Decision making under intuitionistic fuzzy metric distances

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Abstract This article deals with qualitative difference between two intuitionistic fuzzy sets with the help of standard pseudo metric and metric spaces. Some definitions over metric spaces, pseudo metric spaces, intuitionistic fuzzy sets, indeterminacy and the formula of measuring metrices have been incorporated. Numerical illustrations, graphical illustrations, area of applications and ranking for decision making are discussed to show the novelty of this article. Finally, conclusions and scope of future works are mentioned.

Keywords Metric distance; Pseudo metric distance; Intuitionistic fuzzy set; Ranking

1. Introduction

In traditional set theory (classical), the idea of member and non-member of an element in a set was sudden i.e., an element either belongs to a set or not belongs to the set. But there was no knowledge about the transition of an element from member to non-member of the set and vice-versa. Zadeh (1965) has solved these ambiguities through his new invention, the fuzzy set theory. Since then, numerous research articles have been studied over the fuzzy set itself to explain the real-world phenomenon. Bellman and Zadeh (1970) introduced a new concept of Decision making in a fuzzy environment. Piegat (2005) gives us a new definition of fuzzy set. The concepts of dense fuzzy set studied by De and Beg (2016) to discuss the frequent learning effect of the fuzzy parameters. Analysing the behaviour of human thinking process De (2018) developed a new inexact set which is known as Triangular dense fuzzy lock set and its new defuzzification method. After this invention many articles have been made by eminent researchers Maity et al. (2020), Maity et al (2020), De and Mahata (2019) to control the individual or group decision making problems on pollution sensitive inventory modelling. Baez-Sancheza et al. (2012) discussed polygonal fuzzy sets and numbers extensively.
Trapezoidal approximations of fuzzy numbers and their existence as well as uniqueness and continuity are exclusively discussed by Ban and Coroianu (2014). Chutia et al. (2010) contributes to find membership function of a fuzzy number. De and Mahata (2017) designed a fuzzy backorder model where demand rate is considered as cloudy inexactness. Decision making in a bi-objective inventory problem was discussed by eminent researchers like De and Pal (2016). Mahanta et al. (2010) made a new approach of fuzzy arithmetic without using alpha cuts. Mao et al. (2018) extensively analysed about the relation between cloud aggregation operators and multi attribute group decision making in interval valued hesitant fuzzy linguistic environment. However, Atanassov (1986), Atanassov (1983) introduces a new approach of fuzzy set namely Intuitionistic fuzzy sets in terms of membership and non-membership function. Some notable works over the EOQ models on fuzzy environments as well as IFS may be pointed out over here. A step order fuzzy approach discussed by Das et al. (2015). Recently, Maity et al. (2019) studied an intuitionistic dense fuzzy model where the learning- forgetting or agreement-disagreement are considered. De and Sana (2018) discussed a stochastic demand model under aggregation with Bonferroni mean in intuitionistic fuzzy environment. Deli and Broumi (2015) worked in Neutrosophic soft matrices and NSM-decision making. Recently, Kaur et al. (2019) contributed to find relation between interval type intuitionistic trapezoidal fuzzy sets and decision making with incomplete weight information. Liang and Wang (2019) considered a linguistic intuitionistic cloudy fuzzy model with sentiment analysis in E-commerce. Xu (2007) studied about Intuitionistic fuzzy aggregation operators. From the above discussion, it is observed that none of the researchers have been studied over the metric distances of intuitionistic fuzzy numbers. In this study we develop the theory of distances between two non-linear intuitionistic fuzzy sets (numbers) with respect to (pseudo) metrics. We give some definitions of metric spaces and the formula of distance measure of two different sets via cumulative aggregated formula. To show the novelty of this article a numerical illustration has been analysed through the ranking of distances with the existing metrics.

2. Preliminaries

2.1 Here we shall introduce some definitions over metric and pseudo metric spaces.

Definition 2.1.1 Let $A$ be a non-empty set. A function $d:A \times A \rightarrow \mathbb{R}$ is said to be a ‘metric’ or a distance function on $A$ if it satisfies the following properties:

i. $d(x,y) \geq 0$ for all $x, y \in A$;

ii. $d(x,y) = 0$ if and only if $x = y$;

iii. $d(x,y) = d(y,x)$ for all $x, y \in A$;

iv. $d(x,z) = d(x,y) + d(y,z)$ for all $x, y, z \in A$

Any non-empty set $A$ together with a metric $d$ defined on it is said to be a ‘metric space’.

Definition 2.1.2 Let $A$ be a non-empty set. A function $d^P:A \times A \rightarrow \mathbb{R}$ is said to be a ‘pseudo-metric’ on $A$ if it satisfies the following properties:
Any non-empty set $A$ together with a pseudo-metric $d^p$ defined on it is said to be a ‘pseudo-metric space’.

**Definition 2.1.3** Atanassov (1983) An intuitionistic fuzzy set $A$ defined in the universe of discourse $X$ is given by $A = \{(x, \mu_A(x), \nu_A(x)) \, | \, x \in X \}$, where $\mu_A, \nu_A : X \to [0, 1]$ denote the degree of membership and non-membership of $x$ in $A$ respectively, satisfying the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. The indeterminacy degree $\psi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ expresses the lack of knowledge of whether $x$ belongs to $A$ or not, also $0 \leq \psi_A(x) \leq 1$ for $x \in X$. An intuitionistic fuzzy number $\omega = (\mu_\omega, \nu_\omega)$ is an ordered pair which satisfies the conditions: $0 \leq \mu_\omega \leq 1$, $0 \leq \nu_\omega \leq 1$ and $0 \leq \mu_\omega + \nu_\omega \leq 1$, where $\mu_\omega$ and $\nu_\omega$ are called membership degree and non-membership degree respectively.

### 2.2 Pseudo-metrics in Intuitionistic Fuzzy Set

Let us consider $F$ be the set of all intuitionistic fuzzy numbers and each $\omega = (\mu_\omega, \nu_\omega) \in F$ is called a point of $F$. However, $\omega = (\mu_\omega, \nu_\omega, \psi_\omega)$ only has two degrees of freedom because $\mu_\omega + \nu_\omega + \psi_\omega \equiv 1$. So, we observe such a system by keeping one variable constant when the other variable is changing.

**Definition 2.2.1** Xu (2007) Given two Intuitionistic fuzzy numbers (IFN) $\rho$ and $\sigma$, $\rho \cap \sigma = (\min(\mu_\rho, \mu_\sigma), \max(\nu_\rho, \nu_\sigma))$ and $\rho \cup \sigma = (\max(\mu_\rho, \mu_\sigma), \min(\nu_\rho, \nu_\sigma))$.

**Lemma 1** If $(\mu_\rho - \mu_\sigma)(\nu_\rho - \nu_\sigma) \geq 0$, $\rho \cap \sigma \in F$ and $\rho \cup \sigma \in F$.

**Proof**

\[
\begin{align*}
\mu_{\rho \cap \sigma} + \nu_{\rho \cap \sigma} &= \mu_\rho + \nu_\sigma \leq \mu_\sigma + \nu_\sigma \leq 1, \\
\mu_{\rho \cup \sigma} + \nu_{\rho \cup \sigma} &= \mu_\sigma + \nu_\rho \leq \mu_\rho + \nu_\sigma \leq 1, & \text{if } \mu_\rho \leq \mu_\sigma; \\
\mu_{\rho \cap \sigma} + \nu_{\rho \cap \sigma} &= \mu_\sigma + \nu_\rho < \mu_\rho + \nu_\rho \leq 1, \\
\mu_{\rho \cup \sigma} + \nu_{\rho \cup \sigma} &= \mu_\rho + \nu_\sigma < \mu_\rho + \nu_\sigma \leq 1, & \text{if } \mu_\rho > \mu_\sigma;
\end{align*}
\]

**Definition 2.2.2** An ordered pair $(F, d^p_\psi)$ is called the intuitionistic fuzzy indeterminacy pseudo metric space on $F$, where $d^p_\psi : F^2 \to \mathbb{R}$ is the indeterminacy pseudo metric, for any $\rho, \sigma \in F$, $d^p_\psi(\rho, \sigma) = \frac{|\psi_\rho^2 - \psi_\sigma^2|}{2}$.

It is easy to verify that $d^p_\psi$ satisfies the properties of pseudo-metric. Similarly, we can define the intuitionistic fuzzy membership pseudo metric space on $F$, where $d^p_\mu : F^2 \to \mathbb{R}$ is the membership pseudo metric, for any $\rho, \sigma \in F$, $d^p_\mu(\rho, \sigma) = \frac{|\mu_\rho^2 - \mu_\sigma^2|}{2}$ and the
intuitionistic fuzzy non-membership pseudo metric space on $F$, where $d^P_\mu: F^2 \to \mathbb{R}$ is the non-membership pseudo metric, for any $\rho, \sigma \in F$, $d^P_\mu(\rho, \sigma) = \frac{|\mu_\rho - \mu_\sigma|}{2}$.

It is also easy to verify that $d^P_\mu$ and $d^P_\nu$ satisfies the properties of pseudo-metric.

**Definition 2.2.3** An ordered pair $(F, d_\psi)$ is called the intuitionistic fuzzy indeterminacy metric space on $F$, where $d_\psi: F^2 \to \mathbb{R}$ is the indeterminacy metric, for any $\rho, \sigma \in F$, $d_\psi(\rho, \sigma) = |\psi_\rho - \psi_\sigma|$.

It is easy to verify that $d_\psi$ satisfies the properties of metric. Similarly, we can define the intuitionistic fuzzy membership metric space on $F$, where $d_\mu: F^2 \to \mathbb{R}$ is the membership metric, for any $\rho, \sigma \in F$, $d_\mu(\rho, \sigma) = |\mu_\rho - \mu_\sigma|$ and the intuitionistic fuzzy non-membership metric space on $F$, where $d_\nu: F^2 \to \mathbb{R}$ is the non-membership metric, for any $\rho, \sigma \in F$, $d_\nu(\rho, \sigma) = |\nu_\rho - \nu_\sigma|$.

It is also easy to verify that $d_\mu$ and $d_\nu$ satisfies the properties of metric.

**Lemma 2** If $\alpha, \beta \in F$, $d_\Sigma(\alpha, \beta) = \max (d_\mu(\alpha, \beta), d_\nu(\alpha, \beta), d_\psi(\alpha, \beta))$ satisfies the four properties of pseudo-metric as well as metric.

**Lemma 3** Let $\tilde{P}$ and $\tilde{Q}$ be two intuitionistic fuzzy sets defined over the interval $[L, R]$, then the summative metric distance (summative pseudo metric distance) between $\tilde{P}$ and $\tilde{Q}$ is denoted by $d(\tilde{P}, \tilde{Q})$ (for pseudo metric $d_p(\tilde{P}, \tilde{Q})$) and defined as

$$d(\tilde{P}, \tilde{Q}) = \frac{1}{R - L} \int_L^R d_\Sigma \left(\left(\mu_\tilde{P}(x), \nu_\tilde{P}(x)\right), \left(\mu_\tilde{Q}(x), \nu_\tilde{Q}(x)\right)\right) dx$$

**3. Representation of IFSs $\tilde{P}$ and $\tilde{Q}$ over the interval $[a_1, a_3]$** Maity and Mondal (2020).

$$\mu_\tilde{P}(x) = \begin{cases} \left(\frac{x-a_1}{a_2-a_1}\right)^m & \text{if } a_1 \leq x \leq a_2 \\ \left(\frac{a_3-x}{a_3-a_2}\right)^m & \text{if } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases} \quad \mu_\tilde{Q}(x) = \begin{cases} \left(\frac{x-a_1}{a_2-a_1}\right)^n & \text{if } a_1 \leq x \leq a_2 \\ \left(\frac{a_3-x}{a_3-a_2}\right)^n & \text{if } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

$$\nu_\tilde{P}(x) = \begin{cases} \left(\frac{a_2-x}{a_2-a_1}\right)^m & \text{if } a_1 \leq x \leq a_2 \\ \left(\frac{x-a_2}{a_3-a_2}\right)^m & \text{if } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases} \quad \nu_\tilde{Q}(x) = \begin{cases} \left(\frac{a_2-x}{a_2-a_1}\right)^n & \text{if } a_1 \leq x \leq a_2 \\ \left(\frac{x-a_2}{a_3-a_2}\right)^n & \text{if } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

Now as per equation (1), the pseudo metric between two IFSs is given by
\[(3)\]

\[
d^p(\bar{P}, \bar{Q}) = \frac{1}{a_3 - a_1} \int_{a_1}^{a_3} d_\Sigma \left( (\mu_\bar{P}(x), \nu_\bar{P}(x)), (\mu_\bar{Q}(x), \nu_\bar{Q}(x)) \right) dx
\]

\[
= \frac{1}{a_3 - a_1} \int_{a_1}^{a_3} d_\Sigma \left( (\mu_\bar{P}(x), \nu_\bar{P}(x)), (\mu_\bar{Q}(x), \nu_\bar{Q}(x)) \right) dx + \\
\frac{1}{a_3 - a_1} \int_{a_2}^{a_3} d_\Sigma \left( (\mu_\bar{P}(x), \nu_\bar{P}(x)), (\mu_\bar{Q}(x), \nu_\bar{Q}(x)) \right) dx
\]

\[
= \frac{1}{a_3 - a_1} \int_{a_1}^{a_3} d_\Sigma \left( \left( \frac{x-a_1}{a_2-a_1} \right)^m, \left( \frac{x-a_2}{a_3-a_1} \right)^m, \left( \frac{x-a_1}{a_2-a_1} \right)^n, \left( \frac{x-a_2}{a_3-a_1} \right)^n \right) dx + \\
\frac{1}{a_3 - a_1} \int_{a_1}^{a_3} d_\Sigma \left( \left( \frac{x-a_2}{a_3-a_2} \right)^m, \left( \frac{x-a_2}{a_3-a_2} \right)^m, \left( \frac{x-a_3}{a_3-a_2} \right)^n, \left( \frac{x-a_3}{a_3-a_2} \right)^n \right) dx
\]

\[
= \frac{1}{a_3 - a_1} \int_{a_1}^{a_3} \max \left\{ \frac{1}{2} \left( \frac{x-a_1}{a_2-a_1} \right)^{2m} - \left( \frac{x-a_1}{a_2-a_1} \right)^{2n}, \frac{1}{2} \left( \frac{x-a_2}{a_3-a_1} \right)^{2m} - \left( \frac{x-a_2}{a_3-a_1} \right)^{2n} \right\} dx + \\
\frac{1}{a_3 - a_1} \int_{a_1}^{a_3} \max \left\{ \frac{1}{2} \left( \frac{x-a_2}{a_3-a_2} \right)^{2m} - \left( \frac{x-a_2}{a_3-a_2} \right)^{2n}, \frac{1}{2} \left( \frac{x-a_3}{a_3-a_2} \right)^{2m} - \left( \frac{x-a_3}{a_3-a_2} \right)^{2n} \right\} dx
\]

Similarly, for the metric distance we have

\[(4)\]

\[
d(\bar{P}, \bar{Q}) = \frac{1}{a_3 - a_1} \int_{a_1}^{a_3} d_\Sigma \left( (\mu_P(x), \nu_P(x)), (\mu_Q(x), \nu_Q(x)) \right) dx
\]

\[
= \frac{1}{a_3 - a_1} \int_{a_1}^{a_3} d_\Sigma \left( (\mu_\bar{P}(x), \nu_\bar{P}(x)), (\mu_\bar{Q}(x), \nu_\bar{Q}(x)) \right) dx + \\
\frac{1}{a_3 - a_1} \int_{a_2}^{a_3} d_\Sigma \left( (\mu_\bar{P}(x), \nu_\bar{P}(x)), (\mu_\bar{Q}(x), \nu_\bar{Q}(x)) \right) dx
\]

\[
= \frac{1}{a_3 - a_1} \int_{a_1}^{a_3} d_\Sigma \left( \left( \frac{x-a_1}{a_2-a_1} \right)^m, \left( \frac{x-a_1}{a_2-a_1} \right)^m, \left( \frac{x-a_3}{a_3-a_1} \right)^n, \left( \frac{x-a_3}{a_3-a_1} \right)^n \right) dx + \\
\frac{1}{a_3 - a_1} \int_{a_1}^{a_3} d_\Sigma \left( \left( \frac{x-a_2}{a_3-a_2} \right)^m, \left( \frac{x-a_2}{a_3-a_2} \right)^m, \left( \frac{x-a_3}{a_3-a_2} \right)^n, \left( \frac{x-a_3}{a_3-a_2} \right)^n \right) dx
\]
\[ z = \frac{1}{2} \int_{a_1}^{a_2} \max \left\{ \left| \frac{x-a_1}{a_2-a_1} \right|^{m} - \left( \frac{x-a_1}{a_2-a_1} \right)^{n}, \left( \frac{a_2-x}{a_2-a_1} \right)^{m} - \left( \frac{a_2-x}{a_2-a_1} \right)^{n}, \left( 1 - \frac{x-a_1}{a_2-a_1} \right)^{m} - \left( 1 - \frac{x-a_1}{a_2-a_1} \right)^{n}, \left( \frac{a_2-x}{a_2-a_1} \right)^{m} - \left( \frac{a_2-x}{a_2-a_1} \right)^{n} \right\} \, dx \]

4. Formulation of EOQ Model

Let the inventory starts with the order quantity \( Q \) and the demand rate is \( D \) per month. The items are exhausted after time \( T \) and new re-order has been placed. Items are received instantly and the next cycle time begins. Thus, we have the average inventory cost of the classical EOQ model as \( z = \frac{1}{2} hDT + \frac{c}{T} \) with \( i = DT \), where \( h \) is the unit holding cost and \( c \) is the set-up cost.

![Basic EOQ model](image_url)

**Figure 1. Basic EOQ model**

### 4.1 Crisp mathematical model

The crisp mathematical model of the above discussed problem can be formulated as

\[ \begin{aligned}
\text{Minimize} & \quad z = \frac{1}{2} hDT + \frac{c}{T} \\
Q & = DT
\end{aligned} \] (5)

### 4.2 Fuzzy mathematical model

Let the costs associated with the inventory are fuzzy parameters. Then the fuzzy mathematical model can be formulated as

\[ \begin{aligned}
\text{Minimize} & \quad \tilde{z} = \frac{1}{2} \tilde{h}DT + \frac{\tilde{c}}{\tilde{T}} \\
Q & = DT
\end{aligned} \] (6)

Here this will be discussed in the following three cases.
Case 1: The holding cost $\bar{h}$ is considered as fuzzy parameter.

Let the holding cost $\bar{h} = \langle h_1, h_2, h_3 \rangle$ be considered as triangular intuitionistic fuzzy number whose membership and non-membership functions are given by

$$\mu_{\bar{h}}(x) = \begin{cases} \left( \frac{x-h_1}{h_2-h_1} \right)^m & \text{if } h_1 \leq x \leq h_2 \\ \left( \frac{h_3-x}{h_3-h_2} \right)^m & \text{if } h_2 \leq x \leq h_3 \\ 0 & \text{otherwise} \end{cases}$$

and $\nu_{\bar{h}}(x) = \begin{cases} \left( \frac{h_2-x}{h_2-h_1} \right)^m & \text{if } h_1 \leq x \leq h_2 \\ \left( \frac{x-h_2}{h_3-h_2} \right)^m & \text{if } h_2 \leq x \leq h_3 \\ 0 & \text{otherwise} \end{cases}$

Using this we get $\bar{z} = \langle z_1, z_2, z_3 \rangle = \langle \frac{1}{2}DT + \frac{c}{T}h_2DT, \frac{1}{2}DT + \frac{c}{T}h_3DT + \frac{c}{T}DT \rangle$ as a triangular intuitionistic fuzzy number whose membership and non-membership functions are given by

$$\mu_{\bar{z}}(z) = \begin{cases} \left( \frac{z-z_1}{z_2-z_1} \right)^m & \text{if } z_1 \leq z \leq z_2 \\ \left( \frac{z-z_2}{z_3-z_2} \right)^m & \text{if } z_2 \leq z \leq z_3 \\ 0 & \text{otherwise} \end{cases}$$

and $\nu_{\bar{z}}(z) = \begin{cases} \left( \frac{z_2-z}{z_2-z_1} \right)^n & \text{if } z_1 \leq z \leq z_2 \\ \left( \frac{z-z_2}{z_3-z_2} \right)^n & \text{if } z_2 \leq z \leq z_3 \\ 0 & \text{otherwise} \end{cases}$

Case 2: The set-up cost $\bar{c}$ is considered as fuzzy parameter.

Let the set-up cost $\bar{c} = \langle c_1, c_2, c_3 \rangle$ be considered as triangular intuitionistic fuzzy number whose membership and non-membership functions are given by

$$\mu_{\bar{c}}(x) = \begin{cases} \left( \frac{x-c_1}{c_2-c_1} \right)^n & \text{if } c_1 \leq x \leq c_2 \\ \left( \frac{c_3-x}{c_3-c_2} \right)^n & \text{if } c_2 \leq x \leq c_3 \\ 0 & \text{otherwise} \end{cases}$$

and $\nu_{\bar{c}}(x) = \begin{cases} \left( \frac{c_2-x}{c_2-c_1} \right)^n & \text{if } c_1 \leq x \leq c_2 \\ \left( \frac{x-c_2}{c_3-c_2} \right)^n & \text{if } c_2 \leq x \leq c_3 \\ 0 & \text{otherwise} \end{cases}$

Using this we get $\bar{z} = \langle z_1, z_2, z_3 \rangle = \langle \frac{1}{2}DT + \frac{c_1}{T}hDT, \frac{1}{2}DT + \frac{c_2}{T}hDT + \frac{c_3}{T}DT \rangle$ as a triangular intuitionistic fuzzy number whose membership and non-membership functions are given by

$$\mu_{\bar{z}}(z) = \begin{cases} \left( \frac{z-z_1}{z_2-z_1} \right)^m & \text{if } z_1 \leq z \leq z_2 \\ \left( \frac{z-z_2}{z_3-z_2} \right)^m & \text{if } z_2 \leq z \leq z_3 \\ 0 & \text{otherwise} \end{cases}$$

and $\nu_{\bar{z}}(z) = \begin{cases} \left( \frac{z_2-z}{z_2-z_1} \right)^n & \text{if } z_1 \leq z \leq z_2 \\ \left( \frac{z-z_2}{z_3-z_2} \right)^n & \text{if } z_2 \leq z \leq z_3 \\ 0 & \text{otherwise} \end{cases}$

Case 3: Both holding cost $\bar{h}$ and set-up cost $\bar{c}$ are considered as fuzzy parameter.

Let the holding cost $\bar{h} = \langle h_1, h_2, h_3 \rangle$ and the set-up cost $\bar{c} = \langle c_1, c_2, c_3 \rangle$ are considered as triangular intuitionistic fuzzy number whose membership and non-membership functions are given by
\[ \mu_h(x) = \begin{cases} \frac{(x-h_1)^m}{h_2-h_1} & \text{if } h_1 \leq x \leq h_2 \\
 \frac{(h_3-x)^m}{h_3-h_2} & \text{if } h_2 \leq x \leq h_3 \\
 0 & \text{otherwise} \end{cases} \quad \mu_c(x) = \begin{cases} \frac{(x-c_1)^n}{c_2-c_1} & \text{if } c_1 \leq x \leq c_2 \\
 \frac{(c_3-x)^n}{c_3-c_2} & \text{if } c_2 \leq x \leq c_3 \\
 0 & \text{otherwise} \end{cases} \]

\[ v_h(x) = \begin{cases} \frac{(h_2-x)^m}{h_2-h_1} & \text{if } h_1 \leq x \leq h_2 \\
 0 & \text{otherwise} \end{cases} \quad v_c(x) = \begin{cases} \frac{(c_3-x)^n}{c_3-c_2} & \text{if } c_2 \leq x \leq c_3 \\
 0 & \text{otherwise} \end{cases} \]

Using this we get \( \tilde{z} = (z_1, z_2, z_3) = \left( \frac{1}{2} h_1 DT + \frac{c_1}{T}, \frac{1}{2} h_2 DT + \frac{c_2}{T}, \frac{1}{2} h_3 DT + \frac{c_3}{T} \right) \) as a triangular intuitionistic fuzzy number whose membership and non-membership functions are given by

\[ \mu_{\tilde{z}}(z) = \begin{cases} \frac{(z-z_1)^m}{z_2-z_1} & \text{if } z_1 \leq z \leq z_2 \\
 \frac{(z_3-z)^n}{z_3-z_2} & \text{if } z_2 \leq z \leq z_3 \\
 0 & \text{otherwise} \end{cases} \quad v_{\tilde{z}}(z) = \begin{cases} \frac{(z-z_2)^n}{z_3-z_2} & \text{if } z_2 \leq z \leq z_3 \\
 0 & \text{otherwise} \end{cases} \]

4.3 Defuzzification of the fuzzy mathematical model

Here we consider the third case of fuzzy mathematical model for defuzzification because in this case both costs are intuitionistic fuzzy numbers. Using the formula of \( \alpha \) and \( \beta \) cut from De et al. (2016) we solve the model as

\[
\begin{align*}
\text{Maximize } & \alpha - \beta \\
\alpha & \geq \beta, \alpha + \beta \leq 1, \alpha, \beta \geq 0 \\
\mu_{\tilde{z}}(z) & \geq \alpha, v_{\tilde{z}}(z) \leq \beta, \\
\mu_h(h) & \geq \alpha, v_h(h) \leq \beta, \mu_c(c) \geq \alpha, v_c(c) \leq \beta \\
z & = \frac{1}{2} h DT + \frac{c}{T}
\end{align*}
\]
**Table-3:** we consider the holding cost $\tilde{h}$ as intuitionistic fuzzy set

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Pseudo metric distance</th>
<th>Metric distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(m, n integer)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>m=2, n=4</td>
<td>0.146</td>
<td>0.267</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(m integer n fraction)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>m=2, n=1/2</td>
<td>0.150</td>
<td>0.667</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(m, n fraction)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>m=1/2, n=1/4</td>
<td>0.125</td>
<td>0.267</td>
</tr>
</tbody>
</table>

**Table-4:** we consider the set-up cost $\tilde{c}$ as intuitionistic fuzzy set

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Pseudo metric distance</th>
<th>Metric distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(m, n integer)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>m=2, n=4</td>
<td>0.146</td>
<td>0.267</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(m integer n fraction)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>m=2, n=1/2</td>
<td>0.150</td>
<td>0.667</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(m, n fraction)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>m=1/2, n=1/4</td>
<td>0.125</td>
<td>0.267</td>
</tr>
</tbody>
</table>
### Table-5: we consider the holding cost $\tilde{h}$ and set up cost $\tilde{c}$ as intuitionistic fuzzy set

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Pseudo metric distance</th>
<th>Metric distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (m, n integer)</td>
<td>0.146</td>
<td>0.267</td>
</tr>
<tr>
<td>m=2, n=4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B (m integer n fraction)</td>
<td>0.150</td>
<td>0.667</td>
</tr>
<tr>
<td>m=2, n=1/2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C (m, n fraction)</td>
<td>0.125</td>
<td>0.267</td>
</tr>
<tr>
<td>m=1/2, n=1/4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table-6: Ranking Table

<table>
<thead>
<tr>
<th>Type of cost</th>
<th>Pseudo metric distance</th>
<th>Metric distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuzzy holding cost</td>
<td>$B &gt; A &gt; C$</td>
<td>$B &gt; A = C$</td>
</tr>
<tr>
<td>Fuzzy set up cost</td>
<td>$B &gt; A &gt; C$</td>
<td>$B &gt; A = C$</td>
</tr>
<tr>
<td>Fuzzy holding cost and Fuzzy set up cost</td>
<td>$B &gt; A &gt; C$</td>
<td>$B &gt; A = C$</td>
</tr>
</tbody>
</table>

### 4.4 Numerical Illustration

Let us assume the holding cost $h = 2.5$, set-up cost $c = 1200$, constant demand rate $D = 100$. Also let us consider the fuzzy (holding and set up) costs as $\tilde{h} = \langle 1.75, 2.5, 3 \rangle$ and $\tilde{c} = \langle 1000, 1200, 1300 \rangle$. The following table shows that the result in different scenarios:
Table: 7 Result in different scenarios under pseudo metric distances

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Order quantity</th>
<th>Cycle time</th>
<th>Average inventory cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crisp</td>
<td>309.84</td>
<td>3.098</td>
<td>774.596</td>
</tr>
<tr>
<td>Intuitionistic fuzzy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case-B ( (m=2, n=1/2) )</td>
<td>311.69</td>
<td>3.117</td>
<td>770.000</td>
</tr>
<tr>
<td>Case-A ( (m=2, n=4) )</td>
<td>310.54</td>
<td>3.105</td>
<td>770.053</td>
</tr>
<tr>
<td>Case-C ( (m=1/2,n=1/4) )</td>
<td>311.62</td>
<td>3.116</td>
<td>770.090</td>
</tr>
</tbody>
</table>

Table 7 shows that the average inventory cost is minimum when we consider fuzzy cost parameters. Moreover, in fuzzy case we see that the ranking of the costs is exactly same manner with respect to pseudo metric distances. From Table 6 we see that the pseudo metric distance is maximum in case B and minimum in case C. In case of decision making, Table 7 shows that the average inventory cost is minimum in case B and maximum in case C. So we can conclude that when we consider minimization problem maximum pseudo metric distance implies minimum average inventory cost and minimum pseudo metric distance implies maximum average inventory cost. Here we do not consider metric distances because we see that the metric distances are always greater than pseudo metric distances. But the distance between two fuzzy sets should not be large that is why we consider here pseudo metric distances.

3.2 Graphical Illustration
Figure 2. shows that the pseudo metric distance is maximum in case B and that is minimum in case C.

Figure 3. Shows that the average inventory cost is minimum in intuitionistic fuzzy environment rather than crisp environment.
5. Area of application

The area of applications of this proposed approach are stated below:

  i. It is used to measure the qualitative differences among various subjects like any supply chain modelling or decision-making problem.
  ii. To know the degrees of membership, non-membership and indeterminacy, this method can be applied.
  iii. Any kind of ranking of different subjects over several disciplines is possible with its help.

5.1 Merits and Demerits

After the study of numerical and graphical illustrations, we see that there exists some merits and demerits of the proposed approach. They are stated as follows:

5.1.1 Merits

  i. This approach is very useful to find difference between two IFSs whose membership, non-membership and indeterminacy functions are non-linear.
  ii. This method told us that the pseudo metric distance is more user friendly for decision making.
  iii. This method helps us to study different nature of intuitionistic fuzzy sets drawn over physical problems with more detailing.
5.1.2 Demerits

i. This method is not applicable for the IFSs which are defined in a discrete space.

ii. This method is silent for higher dimensional intuitionistic fuzzy sets.

iii. This method might be complicated to handle when the membership function is complicated.

iv. The results may vary when the IFSs are assumed to be different.

6. Conclusion

In this study we have discussed about the qualitative differences of the subjects of real-world problem by means of metric distances between two non-linear intuitionistic fuzzy sets. Here we see that the distance under pseudo metric is more effective and extensive rather than the distance under conventional standard metric. The ranking of (pseudo) metric distances gives us a clear idea of quality measurement of two IFSs with non-linear membership and non-membership function. This idea will be very useful to solve a decision-making problem. Also, graphical illustrations show the fluctuation of differences in several criteria of exponents in membership and non-membership function.

Scope of future work

This study of IFS with the help of metric and pseudo metric is innovative. In future, various types of works can be done using this approach. This method can be applied for decision making problems such as supply chain modelling or inventory modelling.

Conflicts of interest

It is declared by the authors that there is no conflict of interest regarding the publication of this article.

References


