + p_2 q_1, q_1 q_2 + p_1 p_2, \eta r_2, 2 s_1 s_2 - s_1 s_2 - s_1 s_2 + \eta r_2 \] in equation (12), we have
\[ \text{I-IFEHA}_{\theta, \alpha} \left( \langle u_1, \theta_1 \rangle, \ldots, \langle u_{k+1}, \theta_{k+1} \rangle \right) = \left( \prod_{j=1}^{k+1} \left( 1 + \varepsilon_{j\theta_{j+1}} \right)^{\alpha_j} - \prod_{j=1}^{k+1} \left( 1 - \varepsilon_{j\theta_{j+1}} \right)^{\alpha_j} \right)^{\alpha_{k+1}} \prod_{j=1}^{k+1} \left( \kappa_{j\theta_{j+1}} \right)^{\alpha_j}. \]

Thus the given result is true for \( n = k + 1 \). Thus equation (10), true for all \( n \).

**Lemma 1:** [31] Let \( R_j > 0, J_j > 0 (j = 1, 2, \ldots, n) \) and \( \sum_{j=1}^{n} J_j = 1 \). then
\[ \prod_{j=1}^{n} R_j J_j \leq \sum_{j=1}^{n} J_j R_j \]

(13)

where the equality holds if and only if \( R_j (j = 1, 2, \ldots, n) = R \)

**Theorem 5:** Let \( \langle u_j, R_j \rangle (j = 1, 2, \ldots, n) \) be a family of 2-tuples, then
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I-IFEHA_{\theta,\overline{\zeta}}\left(\langle u_1, r_1 \rangle, \langle u_2, r_2 \rangle, \ldots, \langle u_n, r_n \rangle\right) \leq I-IFHA_{\theta,\overline{\zeta}}\left(\langle u_1, r_1 \rangle, \langle u_2, r_2 \rangle, \ldots, \langle u_n, r_n \rangle\right) \quad (14)

Proof: As

\[ \prod_{j=1}^{n} \left(1 + \frac{\zeta}{\theta} r_{\delta(j)}\right)^{3_j} + \prod_{j=1}^{n} \left(1 - \frac{\zeta}{\theta} r_{\delta(j)}\right)^{3_j} \leq \sum_{j=1}^{n} 3_j \left(1 + \frac{\zeta}{\theta} r_{\delta(j)}\right) + \sum_{j=1}^{n} 3_j \left(1 - \frac{\zeta}{\theta} r_{\delta(j)}\right) \]

Now

\[ \sum_{j=1}^{n} 3_j \left(1 + \frac{\zeta}{\theta} r_{\delta(j)}\right) + \sum_{j=1}^{n} 3_j \left(1 - \frac{\zeta}{\theta} r_{\delta(j)}\right) = 2 \]

Hence

\[ \prod_{j=1}^{n} \left(1 + \frac{\zeta}{\theta} r_{\delta(j)}\right)^{3_j} + \prod_{j=1}^{n} \left(1 - \frac{\zeta}{\theta} r_{\delta(j)}\right)^{3_j} \leq 2 \]

Also

\[ \prod_{j=1}^{n} \left(1 + \frac{\zeta}{\theta} r_{\delta(j)}\right)^{3_j} - \prod_{j=1}^{n} \left(1 - \frac{\zeta}{\theta} r_{\delta(j)}\right)^{3_j} = 1 - \frac{2 \prod_{j=1}^{n} \left(1 - \frac{\zeta}{\theta} r_{\delta(j)}\right)^{3_j}}{\prod_{j=1}^{n} \left(1 + \frac{\zeta}{\theta} r_{\delta(j)}\right)^{3_j}} \leq 1 - \prod_{j=1}^{n} \left(1 - \frac{\zeta}{\theta} r_{\delta(j)}\right)^{3_j} \quad (15) \]

If \( \frac{\zeta}{\theta} r_{\delta(j)} \) are equal for all \( n \), then the equality holds. Again

\[ \prod_{j=1}^{n} \left(2 - \kappa r_{\delta(j)}\right)^{3_j} + \prod_{j=1}^{n} \left(\kappa r_{\delta(j)}\right)^{3_j} \leq \sum_{j=1}^{n} 3_j \left(2 - \kappa r_{\delta(j)}\right) + \sum_{j=1}^{n} 3_j \left(\kappa r_{\delta(j)}\right) \]

As

So

Thus

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If $\xi$ are equal for all $n$, then the equality holds. Let

$$
(16)
$$

Then equations (15) and (16) can be transformed into the following forms:

$$
(17)
$$

$$
(18)
$$

There are two cases in this direction.

Case 1: If $\text{and}$, then by Definition 4 we have

$$
(19)
$$

Case 2: If $\text{and}$, then we have $\kappa_{\tilde{\rho}} = \kappa_{\tilde{\rho}} \varepsilon$ this implies that $\xi + \kappa_{\tilde{\rho}} = \xi + \kappa_{\tilde{\rho}} \varepsilon$ hence $H(\tilde{\rho}) = H(\tilde{\rho} \varepsilon)$. Thus by Definition 4 we have

$$
(20)
$$

Hence from equations (19) and (20), we have

$$
I-\text{IFEHA}_{\tilde{\rho}, 3}(\langle u_1, \mathcal{R}_1 \rangle, \langle u_2, \mathcal{R}_2 \rangle, \ldots, \langle u_n, \mathcal{R}_n \rangle) \leq I-\text{IFEHA}_{\tilde{\rho}, 3}(\langle u_1, \mathcal{R}_1 \rangle, \langle u_2, \mathcal{R}_2 \rangle, \ldots, \langle u_n, \mathcal{R}_n \rangle)
$$

The proof is completed.

**Example 1:** Let
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\[ \langle u_1, R_1 \rangle = \langle 0.30, (0.70, 0.20) \rangle, \langle u_2, R_2 \rangle = \langle 0.80, (0.60, 0.30) \rangle, \langle u_3, R_3 \rangle = \langle 0.70, (0.50, 0.50) \rangle \] and

\[ \langle u_4, R_4 \rangle = \langle 0.50, (0.40, 0.60) \rangle \]

By ordering with respect to the first element, then we have

\[ \langle u_2, R_2 \rangle = \langle 0.80, (0.60, 0.30) \rangle, \langle u_3, R_3 \rangle = \langle 0.70, (0.50, 0.50) \rangle, \langle u_4, R_4 \rangle = \langle 0.50, (0.40, 0.60) \rangle, \langle u_1, R_1 \rangle = \langle 0.30, (0.70, 0.20) \rangle \]

Hence

Again using the I-IFEHA operator, then we have

Hence

Thus

**Theorem 6:** Induced intuitionistic fuzzy Einstein hybrid averaging operator satiates the following properties.
1. **Idempotency**: If \( \underset{j}{\gamma} \) for all \( j \), then

\[
\begin{align*}
\text{(21)}
\end{align*}
\]

**Proof**: As \( \underset{j}{\gamma} \) for all \( j \), then we have

\[
\begin{align*}
\text{(22)}
\end{align*}
\]

2. **Boundedness**: Let \( \underset{j}{\gamma} \) then

\[
\begin{align*}
\text{(22)}
\end{align*}
\]

**Proof**: From the above conditions, we have \( \underset{j}{\gamma} \); this implies that

\[
\begin{align*}
\text{(22)}
\end{align*}
\]

then we have
Again, i.e., then we have

Let

Then equations (23), (24) can be written as: and . Then

Now there are three conditions.

1. If and Then by Definition 4, we have
2. If \( \frac{\varepsilon_{j\hat{t}}}{\kappa_{j\hat{t}} - \varepsilon_{j\hat{t}} + \kappa_{j\hat{t}}} \) then \( \frac{\varepsilon_{j\hat{t}}}{\kappa_{j\hat{t}}} = \frac{\varepsilon_{j\hat{t}}}{\kappa_{j\hat{t}} \max} \) hence \( H(\hat{R}) = \frac{\varepsilon_{j\hat{t}}}{\kappa_{j\hat{t}}} = \frac{\varepsilon_{j\hat{t}}}{\kappa_{j\hat{t}} \max} + \kappa_{j\hat{t}} \max = H(\hat{R} \max) \). By Definition 4, we have

\[
\text{I-IFEHA}_{\hat{R}, \hat{\gamma}}(\{u_1, R_1\}, \{u_2, R_2\}, ..., \{u_n, R_n\}) = R_{\max}^{\gamma}
\] (26)

3. If \( S(\hat{R}) = S(\hat{R}_{\min}) \), then \( \frac{\varepsilon_{j\hat{t}}}{\kappa_{j\hat{t}}} - \frac{\varepsilon_{j\hat{t}}}{\kappa_{j\hat{t}} \min} = \frac{\varepsilon_{j\hat{t}}}{\kappa_{j\hat{t}} \min} \) hence \( H(\hat{R}) = \frac{\varepsilon_{j\hat{t}}}{\kappa_{j\hat{t}}} = \frac{\varepsilon_{j\hat{t}}}{\kappa_{j\hat{t}} \min} + \kappa_{j\hat{t}} \min = H(\hat{R}_{\min}) \).

By Definition 4, we have

\[
\text{I-IFEHA}_{\hat{R}, \hat{\gamma}}(\{u_1, R_1\}, \{u_2, R_2\}, ..., \{u_n, R_n\}) = R_{\min}^{\gamma}
\] (28)

Thus from equations (26) to (28) equation (22) always holds.

4. **Monotonicity:** If \( \{u_j, R_j^*\} \) \((j = 1, 2, ..., n)\) be a family 2-tuples, where \( R_j \leq R_j^* \), then

\[
\text{I-IFEHA}_{\hat{R}, \hat{\gamma}}(\{u_1, R_1\}, \{u_2, R_2\}, ..., \{u_n, R_n\}) \leq \text{I-IFEHA}_{\hat{R}, \hat{\gamma}}(\{u_1, R_1^*\}, \{u_2, R_2^*\}, ..., \{u_n, R_n^*\})
\] (29)

**Proof:** The proof is similar to above.

**Theorem 7:** Intuitionistic fuzzy Einstein weighted averaging operator is a specific case of the induced intuitionistic fuzzy Einstein hybrid averaging operator.

**Proof:** Let \( \mathfrak{I} = \left(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right)^T \), then we have

\[
\text{I-IFEHA}_{\hat{R}, \hat{\gamma}}(\{u_1, R_1\}, \{u_2, R_2\}, ..., \{u_n, R_n\}) = \mathfrak{I}^T R_{\delta}^{(1)} \oplus \mathfrak{I}^T R_{\delta}^{(2)} \oplus \mathfrak{I}^T R_{\delta}^{(3)} \oplus \cdots \oplus \mathfrak{I}^T R_{\delta}^{(n)}
\]

\[
= \frac{1}{n} \left( R_{\delta}^{(1)} \oplus \frac{1}{n} \left( R_{\delta}^{(2)} \oplus \cdots \oplus \frac{1}{n} \left( R_{\delta}^{(n)} \right) \right) \right).
\]

\[
= \text{IFEWA}_{\hat{R}}(R_1, R_2, ..., R_n)
\]

**Theorem 8:** Induced intuitionistic fuzzy Einstein ordered weighted averaging operator is a specific case of the induced intuitionistic fuzzy Einstein hybrid averaging operator.
Proof: Let \( \partial = \left( \frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n} \right)^T \), \( \Re \delta(j) = n \partial \Re \delta(j) = \Re \delta(j) \), then

\[
\text{I-IFEHA}_{\partial, \Im} \left( \langle u_1, \Re_1 \rangle, \langle u_2, \Re_2 \rangle, \ldots, \langle u_n, \Re_n \rangle \right) = \sum_{j=1}^{n} \Re \delta(j) \oplus \Re \delta(j) \oplus \ldots \oplus \Re \delta(n) \\
= \text{I-IFEOWA}_{\partial, \Im} \left( \langle u_1, \Re_1 \rangle, \langle u_2, \Re_2 \rangle, \ldots, \langle u_n, \Re_n \rangle \right)
\]

**Definition 9:** An induced intuitionistic fuzzy Einstein hybrid geometric operator can be defined as:

\[
\text{I-IFEHG}_{\partial, \Im} \left( \langle u_1, \Re_1 \rangle, \langle u_2, \Re_2 \rangle, \ldots, \langle u_n, \Re_n \rangle \right) = \left( \sum_{j=1}^{n} \Re \delta(j) \right)^{\frac{3}{2}} - \sum_{j=1}^{n} \left( \Re \delta(j) \right)^{\frac{3}{2}}
\]

Where \( \Re \delta(j) \) is the weighted intuitionistic fuzzy value \( \Re \delta(j) = \Re_j = \left( \Re_j \right)_{n \partial \Re_j} \), \( j = 1, 2, \ldots, n \) of the intuitionistic fuzzy ordered weighted geometric (IFOWG) pair \( \langle u_j, \Re_j \rangle \) having the jth largest \( u_j \) in \( \langle u, \Re \rangle \) is the order inducing variable and \( \Re_j \) is the intuitionistic fuzzy argument variable, \( \Im = (\Im_1, \Im_2, \ldots, \Im_n)^T \) be the weighted vector of I-IFEHG operator with \( \Im_j \in [0,1] \) and \( \sum_{j=1}^{n} \Im_j = 1 \). \( \partial = (\partial_1, \partial_2, \ldots, \partial_n)^T \) with \( \partial_j \in [0,1] \) and \( \sum_{j=1}^{n} \partial_j = 1 \).

**Theorem 9:** Let \( \langle u_j, \Re_j \rangle \) \( (j = 1, 2, \ldots, n) \) be a family of 2-tuples, then their aggregated value by using the I-IFEHG is also a IFV and

\[
\text{I-IFEHG}_{\partial, \Im} \left( \langle u_1, \Re_1 \rangle, \langle u_2, \Re_2 \rangle, \ldots, \langle u_n, \Re_n \rangle \right) = \left( \sum_{j=1}^{n} \Re \delta(j) \right)^{\frac{3}{2}} - \sum_{j=1}^{n} \left( \Re \delta(j) \right)^{\frac{3}{2}}
\]

**Proof:** The proof follows from Theorem 4.

**Theorem 10:** Let \( \langle u_j, \Re_j \rangle \) \( (j = 1, 2, \ldots, n) \) be a family of 2-tuples, then
\[ \text{I-IFHG}_{\hat{\rho}, \hat{\zeta}}(\langle u_1, R_1 \rangle, \langle u_2, R_2 \rangle, \ldots, \langle u_n, R_n \rangle) \leq \text{I-IFEHG}_{\hat{\rho}, \hat{\zeta}}(\langle u_1, R_1 \rangle, \langle u_2, R_2 \rangle, \ldots, \langle u_n, R_n \rangle) \] (32)

**Proof:** The proof follows from Theorem 5.

**Example 2:**

\[ R_1 = \{0.80, (0.70, 0.20)\}, R_2 = \{0.50, (0.60, 0.30)\}, R_3 = \{0.40, (0.50, 0.50)\}, R_4 = \{0.30, (0.40, 0.60)\} \]

are four values and \( \hat{\rho} = (0.10, 0.20, 0.30, 0.40)^T \), \( \hat{\zeta} = (0.10, 0.20, 0.30, 0.40)^T \), then we have

\[ \hat{R}_1 = (0.867, 0.085), \hat{R}_2 = (0.664, 0.248), \hat{R}_3 = (0.435, 0.564), \hat{R}_4 = (0.230, 0.769) \]. Hence

\[ \langle u_{\delta(1)}, \hat{R}_{\delta(1)} \rangle = (0.867, 0.085), \langle u_{\delta(2)}, \hat{R}_{\delta(2)} \rangle = (0.664, 0.248), \langle u_{\delta(3)}, \hat{R}_{\delta(3)} \rangle = (0.435, 0.564), \langle u_{\delta(4)}, \hat{R}_{\delta(4)} \rangle = (0.230, 0.769) \]

I-IFHG\[\langle u_1, R_1 \rangle, \langle u_2, R_2 \rangle, \langle u_3, R_3 \rangle, \langle u_4, R_4 \rangle\]

\[ = (0.867)^{0.10} (0.664)^{0.20} (0.435)^{0.30} (0.230)^{0.40} (1 - (1 - 0.085)^{0.10} (1 - 0.248)^{0.20} (1 - 0.564)^{0.30} (1 - 0.769)^{0.40}) \]

\[ = (0.867)^{0.10} (0.664)^{0.20} (0.435)^{0.30} (0.230)^{0.40} (1 - (0.915)^{0.10} (0.752)^{0.20} (0.436)^{0.30} (0.231)^{0.40}) = (0.393, 0.594) \]

Again using the I-IFEHG geometric operator, then we have

\[ \hat{R}_1 = (0.876, 0.080), \hat{R}_2 = (0.673, 0.242), \hat{R}_3 = (0.422, 0.577), \hat{R}_4 = (0.196, 0.803) \]. Hence

\[ \langle u_{\delta(1)}, \hat{R}_{\delta(1)} \rangle = (0.876, 0.080), \langle u_{\delta(2)}, \hat{R}_{\delta(2)} \rangle = (0.673, 0.242), \langle u_{\delta(3)}, \hat{R}_{\delta(3)} \rangle = (0.422, 0.577), \langle u_{\delta(4)}, \hat{R}_{\delta(4)} \rangle = (0.196, 0.803) \]

Thus

\[ \text{I-IFEHWG}_{\hat{\rho}, \hat{\zeta}}(\langle u_1, R_1 \rangle, \langle u_2, R_2 \rangle, \langle u_3, R_3 \rangle, \langle u_4, R_4 \rangle) \]

\[ = \begin{pmatrix}
2(0.876)^{0.10} (0.673)^{0.20} (0.422)^{0.30} (0.196)^{0.40} \\
(2 - 0.876)^{0.10} (2 - 0.673)^{0.20} (2 - 0.422)^{0.30} (2 - 0.196)^{0.40} + (0.876)^{0.10} (0.673)^{0.20} (0.422)^{0.30} (0.196)^{0.40} \\
(1 + 0.080)^{0.10} (1 + 0.242)^{0.20} (1 + 0.577)^{0.30} (1 + 0.803)^{0.40} - (1 - 0.080)^{0.10} (1 - 0.242)^{0.20} (1 - 0.577)^{0.30} (1 - 0.803)^{0.40} \\
(1 + 0.080)^{0.10} (1 + 0.242)^{0.20} (1 + 0.577)^{0.30} (1 + 0.803)^{0.40} + (1 - 0.080)^{0.10} (1 - 0.242)^{0.20} (1 - 0.577)^{0.30} (1 - 0.803)^{0.40}
\end{pmatrix} = \begin{pmatrix}
0.404, 0.585
\end{pmatrix} \]

**Theorem 11:** Intuitionistic fuzzy Einstein weighted geometric operator is a specific case of the induced intuitionistic fuzzy Einstein hybrid geometric operator.

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**Proof:** Let \( Z = \left( \frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n} \right)^T \), then we have

\[ 1\text{-IFEGH}_{\vartheta, Z}(\langle u_1, \mathcal{R}_1 \rangle, \langle u_2, \mathcal{R}_2 \rangle, ..., \langle u_n, \mathcal{R}_n \rangle) = \left( \hat{\mathcal{R}}_{\vartheta(1)} \right)^{\frac{1}{n}} \otimes_E \left( \hat{\mathcal{R}}_{\vartheta(2)} \right)^{\frac{1}{n}} \otimes_E \cdots \otimes_E \left( \hat{\mathcal{R}}_{\vartheta(n)} \right)^{\frac{1}{n}} \]

\[ = \left( \hat{\mathcal{R}}_{\vartheta(1)} \right)^{1} \otimes_E \left( \hat{\mathcal{R}}_{\vartheta(2)} \right)^{1} \otimes_E \cdots \otimes_E \left( \hat{\mathcal{R}}_{\vartheta(n)} \right)^{1} \]

\[ = \text{IFEWG}_{\vartheta}(\mathcal{R}_1, \mathcal{R}_2, ..., \mathcal{R}_n) \]

**Theorem 12:** Induced intuitionistic fuzzy Einstein ordered weighted geometric operator is a specific case of the induced intuitionistic fuzzy Einstein hybrid geometric operator

**Proof:** Let \( \vartheta = \left( \frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n} \right)^T \) and \( \hat{\mathcal{R}}_{\vartheta(j)} = \left( \hat{\mathcal{R}}_{\vartheta(j)} \right)^{1} = \mathcal{R}_{\vartheta(j)} \), then

\[ 1\text{-IFEGH}_{\vartheta, Z}(\langle u_1, \mathcal{R}_1 \rangle, \langle u_2, \mathcal{R}_2 \rangle, ..., \langle u_n, \mathcal{R}_n \rangle) = \left( \hat{\mathcal{R}}_{\vartheta(1)} \right)^{1} \otimes_E \left( \hat{\mathcal{R}}_{\vartheta(2)} \right)^{1} \otimes_E \cdots \otimes_E \left( \hat{\mathcal{R}}_{\vartheta(n)} \right)^{1} \]

\[ = 1\text{-IFOWG}_{\vartheta}(\langle u_1, \mathcal{R}_1 \rangle, \langle u_2, \mathcal{R}_2 \rangle, ..., \langle u_n, \mathcal{R}_n \rangle) \]

**Definition 10:** The Induced generalized intuitionistic fuzzy Einstein hybrid geometric (1-GIFEHG) operator can be defined as:
I-GIFEHG\(\bar{\mathcal{A}}\), \(\mathcal{A} (\{u_1, \mathfrak{R}_1\}, \{u_2, \mathfrak{R}_2\}, \{u_3, \mathfrak{R}_3\}, \ldots, \{u_n, \mathfrak{R}_n\})\)

\[
\begin{align*}
&\left(\prod_{j=1}^{n} \left(1+\xi_{\mathfrak{R}_j}(\delta_j)\right)^{\sigma} + 3\left(1-\xi_{\mathfrak{R}_j}(\delta_j)\right)^{\sigma}\right)^{3_j} + \frac{1}{\sigma} \\
&- \left(\prod_{j=1}^{n} \left(1+\xi_{\mathfrak{R}_j}(\delta_j)\right)^{\sigma} + 3\left(1-\xi_{\mathfrak{R}_j}(\delta_j)\right)^{\sigma}\right)^{3_j} - \frac{1}{\sigma} \\
&+ \left(\prod_{j=1}^{n} \left(2-\kappa_{\mathfrak{R}_j}(\delta_j)\right)^{\sigma} + 3\left(\kappa_{\mathfrak{R}_j}(\delta_j)\right)^{\sigma}\right)^{3_j} - \frac{1}{\sigma} \\
&+ \left(\prod_{j=1}^{n} \left(2-\kappa_{\mathfrak{R}_j}(\delta_j)\right)^{\sigma} + 3\left(\kappa_{\mathfrak{R}_j}(\delta_j)\right)^{\sigma}\right)^{3_j} - \frac{1}{\sigma}
\end{align*}
\]

Where \(\mathfrak{R}_j(\delta_j)\) is the weighted intuitionistic fuzzy value \(\mathfrak{R}_j\left(\mathfrak{R}_j = (\mathfrak{R}_j)^{\nu_{\mathfrak{R}_j}}, j = 1, 2, \ldots, n\right)\) of the intuitionistic fuzzy ordered weighted geometric (IFOWG) pair \(\{u_j, \mathfrak{R}_j\}\) having the jth largest \(u_j\) in \(\{u_j, \mathfrak{R}_j\}\), \(\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n)^T\) be the weighted vector of I-GIFEHG operator with \(\mathcal{A}_j \in [0, 1]\) and \(\sum_{j=1}^{n} \mathcal{A}_j = 1\). \(\mathfrak{R}_j = (\mathfrak{R}_1, \mathfrak{R}_2, \ldots, \mathfrak{R}_n)^T\) with \(\mathfrak{R}_j \in [0, 1]\) and \(\sum_{j=1}^{n} \mathfrak{R}_j = 1\). And \(\sigma\) is a real number greater than zero.
**Theorem 13:** Let \( \langle u_j, \mathfrak{R}_j \rangle \ (j = 1, 2, \ldots, n) \) be a family of 2-tuples, then their aggregated IFV by using the I-GIFEHG operator can be expressed as:

\[
\text{I-GIFEHG}_{\sigma, 3} \left( \langle u_1, \mathfrak{R}_1 \rangle, \langle u_2, \mathfrak{R}_2 \rangle, \langle u_3, \mathfrak{R}_3 \rangle, \ldots, \langle u_n, \mathfrak{R}_n \rangle \right) = \sum_{j=1}^{n} \left[ \prod_{i=1}^{n} \left( 1 + \frac{\xi_j \delta_j}{\sigma} \right) + 3 \left( 1 - \frac{\xi_j \delta_j}{\sigma} \right) \right]^{3-j} \frac{1}{\sigma}
\]

**Proof:** Since

\[
\sigma \cdot \bar{\mathfrak{R}} = \left( \frac{(1 + \xi_j \delta_j) - (1 - \xi_j \delta_j)}{(1 + \xi_j \delta_j) + (1 - \xi_j \delta_j)} \right)
\]

Then
\[
\bigotimes_{j=1}^{n} e^{\kappa \delta(j)} = \prod_{j=1}^{n} \left( \frac{1 + \xi \delta(j)}{1 - \xi \delta(j)} \right)^{s_j} + \sum_{j=1}^{n} \left( \frac{\kappa \delta(j)}{1 - \kappa \delta(j)} \right)^{s_j}
\]
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Thus
Some induced generalized Einstein aggregating operators and their application to...

I-GIFEHG,\(\mathcal{J}\)\((u_1, R_1), (u_2, R_2), (u_3, R_3), \ldots, (u_n, R_n)\)

\[
\sum_{j=1}^{n} \left[ \left( 1 + \frac{\sigma}{3} \right) \left( 1 + \frac{\sigma}{3} \right) \left( 1 + \frac{\sigma}{3} \right) \right] - \sum_{j=1}^{n} \left[ \left( 1 - \frac{\sigma}{3} \right) \left( 1 - \frac{\sigma}{3} \right) \left( 1 - \frac{\sigma}{3} \right) \right] 
\]

\[
= \prod_{j=1}^{n} \left[ \left( 1 - \frac{\sigma}{2} \right) \left( 1 - \frac{\sigma}{2} \right) \right] - \prod_{j=1}^{n} \left[ \left( 1 + \frac{\sigma}{2} \right) \left( 1 + \frac{\sigma}{2} \right) \right] 
\]

**Definition 11:** The Induced generalized intuitionistic fuzzy Einstein hybrid averaging (I-GIFEHA) operator can be defined as follows:
I-GIFEHA\[\mathcal{E}_{\mathcal{G}}(\langle u_1, R_1 \rangle, \langle u_2, R_2 \rangle, \langle u_3, R_3 \rangle, ..., \langle u_n, R_n \rangle)\]

\[
\begin{align*}
&\frac{2}{\mathcal{E}_{\mathcal{G}}} \left[ \prod_{j=1}^{n} \left( 2 - \varepsilon_{\mathcal{G}}(j) \right)^{\sigma} + 3 \left( \varepsilon_{\mathcal{G}}(j) \right)^{\sigma} \right]^{3} - \prod_{j=1}^{n} \left( 2 - \varepsilon_{\mathcal{G}}(j) \right)^{\sigma} - \left( \varepsilon_{\mathcal{G}}(j) \right)^{\sigma} \right]^{3} \right]^{\frac{1}{\sigma}} \\
&+ \left[ \prod_{j=1}^{n} \left( 2 - \varepsilon_{\mathcal{G}}(j) \right)^{\sigma} + 3 \left( \varepsilon_{\mathcal{G}}(j) \right)^{\sigma} \right]^{3} - \prod_{j=1}^{n} \left( 2 - \varepsilon_{\mathcal{G}}(j) \right)^{\sigma} - \left( \varepsilon_{\mathcal{G}}(j) \right)^{\sigma} \right]^{3} \right]^{\frac{1}{\sigma}} \\
\end{align*}
\]

(35)

where \( \varepsilon_{\mathcal{G}}(j) \) is the weighted intuitionistic fuzzy value \( \varepsilon_{\mathcal{G}}(j) = n \varepsilon_{\mathcal{G}}(j)R_j, j = 1, 2, ..., n \) of the intuitionistic fuzzy ordered weighted averaging (IFOWA) pair \( \langle u_j, \mathcal{R}_j \rangle \) having the jth largest \( u_j \) in \( \langle u_j, \mathcal{R}_j \rangle \), \( \mathcal{R} = (\mathcal{R}_1, \mathcal{R}_2, ..., \mathcal{R}_n)^T \) be the associated vector of I-GIFEHA operator with \( \partial_j \in [0,1] \) and \( \sum_{j=1}^{n} \partial_j = 1 \). \( \partial = (\partial_1, \partial_2, ..., \partial_n)^T \) be the weighted with \( \partial_j \in [0,1] \) and \( \sum_{j=1}^{n} \partial_j = 1 \). And \( \sigma \) is a real number greater than zero.

5. Application of the Proposed Methods

In this section, a MAGDM approach has been developed to show the applications and advantages of the new developed methods. For this purpose, we construct and algorithms and also practical example for the selection of new system of information.
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Algorithms: Let the set of n finite alternatives with \( p = \{ p_1, p_2, p_3, ..., p_n \} \), set of m criteria/attributes with \( \Psi = \{ \Psi_1, \Psi_2, \Psi_3, ..., \Psi_m \} \) and the set of k experts with \( E = \{ E_1, E_2, ..., E_k \} \).

Let \( \Xi = (\Xi_1, \Xi_2, ..., \Xi_n)^T \) be the associated, \( \partial = (\partial_1, \partial_2, ..., \partial_n)^T \) be the weighted vector and \( \varphi = (\varphi_1, \varphi_2, ..., \varphi_k)^T \) be weight of decision makers, all have the same conditions that is belongs to the close interval and their sum is equal to 1.

**Step 1:** Construct decision-making matrices, \( E^s = \left[ a_{ij}^{(h)} \right]_{m \times n} \) (\( h = 1, 2, ..., k \)) for decision.

**Step 2:** If the criteria have two types such as, benefit criteria and cost criteria, then the interval-valued intuitionistic fuzzy decision-making matrices, \( R^h = \left[ r_{ij}^{(h)} \right]_{m \times n} \) (\( h = 1, 2, ..., k \)) can be converted into the normalized Pythagorean fuzzy decision-making matrices, \( R^h = \left[ r_{ij}^{(h)} \right]_{m \times n} \) (\( h = 1, 2, ..., k \)), where

\[
\begin{align*}
    r_{ij}^{(h)} &= \begin{cases}
    a_{ij}^{(h)} & \text{for benefit criteria } C_j (j = 1, 2, ..., n) \\
    \bar{a}_{ij}^{(h)} & \text{for cost criteria } C_j (i = 1, 2, ..., m)
    \end{cases}
\end{align*}
\]

and \( a_{ij}^{(s)} \) is the complement of \( a_{ij}^{s} \). If all the criteria have the same type, then there is no need of normalization.

**Step 3:** Utilize the proposed aggregation operator to aggregate all the individual normalized intuitionistic fuzzy decision-making matrices, \( R^h = \left[ r_{ij}^{(h)} \right]_{m \times n} \) (\( h = 1, 2, ..., k \)) into a single intuitionistic fuzzy decision-making matrix, \( R = \left[ r_{ij} \right]_{m \times n} \).

**Step 4:** Calculate \( \hat{R}_{ij} = n \hat{\partial}_j r_{ij} \).

**Step 5:** Again utilize the proposed operators to derive the overall preference values, and then calculate the scores of all preference values

**Step 6:** That alternative/choice will be consider having the high score function.

6. Illustrative Example
Supposing in Abdul Wali Khan University Mardan, the department of computer science wishes to introduce a new system for information in the university. In the first collection, only four systems of information (alternatives) \( p_m (m = 1, 2, 3, 4) \) have been considered for further process. For the selection of best option three experts from a group to act as decision makers, and their weight is \( \varphi = (0.30, 0.30, 0.40)^T \). For the selecting of most suitable system the experts considered only four criteria, whose weighted vector is \( \mathcal{A} = (0.10, 0.20, 0.30, 0.40)^T \). \( \mathcal{V}_1 \) : Prices of instrument, \( \mathcal{V}_2 \) : Funding of the university, \( \mathcal{V}_3 \) : Effort to transform from current systems, \( \mathcal{V}_4 \) : Outsourcing software developer reliability, where \( \mathcal{V}_1, \mathcal{V}_3 \) are cost criteria and \( \mathcal{V}_2, \mathcal{V}_4 \) benefit criteria.

Step 1: Construct decision-making matrices

<table>
<thead>
<tr>
<th>( \mathcal{V}_1 )</th>
<th>( \mathcal{V}_2 )</th>
<th>( \mathcal{V}_3 )</th>
<th>( \mathcal{V}_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_1 )</td>
<td>( 0.80, (0.50, 0.40) )</td>
<td>( 0.70, (0.50, 0.40) )</td>
<td>( 0.60, (0.60, 0.40) )</td>
</tr>
<tr>
<td>( h_2 )</td>
<td>( 0.90, (0.60, 0.30) )</td>
<td>( 0.80, (0.40, 0.50) )</td>
<td>( 0.60, (0.60, 0.40) )</td>
</tr>
<tr>
<td>( h_3 )</td>
<td>( 0.70, (0.50, 0.40) )</td>
<td>( 0.60, (0.50, 0.30) )</td>
<td>( 0.50, (0.50, 0.30) )</td>
</tr>
<tr>
<td>( h_4 )</td>
<td>( 0.60, (0.60, 0.30) )</td>
<td>( 0.50, (0.50, 0.40) )</td>
<td>( 0.40, (0.50, 0.40) )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \mathcal{V}_1 )</th>
<th>( \mathcal{V}_2 )</th>
<th>( \mathcal{V}_3 )</th>
<th>( \mathcal{V}_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_1 )</td>
<td>( 0.70, (0.60, 0.30) )</td>
<td>( 0.60, (0.50, 0.40) )</td>
<td>( 0.50, (0.60, 0.30) )</td>
</tr>
<tr>
<td>( h_2 )</td>
<td>( 0.80, (0.50, 0.30) )</td>
<td>( 0.70, (0.60, 0.40) )</td>
<td>( 0.60, (0.40, 0.50) )</td>
</tr>
<tr>
<td>( h_3 )</td>
<td>( 0.60, (0.60, 0.30) )</td>
<td>( 0.50, (0.60, 0.30) )</td>
<td>( 0.40, (0.60, 0.40) )</td>
</tr>
<tr>
<td>( h_4 )</td>
<td>( 0.50, (0.40, 0.50) )</td>
<td>( 0.40, (0.60, 0.20) )</td>
<td>( 0.30, (0.40, 0.50) )</td>
</tr>
</tbody>
</table>

Table 1. Decision Matrix E1.

Table 2. Decision Matrix E2.

Table 3. Decision Matrix of E3.
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\[ h_1 \langle 0.60, (0.50, 0.40) \rangle \langle 0.50, (0.50, 0.40) \rangle \langle 0.40, (0.50, 0.40) \rangle \langle 0.30, (0.40, 0.40) \rangle \\
\]

\[ h_2 \langle 0.70, (0.50, 0.30) \rangle \langle 0.60, (0.40, 0.30) \rangle \langle 0.50, (0.50, 0.30) \rangle \langle 0.40, (0.60, 0.30) \rangle \\
\]

\[ h_3 \langle 0.50, (0.60, 0.30) \rangle \langle 0.40, (0.60, 0.30) \rangle \langle 0.30, (0.50, 0.40) \rangle \langle 0.20, (0.60, 0.30) \rangle \\
\]

\[ h_4 \langle 0.40, (0.60, 0.20) \rangle \langle 0.30, (0.50, 0.40) \rangle \langle 0.20, (0.70, 0.10) \rangle \langle 0.10, (0.60, 0.30) \rangle \\
\]

**Step 2:** Construct normalized decision-making matrices

### Table 4. Normalized Decision Matrix R1.

<table>
<thead>
<tr>
<th>( \mathcal{Y}_1 )</th>
<th>( \mathcal{Y}_2 )</th>
<th>( \mathcal{Y}_3 )</th>
<th>( \mathcal{Y}_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_1 )</td>
<td>( 0.80,(0.40,0.50) )</td>
<td>( 0.70,(0.50,0.40) )</td>
<td>( 0.60,(0.40,0.60) )</td>
</tr>
<tr>
<td>( h_2 )</td>
<td>( 0.90,(0.30,0.60) )</td>
<td>( 0.80,(0.40,0.50) )</td>
<td>( 0.70,(0.50,0.40) )</td>
</tr>
<tr>
<td>( h_3 )</td>
<td>( 0.70,(0.40,0.50) )</td>
<td>( 0.60,(0.50,0.30) )</td>
<td>( 0.50,(0.30,0.50) )</td>
</tr>
<tr>
<td>( h_4 )</td>
<td>( 0.60,(0.30,0.60) )</td>
<td>( 0.50,(0.50,0.40) )</td>
<td>( 0.40,(0.40,0.50) )</td>
</tr>
</tbody>
</table>

### Table 5. Normalized Decision Matrix R2.

<table>
<thead>
<tr>
<th>( \mathcal{Y}_1 )</th>
<th>( \mathcal{Y}_2 )</th>
<th>( \mathcal{Y}_3 )</th>
<th>( \mathcal{Y}_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_1 )</td>
<td>( 0.70,(0.30,0.60) )</td>
<td>( 0.60,(0.50,0.40) )</td>
<td>( 0.50,(0.30,0.60) )</td>
</tr>
<tr>
<td>( h_2 )</td>
<td>( 0.80,(0.30,0.50) )</td>
<td>( 0.70,(0.60,0.40) )</td>
<td>( 0.60,(0.50,0.40) )</td>
</tr>
<tr>
<td>( h_3 )</td>
<td>( 0.60,(0.30,0.50) )</td>
<td>( 0.50,(0.60,0.30) )</td>
<td>( 0.40,(0.40,0.60) )</td>
</tr>
<tr>
<td>( h_4 )</td>
<td>( 0.50,(0.50,0.40) )</td>
<td>( 0.40,(0.60,0.20) )</td>
<td>( 0.30,(0.50,0.40) )</td>
</tr>
</tbody>
</table>

### Table 6. Normalized Decision Matrix R3.
\begin{align*}
\forall_1 & \quad \forall_2 \quad \forall_3 \quad \forall_4 \\
\rho_1 & \quad \langle 0.60, (0.40, 0.50) \rangle \quad \langle 0.50, (0.50, 0.40) \rangle \quad \langle 0.40, (0.40, 0.50) \rangle \quad \langle 0.30, (0.40, 0.40) \rangle \\
\rho_2 & \quad \langle 0.70, (0.30, 0.50) \rangle \quad \langle 0.60, (0.40, 0.30) \rangle \quad \langle 0.40, (0.30, 0.50) \rangle \quad \langle 0.40, (0.60, 0.30) \rangle \\
\rho_3 & \quad \langle 0.50, (0.30, 0.60) \rangle \quad \langle 0.40, (0.60, 0.30) \rangle \quad \langle 0.30, (0.40, 0.50) \rangle \quad \langle 0.20, (0.60, 0.3) \rangle \\
\rho_4 & \quad \langle 0.40, (0.20, 0.60) \rangle \quad \langle 0.30, (0.50, 0.40) \rangle \quad \langle 0.20, (0.10, 0.70) \rangle \quad \langle 0.10, (0.60, 0.30) \rangle \\
\end{align*}

**Step 3:** Utilize the I-IFEWA operator, where \( \varphi = (0.30, 0.30, 0.40)^T \). Then we have

<table>
<thead>
<tr>
<th>( \forall_1 )</th>
<th>( \forall_2 )</th>
<th>( \forall_3 )</th>
<th>( \forall_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_1 )</td>
<td>( 0.3716, 0.5288 )</td>
<td>( 0.5000, 0.4000 )</td>
<td>( 0.3716, 0.5581 )</td>
</tr>
<tr>
<td>( \rho_2 )</td>
<td>( 0.3000, 0.5288 )</td>
<td>( 0.4657, 0.3833 )</td>
<td>( 0.4246, 0.4380 )</td>
</tr>
<tr>
<td>( \rho_3 )</td>
<td>( 0.3308, 0.5688 )</td>
<td>( 0.5717, 0.3000 )</td>
<td>( 0.3912, 0.5288 )</td>
</tr>
<tr>
<td>( \rho_4 )</td>
<td>( 0.3263, 0.5343 )</td>
<td>( 0.5316, 0.3277 )</td>
<td>( 0.2861, 0.5753 )</td>
</tr>
</tbody>
</table>

**Step 4:** Calculate \( \hat{R}_{ij} = n\hat{\varphi} \hat{R}_{ij} \), where \( \hat{\varphi} = (0.10, 0.20, 0.30, 0.40)^T \). Then we have

\[
\begin{align*}
\hat{R}_{11} &= (0.154, 0.798), \hat{R}_{12} = (0.415, 0.496), \hat{R}_{13} = (0.436, 0.466), \hat{R}_{14} = (0.733, 0.196), \hat{R}_{21} = (0.123, 0.798) \\
\hat{R}_{22} &= (0.383, 0.480), \hat{R}_{23} = (0.450, 0.357), \hat{R}_{24} = (0.748, 0.117), \hat{R}_{31} = (0.136, 0.817), \hat{R}_{32} = (0.477, 0.399) \\
\hat{R}_{33} &= (0.458, 0.453), \hat{R}_{34} = (0.758, 0.117), \hat{R}_{41} = (0.134, 0.800), \hat{R}_{42} = (0.441, 0.427), \hat{R}_{43} = (0.339, 0.503) \\
\hat{R}_{44} &= (0.777, 0.196). 
\end{align*}
\]

Now we calculate the score functions:

\[
\begin{align*}
S(\hat{R}_{11}) &= 0.154 - 0.798 = -0.644, S(\hat{R}_{12}) = 0.415 - 0.496 = -0.081, S(\hat{R}_{13}) = 0.436 - 0.466 = -0.030, S(\hat{R}_{14}) = 0.733 - 0.196 = 0.537 \\
S(\hat{R}_{21}) &= 0.123 - 0.798 = -0.675, S(\hat{R}_{22}) = 0.383 - 0.480 = -0.097, S(\hat{R}_{23}) = 0.450 - 0.357 = 0.093, S(\hat{R}_{24}) = 0.748 - 0.117 = 0.631 \\
S(\hat{R}_{31}) &= 0.136 - 0.817 = -0.681, S(\hat{R}_{32}) = 0.477 - 0.399 = 0.078, S(\hat{R}_{33}) = 0.458 - 0.453 = 0.005, S(\hat{R}_{34}) = 0.758 - 0.117 = 0.641 \\
S(\hat{R}_{41}) &= 0.134 - 0.800 = -0.666, S(\hat{R}_{42}) = 0.441 - 0.427 = 0.014, S(\hat{R}_{43}) = 0.339 - 0.503 = -0.164, S(\hat{R}_{44}) = 0.777 - 0.196 = 0.581
\end{align*}
\]

**Table 8. Hybrid Decision Matrix R.**
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<table>
<thead>
<tr>
<th></th>
<th>( Y_1 )</th>
<th>( Y_2 )</th>
<th>( Y_3 )</th>
<th>( Y_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_1 )</td>
<td>(0.733, 0.196)</td>
<td>(0.436, 0.466)</td>
<td>(0.415, 0.496)</td>
<td>(0.154, 0.798)</td>
</tr>
<tr>
<td>( p_2 )</td>
<td>(0.748, 0.117)</td>
<td>(0.450, 0.357)</td>
<td>(0.383, 0.480)</td>
<td>(0.123, 0.798)</td>
</tr>
<tr>
<td>( p_3 )</td>
<td>(0.758, 0.117)</td>
<td>(0.477, 0.399)</td>
<td>(0.458, 0.453)</td>
<td>(0.136, 0.817)</td>
</tr>
<tr>
<td>( p_4 )</td>
<td>(0.777, 0.196)</td>
<td>(0.441, 0.427)</td>
<td>(0.339, 0.503)</td>
<td>(0.134, 0.800)</td>
</tr>
</tbody>
</table>

Step 5: Utilize the I-IFEHA operator, where \( \mathbf{Z} = (0.10, 0.20, 0.30, 0.40)^T \). Then we have

\[
\eta = (0.3637, 0.5520), r_2 = (0.3510, 0.5024), r_3 = (0.3858, 0.5584), r_4 = (0.3042, 0.5494)
\]

\[
S(\eta) = 0.363 - 0.552 = -0.189, S(r_2) = 0.351 - 0.502 = -0.151, S(r_3) = 0.385 - 0.558 = -0.173, S(r_4) = 0.304 - 0.549 = -0.245
\]

Step 6: Thus the best option is \( p_2 \).

Table 9. Comparative analysis with some existing methods.

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>Su et al. [37]</th>
<th>Rahman et al. [39]</th>
<th>Proposed Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Methods</td>
<td>Ranking</td>
<td>Methods</td>
</tr>
<tr>
<td>1</td>
<td>I-IFOWA</td>
<td>(2134)</td>
<td>IFEHA</td>
</tr>
<tr>
<td></td>
<td>I-IFOWG</td>
<td>(2134)</td>
<td>IFEHG</td>
</tr>
<tr>
<td></td>
<td>I-IFHA</td>
<td>(2134)</td>
<td>GIFEHA</td>
</tr>
<tr>
<td></td>
<td>I-IFHG</td>
<td>(2134)</td>
<td>GIFEHG</td>
</tr>
<tr>
<td>2</td>
<td>I-IFOWA</td>
<td>(2134)</td>
<td>IFEHA</td>
</tr>
<tr>
<td></td>
<td>I-IFOWG</td>
<td>(2134)</td>
<td>IFEHG</td>
</tr>
<tr>
<td></td>
<td>I-IFHA</td>
<td>(2134)</td>
<td>GIFEHA</td>
</tr>
<tr>
<td></td>
<td>I-IFHG</td>
<td>(2134)</td>
<td>GIFEHG</td>
</tr>
<tr>
<td>3</td>
<td>I-IFOWA</td>
<td>(2134)</td>
<td>IFEHA</td>
</tr>
<tr>
<td></td>
<td>I-IFOWG</td>
<td>(2134)</td>
<td>IFEHG</td>
</tr>
<tr>
<td></td>
<td>I-IFHA</td>
<td>(2134)</td>
<td>GIFEHA</td>
</tr>
<tr>
<td></td>
<td>I-IFHG</td>
<td>(2134)</td>
<td>GIFEHG</td>
</tr>
<tr>
<td>4</td>
<td>I-IFOWA</td>
<td>(2134)</td>
<td>IFEHA</td>
</tr>
<tr>
<td></td>
<td>I-IFOWG</td>
<td>(2134)</td>
<td>IFEHG</td>
</tr>
<tr>
<td></td>
<td>I-IFHA</td>
<td>(2134)</td>
<td>GIFEHA</td>
</tr>
<tr>
<td></td>
<td>I-IFHG</td>
<td>(2134)</td>
<td>GIFEHG</td>
</tr>
</tbody>
</table>

7. Conclusion

Generalized Einstein operators syndicate Einstein operators with some generalized operators using IFNs. Hence Einstein operational laws play an imperative, significant and important role in our daily problems in the case of best selection. Therefore, here we have presented and explore some new types methods and operators using on IFNs base.
on Einstein operations, such as I-IFEHA operator, I-IFEHG operator, I-GIFEHA operator, I-GIFEHG operator. Additionally, the new methods have been utilized and exploited with decision-making. We have discussed the applicability in a decision-making concerning strategic choice of the best information system and also construct examples and the operational processes were illustrated in detail to show the efficacy and value of the developed new methods. Some of their wanted and structure properties, such as monotonicity, boundedness idempotency, commutativity were developed and some new results were developed. Finally, an illustrative example is given to show the steps of decision process of the proposed operators and methods.

In further research, it is necessary to give the applications of these operators to the other domains such as, Confidence levels, Hamacher operators, Power operators, Symmetric operator, Logarithmic operators, Dombi operators, Linguistic terms, trigonometric operation, ranking method for normal intuitionistic sets.

References


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