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An algorithm for an improved intuitionistic fuzzy correlation measure with medical diagnostic application

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Abstract Correlation measure is a vital measuring operator with vast applications in decision-making. On the other hand, intuitionistic fuzzy set (IFS) is very resourceful in soft computing to tackle embedded fuzziness in decision-making. The extension of correlation measure to intuitionistic fuzzy setting has proven to be useful in multi-criteria decision-making (MCDM). This paper introduces a new intuitionistic fuzzy correlation measure encapsulates in an algorithm by taking into account the complete parameters of IFSs. This new computing technique evaluates the strength of relationship and it is defined within the codomain of IFS. The proposed technique is demonstrated with some theoretical results, and numerically authenticated to be superior in terms of performance index in contrast to some existing correlation measures. We demonstrate the application of the new correlation measure coded with JAVA programming language in medical diagnosis to enhance efficiency since diagnosis is a delicate medical decision-making exercise.

Keywords Algorithmic approach; Correlation measure; Intuitionistic fuzzy set; Medical diagnosis

1. Introduction

Identification and selection of choices on the basis of preferences of decision-makers via relevant assessment criteria is known as decision-making. Decision-making involving multi-criteria approach, i.e., MCDM, is an operational technique for curbing complex engineering problems in machine language, artificial intelligence, etc. Fuzzy data or variables constitute massive problems in decision-making. To this end, Zadeh (1965)

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introduced fuzzy set to model the fuzziness in human knowledge. The idea of correlation measure has been extended to fuzzy environment as discussed in (Chiang and Lin, 1999; Dumitrescu, 1978; Murthy *et al*., 1985; Yu, 1993) to boost the measure of correlation between fuzzy sets. Although the idea of fuzzy models has solved a great deal of problems in decision-making, it is limited because it considered only the grade of membership of the concerned data.

Atanassov (1986) proposed a construct called intuitionistic fuzzy sets (IFSs) to amend the setback of the ordinary fuzzy sets and therefore, well positioned to solve real-life problems with precision. IFS constituents membership degree (MD) η, non-membership degree (NMD) θ and the possibility of the existent of hesitation margin (HM) ϱ in which $\eta + \theta \leq 1$ and the sum of the three parameters is one. The concept of IFSs has been researched and applied to various applicative areas via measuring tools (like distance and similarity measures) because of its flexibility in solving real-life problems (Atanassov, 1999; Boran and Akay, 2014; De *et al*., 2001; Ejegwa, 2015; Ejegwa *et al*., 2014a, 2014b; Ejegwa and Modom, 2015; Ejegwa and Onasanya, 2019, Ejegwa and Onyeke, 2018; Hatzimichailidis *et al*., 2012; Liu and Chen, 2017; Szmidt and Kacprzyk, 2001, 2004; Wang and Xin, 2005; Davvaz and Sadrabadi, 2016).

To measure the correlation between IFSs, Gerstenkorn and Manko (1991) initiated the study on correlation coefficient between IFSs (CCIFSs). Xu *et al.* (2008) extended the approach in (Gerstenkorn and Manko, 1991) by taking cognizance of HM. Zeng and Li (2007) proposed a new method to calculate CCIFSs which modified the approach in (Xu *et al*., 2008). Ejegwa (2020) proposed a new CCIFSs method with applications involving some MCDM problems. Garg and Kumar (2018) presented a CCIFSs method on the basis of set pair analysis and used it to solve MCDM problems. The idea of correlation coefficient and its applications have been considered in complex intuitionistic fuzzy and intuitionistic multiplicative environments (Garg, 2018; Garg and Rani, 2018). Many other methods of computing CCIFSs have been researched and applied to various applicative areas by different researchers as seen in (Garg and Arora, 2020; Hong and Hwang, 1995; Hung, 2001; Hung and Wu, 2002; Liu *et al*., 2016; Mitchell, 2004; Park *et al*., 2009; Szmidt and Kacprzyk, 2010; Thao, 2018; Thao *et al*., 2019; Xu, 2006).

In a critical study of the correlation measures in (Garg, 2016; Gerstenkorn and Manko, 1991; Xu *et al*., 2008; Zeng and Li, 2007) defined in the closed interval [0,1], we spotted some drawbacks which motivated this present research. In this paper, we introduce a new intuitionistic fuzzy correlation measure encapsulate in an algorithm to save time and enhance efficiency with high performance rating in comparison to the measures in (Garg, 2016; Gerstenkorn and Manko, 1991; Xu *et al*., 2008; Zeng and Li, 2007). The objectives of the paper are to; review the performances of the methods for computing CCIFSs in (Garg, 2016; Gerstenkorn and Manko, 1991; Xu *et al*., 2008; Zeng and Li, 2007), introduce a high performance intuitionistic fuzzy correlation measure and its algorithm, mathematically prove the consistency of the new intuitionistic fuzzy correlation measure with the axiomatic description of correlation measure, validate the superiority of the new technique over the approaches in (Garg, 2016; Gerstenkorn and Manko, 1991; Xu *et al*., 2008; Zeng and Li, 2007), codes the new correlation coefficient measure algorithm using JAVA programming language for efficiency, and determine diagnosis in a hypothetical case using the coded new intuitionistic fuzzy correlation coefficient measure.

For easy flow of discussion, we summarize the rest of the paper as follow; In Section 2, we present some basic ideas of IFSs and some existing correlation coefficient measures defined within [0,1]. Section 3 introduces the new intuitionistic fuzzy correlation coefficient measure with numerical verifications to ascertain its supremacy over the approaches in (Garg, 2016; Gerstenkorn and Manko, 1991; Xu *et al.*, 2008; Zeng and Li, 2007). Section 4 discusses the applications of the new intuitionistic fuzzy correlation measure algorithm coded with JAVA programming language in a hypothetical medical diagnosis scenario. In Section 5, the work is concluded with recommendation for future research.

2.Preliminaries

This section presents some basic ideas of IFSs and some correlation coefficient measures between IFSs defined within [0,1].

2.1.Intuitionistic fuzzy set

Let X denotes non-empty set throughout this work.

Definition 2.1 (Atanassov, 2012). An object in X denoted by G is called an intuitionistic fuzzy set if

$$G = \left\{\langle\frac{\eta G(x), \theta G(x)}{x}\rangle \middle| x \in X\right\}$$

where the functions $\eta G(x), \theta G(x)$: X → [0,1] define MD and NMD of $x \in X$ for

$0 \leq \eta\, G(x) + \theta G(x) \leq 1$

For any IFS G in X, $\varrho G(x) = 1 - \eta G(x) - \theta G(x)$ is the IFS index or HM of G.

Definition 2.2 (Atanassov, 1994). Assume G and H are IFSs in X, then we have the following definitions:

(i) G = H iff $\eta G(x) = \eta H(x)$ ηH(x) and $\theta G(x) = \eta H(x)$ for all $x \in X$.

(ii) G ⊆ H iff $\eta G(x) \leq \eta H(x)$ ηH(x) and $\theta G(x) \geq \eta H(x)$ for all $x \in X$.

(iii) $\bar{G} = \left\{\langle\frac{\eta G(x),\theta G(x)}{x}\rangle \middle| x \in X\right\}$

(iv)$G \cup H = \left\{\langle \max(\frac{\eta G(x),\eta H(x)}{x}), \min(\frac{\theta G(x),\theta H(x)}{x})\rangle \middle| x \in X\right\}$

(v) $G \cap H = \left\{\langle \min(\frac{\eta G(x),\eta H(x)}{x}), \max(\frac{\theta G(x),\theta H(x)}{x})\rangle \middle| x \in X\right\}$

Definition 2.3.(Ejegwa and Onasanya, 2019). Intuitionistic fuzzy values (IFVs) or Intuitionistic fuzzy pairs (IFPs) are characterized by the form hx,yi such that x + y ≤ 1 where x,y ∈ [0,1]. IFVs evaluate the IFS for which the components (x and y) are interpreted as MD and NMD.

2.2 Some existing methods of computing correlation coefficient defined within [0,1]

Many methods of calculating CCIFS defined within the range [0,1] have been studied in literature. Now, we review each of these methods before introducing the new method of computing correlation coefficient of IFSs. Suppose G and H are IFSs in $X = \{x_1, \dots, x_n\}$. Then, we present the following approaches of computing correlation coefficient.

2.2.1.Gerstenkorn and Manko's correlation coefficient for IFSs

Correlation coefficient in intuitionistic fuzzy context was initiated by Gerstenkorn and Manko (1991), and defined thus:

$$\sigma_1 = \frac{\phi(G,H)}{\sqrt{\psi(G)\psi(H)}} \quad (1)$$

where $\phi(G,H)$ is the correlation of G and H, $\psi(G) and\, \psi(H)$ are informational energies of G and H, respectively, defined by

$$\phi(G,H) = \sum_{i=1}^{n} [\eta G(x_i)\eta H(x_i) + \theta G(x_i) + \theta H(x_i)] \quad (2)$$

$$\psi(G) = \sum_{i=1}^{n} \eta_G^2\,(x_i) + \theta_G^2(x_i) \quad (3)$$

and

$$\psi(H) = \sum_{i=1}^{n} \eta_H^2\,(x_i) + \theta_H^2(x_i) \quad (4)$$

2.2.2 Xu *et al.*'s correlation coefficient for IFSs

Xu *et al.*, (2008) modified the approach in (Gerstenkorn and Manko, 1991) as follows:

$$\sigma_2 = \frac{\phi(G,H)}{\sqrt{\psi(G)\psi(H)}} \quad (5)$$

where $\phi(G,H)$ is the correlation of G and H, $\psi(G) and\, \psi(H)$ are informational energies of G and H, respectively, defined by

$$\phi(G,H) = \sum_{i=1}^{n} [\eta_G(x_i)\eta_H(x_i) + \theta_G(x_i)\theta_H(x_i) + \varrho_G(x_i)\varrho_H(x_i)] \quad (6)$$

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$$\psi(\mathrm{G}) = \sum_{i=1}^{n} [\eta_G^2(x_i) + \theta_G^2(x_i) + \varrho_G^2(x_i)] \tag{7}$$

and

$$\psi(\mathrm{H}) = \sum_{i=1}^{n} [\eta_H^2(x_i) + \theta_H^2(x_i) + \varrho_H^2(x_i)] \tag{8}$$

2.2.3 Zeng and Li's correlation coefficient for IFSs

In (Zeng and Li, 2007), the approach in (Xu *et al.*, 2008) was improved as follows:

$$\sigma_3 = \frac{\phi(G,H)}{\sqrt{\psi(G)\psi(H)}} \tag{9}$$

where φ(G,H) is the correlation of G and H, $\psi(G) and\ \psi(H)$ are informational energies of G and H, respectively, defined by

$$\phi(G,H) = \frac{\sum_{i=1}^{n}[\eta_G(x_i)\eta_H(x_i) + \theta_G(x_i)\theta_H(x_i) + \varrho_G(x_i)\varrho_H(x_i)]}{n} \tag{10}$$

$$\psi(\mathrm{G}) = \frac{\sum_{i=1}^{n}[\eta_G^2(x_i) + \theta_G^2(x_i) + \varrho_G^2(x_i)]}{n} \tag{11}$$

And

$$\psi(\mathrm{H}) = \frac{\sum_{i=1}^{n}[\eta_H^2(x_i)+\theta_H^2(x_i)+\varrho_H^2(x_i)]}{n} \tag{12}$$

2.2.4 Garg's correlation coefficient method

Here, we present the method of correlation coefficient by Garg (2016) in intuitionistic fuzzy environment as follows:

$$\sigma_4 = \frac{\phi(G,H)}{\sqrt{\psi(G)\psi(H)}} \tag{13}$$

where $\varphi(G,H)$is the correlation of G and H, $\psi(G) and\ \psi(H)$are informational energies of G and H, respectively, defined by

$$\varphi(G,H) = \sum_{i=1}^{n}\left[\eta_G^2(x_i)\eta_H^2(x_i) + \theta_G^2(x_i)\theta_H^2(x_i) + \varrho_G^2(x_i)\varrho_H^2(x_i)\right] \tag{14}$$

$$\psi(G) = \sum_{i=1}^{n}[\eta_G^4(x_i) + \theta_G^4(x_i) + \varrho_G^4(x_i)] \tag{15}$$

And

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$$\psi(H) = \sum_{i=1}^{n} [\eta_H^4(x_i) + \theta_H^4(x_i) + \varrho_H^4(x_i)] \tag{16}$$

2.3. New intuitionistic fuzzy correlation measure

Suppose l is an intuitionistic fuzzy space in $X = \{x_1, \dots, x_n\}$ for $n \in [1, \infty]$ and $G, H \subseteq l$ .Then, the informational energies of G and H are defined by

$$\psi(G) = \frac{\sum_{i=1}^{n} [\eta_G^2(x_i) + \theta_G^2(x_i) + \varrho_G^2(x_i)]}{n-1} \tag{17}$$

And

$$\psi(H) = \frac{\sum_{i=1}^{n} [\eta_H^2(x_i) + \theta_H^2(x_i) + \varrho_H^2(x_i)]}{n-1} \tag{18}$$

Similarly, the correlation of (G,H) can be computed by

$$\varphi(G,H) = \frac{\sum_{i=1}^{n} [\eta_G(x_i)\eta_H(x_i) + \theta_G(x_i)\theta_H(x_i) + \varrho_G(x_i)\varrho_H(x_i)]}{n-1} \tag{19}$$

Now, the correlation coefficient of (G,H) can be computed by

$$\sigma = \frac{\phi(G,H)}{\sqrt{\psi(G)\psi(H)}} \tag{20}$$

Remark 2.4. In fact, if G = H then (i) $\varphi(G,H) = \varphi(G)$ and $\varphi(G,H) = \varphi(H)$, (ii) $\frac{\phi(G,H)}{\sqrt{\psi(G)\psi(H)}}$.

Thus, Eq.(20) can be rewritten as

$$\sigma(G,H) = \frac{\phi(G,H)}{\sqrt{\psi(G)\psi(H)}} \tag{21}$$

Proposition 2.5. Let $G.H \subseteq l \ in \ X = \{x_i\}, i = 1, \dots, n \in [1, \infty]$. Then (i) $\psi(G) = \psi(\bar{G})$ and $\psi(H) = \psi(\bar{H})$ (ii)$\phi(G,H) = \phi(\bar{G}, \bar{H})$ (iii) $\phi(G,H) = \phi(H,G)$

Proof. The results are easy to see, so we omit the details.

Theorem 2.6. If $G.H \subseteq l$ in X. Then $\sigma(G,H)$ satisfies the following conditions (i) $\sigma(G,H) = \sigma(H,G) \in$ [0,1](ii) $\sigma(G,H) \in$ [0,1] (iii) $\sigma(G,H) = 1$ iff G=H

Proof. The proof of (i) is easy because $\phi(G,H) = \phi(H,G)$, and so

$$\sigma(G,H) = \frac{\phi(G,H)}{(\phi(G,G)\phi(H,H))^{\frac{1}{2}}} = \frac{\phi(H,G)}{(\phi(H,H)\phi(G,G))^{\frac{1}{2}}} = \sigma(H,G)$$

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Next, we prove ii. Certainly, $\sigma(G,H) \geq 0$ since $\phi(G,G)$ and $\phi(H,H)$are non-zeros and the $\phi(G,H) \geq 0$. Now, we establish that $\sigma(G,H) \leq 1$ by deploying the principle of Schwarz's inequality. Let us assume that

$$\sum_{i=1}^{n} \eta_G^2(x_i) = \alpha_1, \qquad \sum_{i=1}^{n} \eta_H^2(x_i) = \alpha_2,$$

$$\sum_{i=1}^{n} \theta_G^2(x_i) = \beta_1, \qquad \sum_{i=1}^{n} \theta_H^2(x_i) = \beta_2,$$

$$\sum_{i=1}^{n} \varrho_G^2(x_i) = \gamma_1, \qquad \sum_{i=1}^{n} \varrho_H^2(x_i) = \gamma_2.$$

Since

$$\sigma(G,H) = \frac{\phi(G,H)}{(\phi(G,G)\phi(H,H))^{\frac{1}{2}}}$$

we have

$$\sigma(G,H) = \frac{\frac{\sum_{i=1}^{n}[\eta_G(x_i)\eta_H(x_i)+\theta_G(x_i)\theta_H(x_i)+\varrho_G(x_i)\varrho_H(x_i)]}{n-1}}{\left(\frac{\sum_{i=1}^{n}[\eta_G^2(x_i)+\theta_G^2(x_i)+\varrho_G^2(x_i)]}{n-1}\,\frac{\sum_{i=1}^{n}[\eta_H^2(x_i)+\theta_H^2(x_i)+\varrho_H^2(x_i)]}{n-1}\right)^{\frac{1}{2}}}$$

$$= \frac{\sum_{i=1}^{n}[\eta_G(x_i)\eta_H(x_i) + \theta_G(x_i)\theta_H(x_i) + \varrho_G(x_i)\varrho_H(x_i)]}{\left(\sum_{i=1}^{n}[\eta_G^2(x_i) + \theta_G^2(x_i) + \varrho_G^2(x_i)\right]\sum_{i=1}^{n}[\eta_H^2(x_i) + \theta_H^2(x_i) + \varrho_H^2(x_i)])^{\frac{1}{2}}}$$

$$\leq \frac{(\sum_{i=1}^{n}[\eta_G^2(x_i)\eta_H^2(x_i)])^{\frac{1}{2}} + (\sum_{i=1}^{n}[\theta_G^2(x_i)\theta_H^2(x_i)])^{\frac{1}{2}} + (\sum_{i=1}^{n}[\varrho_G^2(x_i)\varrho_H^2(x_i)])^{\frac{1}{2}}}{(\sum_{i=1}^{n}[\eta_G^2(x_i) + \theta_G^2(x_i) + \varrho_G^2(x_i)]\sum_{i=1}^{n}[\eta_H^2(x_i) + \theta_H^2(x_i) + \varrho_H^2(x_i)])^{\frac{1}{2}}}$$

$$= \frac{(\alpha_1\alpha_2)^{\frac{1}{2}} + (\beta_1\beta_2)^{\frac{1}{2}} + (\gamma_1\gamma_2)^{\frac{1}{2}}}{[(\alpha_1+\beta_1+\gamma_1)(\alpha_2+\beta_2+\gamma_2)]^{\frac{1}{2}}}$$

For simplicity sake, let $= \alpha_1\alpha_2$,$\beta = \beta_1\beta_2$ and $\gamma = \gamma_1\gamma_2$. Thus,

$$\sigma(G,H) \leq \frac{\alpha^{\frac{1}{2}} + \beta^{\frac{1}{2}} + \gamma^{\frac{1}{2}}}{[(\alpha_1+\beta_1+\gamma_1)(\alpha_2+\beta_2+\gamma_2)]^{\frac{1}{2}}}$$

Consequently,

$$\sigma^2(G,H) \leq \frac{(\alpha^{\frac{1}{2}}+\beta^{\frac{1}{2}}+\gamma^{\frac{1}{2}})^2}{(\alpha_1+\beta_1+\gamma_1)(\alpha_2+\beta_2+\gamma_2)} = \frac{\alpha+\beta+\gamma+\alpha^{\frac{1}{2}}\left(\beta^{\frac{1}{2}}+\gamma^{\frac{1}{2}}\right)+\beta^{\frac{1}{2}}\left(\alpha^{\frac{1}{2}}+\gamma^{\frac{1}{2}}\right)+\gamma^{\frac{1}{2}}(\alpha^{\frac{1}{2}}+\beta^{\frac{1}{2}})}{(\alpha_1+\beta_1+\gamma_1)(\alpha_2+\beta_2+\gamma_2)}$$

But,

$$\sigma^2(G,H) - 1 = \frac{\alpha+\beta+\gamma+\alpha^{\frac{1}{2}}\left(\beta^{\frac{1}{2}}+\gamma^{\frac{1}{2}}\right)+\beta^{\frac{1}{2}}\left(\alpha^{\frac{1}{2}}+\gamma^{\frac{1}{2}}\right)+\gamma^{\frac{1}{2}}\left(\alpha^{\frac{1}{2}}+\beta^{\frac{1}{2}}\right)}{(\alpha_1+\beta_1+\gamma_1)(\alpha_2+\beta_2+\gamma_2)} - 1$$

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$$= -\frac{[\alpha_1(\beta_2+\gamma_2)+\beta_1(\alpha_2+\gamma_2)+\gamma_1(\alpha_2+\beta_2)]-[\alpha^{\frac{1}{2}}\left(\beta^{\frac{1}{2}}+\gamma^{\frac{1}{2}}\right)+\beta^{\frac{1}{2}}\left(\alpha^{\frac{1}{2}}+\gamma^{\frac{1}{2}}\right)+\gamma^{\frac{1}{2}}\left(\alpha^{\frac{1}{2}}+\beta^{\frac{1}{2}}\right)]}{(\alpha_1+\beta_1+\gamma_1)(\alpha_2+\beta_2+\gamma_2)}$$

$\leq 0.$

Thus, $\sigma^2(G,H) \leq 1$and so, $\sigma(G,H) \leq 1$ as desired. Hence, $\sigma(G,H) \in [0,1]$, i.e., (ii) holds.

Assume G = H, then

$$\sigma(G,H) = \frac{\phi(G,H)}{(\phi(G,G)\phi(H,H))^{\frac{1}{2}}} = \frac{\phi(G,G)}{(\phi(G,G)\phi(G,G))^{\frac{1}{2}}} = 1$$

Conversely, if$\sigma(G,H) = 1$, it is certain that G = H. Hence, iii holds. These complete the proof.

2.4.Numerical comparisons

Here, we give examples to test whether the performance index of the proposed method is better than the performance indexes of the existing methods.

2.4.1.Example 1

Suppose $G, H \subseteq l$ in $X = \{x_1, x_2, x_3, x_4\}$ are defined as follow:

$$G = \left\{\langle\frac{0.5,0.3,0.2}{x_1}\rangle, \langle\frac{0.3,0.7,0.0}{x_2}\rangle, \langle\frac{0.9,0.1,0.0}{x_3}\rangle, \langle\frac{0.6,0.3,0.1}{x_4}\rangle\right\},$$

$$H = \left\{\langle\frac{0.5,0.5,0.0}{x_1}\rangle, \langle\frac{0.7,0.2,0.1}{x_2}\rangle, \langle\frac{0.8,0.1,0.1}{x_3}\rangle, \langle\frac{0.7,0.1,0.2}{x_4}\rangle\right\}$$

Now, we compute the correlation coefficient (CC) between G and H by using Eqs. (1), (5), (9), (13) and (20) as follow. Table 1 contains the results.

Table 1.Computation of correlation coefficients.

Approaches	$\varphi(G)$	$\varphi(H)$	$\phi(G,H)$	CC
G & M [22]	2.1900	2.1800	1.9300	0.8833
Xu et al. [42]	2.2400	2.2400	1.9500	0.8705
Z & L [45]	0.5600	0.5600	0.4875	0.8705
Garg [17]	1.1144	1.0184	0.8449	0.7931
New method	0.7467	0.7467	0.6500	0.8705

2.4.2 Example 2

Suppose $I, J \subseteq l$ in $X = \{x_1, x_2, x_3\}$ are defined as follow:

By Eqs. (1), (5), (9), (13) and (20), we compute the correlation coefficient between I and J and obtain the results in Table 2.

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Table 2. Computation of correlation coefficients

Approaches	$\varphi(G)$	$\varphi(H)$	$\phi(G,H)$	CC
G & M [22]	1.2000	1.1425	1.1698	0.9991
Xu et al. [42]	1.2600	1.2115	1.9828	>1
Z & L [45]	0.4200	0.4038	0.6609	>1
Garg [17]	0.2982	0.2589	0.2767	0.9958
New method	0.6300	0.6057	0.6164	0.9978

2.4.3. Discussions

In Example 1, the new method and the methods in (Xu *et al.*, 2008; Zeng and Li, 2007) yield the same results, which are better than the result given by the method in (Garg, 2016). Although the result given by the method in (Gerstenkorn and Manko, 1991) shows better correlation between (G,H) obviously, it is not reliable because it omitted the influence of HM in the computation.

In Example 2, the new method gives a precise result that shows that I and J have almost perfect linear relationship. The methods in (Xu *et al.*, 2008; Zeng and Li, 2007) give results that are not defined within the closed interval [0,1] hence, they are not reliable in measuring correlation coefficient between IFSs.

Although the method in (Gerstenkorn and Manko, 1991) gives the most reasonable results in both cases, it does not measure all the parameters of the considered IFSs. The methods in (Xu *et al.*, 2008; Zeng and Li, 2007) give the same result in both examples. The method in (Garg, 2016) is a reliable correlation coefficient measure among the existing methods. In conclusion, the new correlation coefficient measure yields a better result with high performance index in comparison with the methods in (Garg, 2016; Gerstenkorn and Manko, 1991; Xu *et al.*, 2008; Zeng and Li, 2007).

2.4.4 Advantages of the new approach over the existing approaches

The following are the advantages of the new method of computing CCIFSs over the existing approaches discussed in (Garg, 2016; Gerstenkorn and Manko, 1991; Xu *et al.*, 2008; Zeng and Li, 2007).

(i) It is very precise and reliable because it satisfies the axiomatic description of correlation coefficient measure different from the approaches in (Xu *et al.*, 2008; Zeng and Li, 2007).

(ii) It incorporates the complete parameters of IFS to minimize error of computation distinct from the methods in (Gerstenkorn and Manko, 1991).

(iii) It provides a better interpretation between the IFSs which corroborates with the classical sense of correlation coefficient.

3. Application of the new method in medical diagnostic analysis

Supposing we have m alternatives exemplified in IFPs Ii for i = 1,...,m, defined in a universal set X. If there is an alternative sample exemplified in IFP J to be associated with any of Ii. The value

$$\sigma(I_i, J) = \max(\sigma(I_1, J), \dots, \sigma(I_m, J))$$

where $\sigma(I_i, J)$ states the grade of correlation index between (I_i, J).The maximum of $\sigma(I_1, J), \dots, \sigma(I_m, J)$indicates that the alternative J is associated with any of such I_i for $i = 1, \dots, m$.

3.1. Experimental example

In a hypothetical case of medical diagnosis, let us assume that four patients namely; Lil, Ani, Jo and Sam visit a medical facility for medical diagnosis. We exemplified the patients by IFPs P_j for1,2,3,4. After taken their vital signs and samples for critical analysis, the following major symptoms were observed;

$$S = \{s_1, s_2, s_3, s_4, s_5\}$$

where s_1 = temperature, s_2 = headache, s_3 = abdominal pain, s_4 = cough, and s_5 = thoracic pain.

From the observed symptoms, the suspected diseases that P1, P2, P3, and P4 are likely to be infected with are represented by a set of IFPs

$$D = \{D_1, D_2, D_3, D_4, D_5\}$$

Where D_1 = viral fever, D_2 = malaria, D_3 = typhoid fever, D_4 = peptic ulcer, and D_5 = cardiac problem.

We present the diseases' intuitionistic fuzzy medical information hypothetically in Table 3 based on the medical knowledge of the enlisted diseases. The patients' intuitionistic fuzzy medical information are hypothetically given in Table 4 based on the medical analysis on $P = \{P_1, P_2, P_3, P_4\}$

Table 3.Diseases' intuitionistic fuzzy medical information

	D_1	D_2	D_3	D_4	D_5
s_1	(0.4,0.0)	(0.7, 0.0)	(0.3,0.3)	(0.1,0.7)	(0.1,0.8)
s_2	(0.3,0.5)	(0.2,0.6)	(0.6,0.1)	(0.2,0.4)	(0.0,0.8)
s_3	(0.1,0.7)	(0.0,0.9)	(0.2,0.7)	(0.8,0.0)	(0.2,0.8)
s_4	(0..4,0.3)	(0.7,0.0)	(0.2,0.6)	(0.2,0.7)	(0.2,0.8)
s_5	(0.1,0.7)	(0.1,0.8)	(0.1,0.9)	(0.2,0.7)	(0.8.0.1)

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Table 4. Patients' intuitionistic fuzzy medical information.

	s_1	s_2	s_3	s_4	s_5
P_1	(0.8,0.1)	(0.6,0.1)	(0.2,0.8)	(0.6,0.1)	(0.1,0.6)
P_2	(0.0,0.8)	(0.4,0.4)	(0.6,0.1)	(0.1,0.7)	(0.1,0.8)
P_3	(0.8,0.1)	(0.8,0.1)	(0.0,0.6)	(0.2,0.7)	(0.0,0.5)
P_4	(0.6,0.1)	(0.5,0.4)	(0.3,0.4)	(0.7,0.2)	(0.3,0.4)

Using the formula for computing HM in Definition 2.1, we get the HMs of the IFPs in Tables 3 and 4. The task before us is to determine the likely disease each of the patients is suffering from by calculating the correlation coefficients between each of the diseases with each of the patients. The algorithm that incorporates the new correlation measure, i.e., Eq. (21) for the computation of the correlation coefficients between Di and Pj follows:

3.2.Algorithm

PRE:

Eta $P_j[s_i]$is membership degree (MD) of patients (P_j) where j = 1, 2, 3, 4; theta $P_j[s_i]$is non-membership Degree (NMD) of P_j where j = 1, 2, 3, 4; varrho $P_j[s_i]$is hesitation margin(HM) of Pj where j = 1, 2, 3, 4.

Eta $D_i[s_i]$ is membership degree (MD) of diseases (D_i) where i = 1, 2, 3, 4 , 5; thetaDi[si] is non- membership degree (NMD) of Di where i = 1, 2, 3 , 4, 5; varrho $D_i[s_i]$ is hesitation margin (HM) of D_i where i = 1, 2, 3, 4, 5.

Phi P_jD_i is the correlation between the $P_j$and D_i; phi P_jP_i is the informational energy of the patient P_j; phi D_iD_i is the informational energy of the disease $D_i, n = 5$ is the number of symptoms.

POST: this algorithm finds the correlation coefficient (sigma P_jD_i), where P_j is the set of patients and D_i is the set of diseases.

STEPS:

i: Set value for n

ii: Initialize values for P_j

iii: Initialize values for D_i

iv: Repeat steps v to viii for each P_j

v For i = 1 to n

vi: Set sum P_j=sum P_j + ((eta $P_j[s_i]$* eta $P_j[s_i]$) + (theta $P_j[s_i]$* theta $P_j[s_i]$]) + (varrho $P_j[s_i]$))

vii: Next i

viii: Set phi P_jP_j = sum $^{P_j}/_{n-1}$

ix: Repeat steps x to xvii for each D_i

x: For i = 1 to n

xi: Set sum D_i=sum D_i + ((eta $D_i[s_i]$* eta $D_i[s_i]$) + (theta $D_i[s_i]$* theta $D_i[s_i]$]) + (varrho $D_i[s_i]$))

xii: Next i

xiii: Set phi D_iD_i = sum $^{D_i}/_{n-1}$

xix: For i = 1 to n

xv: sum P_jD_i=sum P_jD_i + ((eta $P_j[i]$* eta $D_i[i]$) + (theta $P_j[i]$* theta $D_i[i]$]) + (varrho $P_i[i]$))

xvi: Next i

xvii: Set phi P_jD_i = sum$P_jD_i/n-1$

Set sigma P_jD_i = phiP_jD_i/sqrt(phiP_jP_j * phi D_iD_i)

xviii: Repeat step ix for next D_i

xix: Repeat step iv for next P_j

xx: Exit

We now use JAVA to code the information in Tables 3 and 4 using the algorithm and obtain the result in Table 5.

Table 5. Diagnostic results

Patients	viral fever	malaria	typhoid fever	peptic ulcer	cardiac problem
Lil	0.8360	0.8914	0.8122	0.4833	0.4499
Ani	0.6273	0.4612	0.7888	0.9627	0.6541
Jo	0.7583	0.6816	0.8113	0.5167	0.4661
Sam	0.8222	0.8419	0.7243	0.6083	0.5681

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3.3 Medical decision

From the results display in Table 5, Lil is mainly diagnosed with malaria, but should be treated for viral fever and typhoid fever in that order. Ani is suffering severely from peptic ulcer coupled with typhoid fever, cardiac problem and viral fever. Jo is diagnosed with typhoid fever, viral fever, malaria and a case of peptic ulcer. Sam is suffering mainly from malaria and viral fever, but should be treated for typhoid fever, peptic ulcer and cardiac problem. In the diagnostic analysis, we see that only Sam shows positive for all the diseases but with different degrees of severity. These information give clue on how the treatments should be carried out since the diagnosis give degree of severity. This approach clearly shows the interrelationship among diseases. The analysis will direct the physician on the drugs dosage needed for each of the patients since all of them are suffering from more than one diseases.

4.Conclusion

In this paper, we have discussed some existing correlation coefficient measures in intuitionistic fuzzy environment defined within closed interval [0,1]. The aim of the discussion was to propose a more reliable and efficient method of computing CCIFSs. The new method in contrast to the pioneer work on correlation in intuitionistic fuzzy environment (Gerstenkorn and Manko, 1991) considered the complete traditional features of IFSs. We comparatively verified the performance index of the new method with existing methods discussed in (Garg, 2016; Gerstenkorn and Manko, 1991; Xu *et al.*, 2008; Zeng and Li, 2007), and found that our method gives a better performance index with precision and ease of computation. To authenticate the new method in terms of applicability, we determined medical diagnosis problem with the aid of the proposed intuitionistic fuzzy correlation algorithm coded with JAVA programming language. We look forward to develop some applications of the new method in multi-attributes decision-making (MADM) and multi-objectives decision-making (MODM) via cluster algorithm in the future research.

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