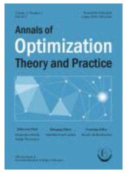
Annals of Optimization Theory and Practice

Volume 3, Number 3, 93-115
December 2020

DOI: 10.22121/aotp.2020.252845.1047



Computation of three-stage stochastic transportation planning under an uncertain environment

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(Received: October 14, 2020 / Accepted: November 13, 2020)

Abstract Stochastic programming is often used to solve optimization problems where parameters are uncertain. In this article, we have proposed a mathematical model for a three-stage transportation problem, where the parameters, namely transport costs, demand, unload capacity and external purchasing costs are uncertain. In order to remove the uncertainty, we have proposed a new transformation technique to reformulate the uncertain model deterministically with the help of Essen inequality. The obtained equivalent deterministic model is nonlinear. Furthermore, we have provided a theorem to ensure that the deterministic model gives a feasible solution. Finally, a numerical example, following uniform random variables, is presented to illustrate the model and methodology.

Keywords Stochastic optimization; Chance constraints programming; Essen inequality

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Abstract

Stochastic programming is often used to solve optimization problems where parameters are uncertain. In this article, we have proposed a mathematical model for a three-stage transportation problem, where the parameters, namely transport costs, demand, unload capacity and external purchasing costs are uncertain. In order to remove the uncertainty, we have proposed a new transformation technique to reformulate the uncertain model deterministically with the help of Essen inequality. The obtained equivalent deterministic model is nonlinear. Furthermore, we have provided a theorem to ensure that the deterministic model gives a feasible solution. Finally, a numerical example, following uniform random variables, is presented to illustrate the model and methodology.

Keywords: Stochastic optimization, Chance constraints programming, Essen inequality

1. Introduction

General formulation of linear programming problem is given by:

$$\min_{x} \left\{c^T x : Ax \leq b, x \geq 0\right\} \tag{1.1}$$

where $x \in \mathcal{R}^n$, $A \in \mathcal{R}^m \times \mathcal{R}^n, c \in \mathcal{R}^n$ and $b \in \mathcal{R}^m$. In real-world, the parameters c, A, b are not certain. Uncertainty can be considered as probability, probabilistic or interval. In the case of possibilistic, the decision-maker has several options for choosing parameters, in which he can choose the best among them. If any of the parameters in (1.1) followed a random variable, the problem is known as the stochastic programming (SP) problem. In SP, if the constraints are probabilistic, then such problems are related to probabilistic types of situations. A chance constraint programming technique is used to reformulate the model into a deterministic form. It was first introduced by Charnes [1] and derived the corresponding equivalent deterministic model. In most cases, the author has assumed that the right-hand side parameter is uncertain, which is easy to reformulate the model deterministically. But if the coefficient parameter ($a_{ij} \in A, i = 1, 2..., m, j = 1, 2, ..., n$) are independent other than normal random variables then we need to define the joint probability distribution function. It is the main drawback to defining the joint probability distribution function for big data. To avoid this situation, we have proposed an alternative technique based on Essen inequality to derive deterministic models.

There are several articles related to stochastic programming technique with different scenarios have been discussed in the literature. Then by using chance constraint programming technique and obtained the corresponding equivalent deterministic form.

For details on stochastic optimization, we refer to Prékopa [2], Birge and Louveaux [3] and Ruszezyński and Shapiro [4] and these articles [5, 6] are related to transportation problem with the uncertain parameter. Symonds [7] has studied uncertain linear programming problem and obtained the deterministic solutions for the class of chance-constraint programming problem. Olson [8] studied Chance-Constrained Programming for linear approximation. Atalaya [9] studied chance-constrained stochastic programming model by considering the coefficient parameters follow independent gamma random variables. Liu and Iwamura [10] extended chance constrained programming from stochastic to fuzzy environments and also proposed a fuzzy simulation technique to obtain their crisp equivalents. Hulsurkar *et al* [11] studied multi-objective stochastic linear programming problem by assuming the coefficients parameters in the objective functions and in the constraints, and the right-hand-side parameters in the constraints as normal random variables. Yang and Liu [12] presented fixed charge solid transportation problem, in which the parameters namely direct costs, the fixed charges, the supplies, the demands and the conveyance capacities are supposed to be fuzzy variables. Roy [13] studied multi-choice stochastic transportation problem where the parameters namely supplies and demands follow Weibull distribution. Roy *et al* [14] obtained an equivalent deterministic model of a multi-choice stochastic transportation problem where supplies and demands follow exponential distribution. Xiao and Yufu [15] presented Uncertain chance-constrained programming model for project scheduling problem. Roy *et al* [16] sudied a multi-objective multi-item fixed-charge solid transportation problem where the uncertain parameters are fuzzy-rough in nature. Shubham *et al* [17] studied multiobjective solid transportation problem by considering the parameters (demand, supply and conveyance) are uncertain and follows independent gamma random variables. Recently, Sagratella *et al* [21] extended fixed charged transportation problem into anon-cooperative fixed charge transportation problem. Shen and Zhu [22] studied fixed charge transportation problem, where the demands, supplies, availabilities, fixed charges, and transported quantities are assumed as uncertain variables. Genetic algorithm and particle swarm optimization are proposed to solve the problem.

From the literature of stochastic programming problems, it has been observed that most of the work has been done by considering the right-hand side parameters that are either random variables or fuzzy. To solve the problem with random uncertainty, Chebyshev inequality based chance constraint programming technique has been applied. In this present study, we propose a different approach to reformulate the deterministic model in deterministic form. The major contributions of the present study are as follows:

- A mathematical formulation of a three-stage stochastic sugarcane transport planning model has been proposed.
- Further, we consider that transportation costs, external purchasing costs, demand and unloaded capacities follow some independent random variables.
- To obtain the deterministic model of the proposed problem, we propose chance constraint programming technique based on Essen inequality.

- In addition, feasibility condition is derived to obtain an optimal solution to the proposed problem.

This article is organized as follows. In Section 2, a brief description of the problem and a mathematical formulation for the three stage transportation planning problem with uncertain parameters is presented. In Section 3, an equivalent deterministic model is derived. The uniform distribution approach and a numerical example are set out in Sections 4 and 5, respectively. Finally, Section 6 draws conclusions.

2. Problem Description

This problem is related to the real-life problem of three stages of transportation planning for sugarcane from plants to the suppliers to the destinations. In this planning, we consider that every supplier has a different number of plants, which is shown in Fig. 1. Let us consider E is a set of suppliers and each supplier has the set of several

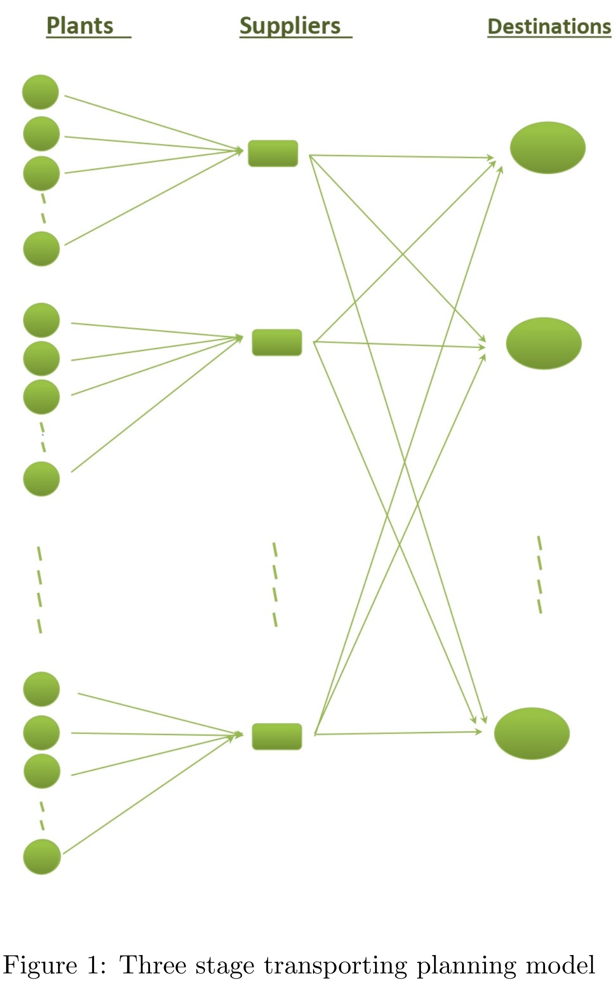

Figure 1: Three stage transporting planning model

plants $O_k, k \in E$ (origins) and satisfied the set of demand D (destination) of sugarcane of the sugar factories. Here, we have considered the demand d_j of sugar cane at sugar factory $j \in D$ is stochastic and considered a uniform fleet of conveyances with capacity q which is allowed to fully loaded shipment. Since the demand is unknown and shipments are performed by the capacitated conveyance which has to be booked in advance before knowing the demand. If the demand is known, then there is an option to discount vehicles booked i.e, according to demand some conveyances can be canceled. Conveyances z_{ijk} which have been actually used from suppliers $i \in O_k, k \in E$ to sugar factory $j \in D$. The cancellation fee of conveyance $\alpha, (0 \leq \alpha \leq 1)$ is proportional to the transportation cost. If the conveyance is booked and then used, then the transportation cost of conveyance is qc_{ijk} and if the conveyance is booked and later canceled then transportation cost will be $\alpha q c_{ijk}$ from the suppliers $i \in O_k, k \in E$ to the destination $j \in D$. If the demand of sugarcane in sugar factories are not fulfilling, then remaining sugarcane $q\ y_j, j \in D$, can be purchased from the external sources at a higher price $b_j, j \in D$ that can also be uncertain. From the above-described problem, the aim is to determine how many numbers of conveyance x_{ijk} need to book by which, the factory manager can transport sugarcane according to the demand. In order, we also minimize the total transportation cost and the cost buying from the external sources in extreme situations.

2.1. Mathematical formulation of transportation planning

In this Section, we have discuss the mathematical formulation of the above described problem. Consider there are ℓ suppliers $(E_k,\ k = 1, 2, ..., \ell)$, m plants $(O_i,\ i = 1, 2, ..., m)$ and n destination $(D_j,\ j = 1, 2, ..., n)$ point. The mathematical model of the above defined problem is given as follows:

$$\min \quad q\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{\ell} c_{ijk}x_{ijk} + \left[q\sum_{j=1}^{n} b_j y_j - \alpha q \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{\ell} c_{ijk}(x_{ijk} - z_{ijk}) \right], \tag{2.1}$$

subject to

$$q\sum_{i=1}^{m}\sum_{k=1}^{\ell} x_{ijk} \leq g_j, \qquad j = 1, 2, ..., n, \tag{2.2}$$

$$L_j^0 + q\left(\sum_{i=1}^{m}\sum_{k=1}^{\ell} z_{ijk} + y_j\right) \geq d_j, \qquad j = 1, 2, ..., n, \tag{2.3}$$

$$r_k \leq q\sum_{i=1}^{m}\sum_{j=1}^{n} z_{ijk} \leq v_k, \qquad k = 1, 2, ..., \ell, \tag{2.4}$$

$$z_{ijk} \leq x_{ijk}, \quad i = 1, 2, ..., m; j = 1, 2, ..., n; \; k = 1, 2, ..., \ell, \tag{2.5}$$

$$x_{ijk}, z_{ijk} \in \mathbb{N}, \quad i = 1, 2, ..., m; j = 1, 2, ..., n; \; k = 1, 2, ..., \ell, \tag{2.6}$$

$$y_j \in \mathbb{R}^+, \tag{2.7}$$

where

decision variables are

x_{ijk}	number of conveyance booked for the i-th plant of the k-th suppliers to the j-th destination
z_{ijk}	number of conveyance actually used for the i-th plant of the k-th suppliers to the j-th destination
y_j	volume of sugarcane to purchase from external source for j-th destination point

parameters are

c_{ijk}	unit transportation cost for the i-th plant of the k-th suppliers to the j-th destination
b_j	purchasing cost from external sources for j-th destination point
g_j	unloading capacity at the j-th destination point
q	the conveyance capacity
v_k	maximum requirement capacity of k-th suppliers
r_k	minimum requirement capacity of k-th suppliers
L_j^0	initial inventory of the sugarcane at j-th destination point
α	discount
d_j	demand of sugarcane at j-th destination point

In the objective function (2.1) the first sum represents the expected booking costs of the conveyance, while the second sum denotes expected recourse actions, consisting of buying sugarcane from the external sources. Constraints (2.2) ensure that the total quantity of sugarcane delivered from suppliers to destination j cannot greater than the j-destination's unloaded capacity g_j. Constraints (2.3) guarantees that the demand of j-destination's point is fulfilled. Constraints (2.5) ensure that the number of conveyance serving to the suppliers does not exceed the maximum production capacity and also satisfied the minimum required capacity of sugarcane. Constraints (2.5) ensure that the number of actually used conveyances is at most equal to the number of conveyance booked in advance.

If the number of actually used conveyance corresponding to the number of pre-booked conveyance is equal i.e, $(x_{ijk} = z_{ijk})$. Then the above model can be reformulated as follows:

$$\min \quad q \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{\ell} c_{ijk} x_{ijk} + q \sum_{j=1}^{n} b_j y_j, \tag{2.8}$$

subject to

$$q\sum_{i=1}^{n}\sum_{k=1}^{\ell} x_{ijk} \leq g_j, \qquad j=1,2,...,n, \tag{2.9}$$

$$L_j^0 + q\left(\sum_{i=1}^{n}\sum_{k=1}^{\ell} z_{ijk} + y_j\right) \geq d_j, \qquad j=1,2,...,n, \tag{2.10}$$

$$r_k \leq q\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \leq v_k, \qquad k=1,2,...,\ell, \tag{2.11}$$

$$x_{ijk} \in \mathbb{N}, \qquad i=1,2,...,m; j=1,2,...,n; \quad k=1,2,...,\ell, \tag{2.12}$$

$$y_j \in \mathbb{R}^+, \qquad j=1,2,...,n. \tag{2.13}$$

3. Stochastic programming problem

The occurrence of randomness in the model parameters can be formulated as stochastic programming (SP) within a general optimization framework. Stochastic programming is widely used in many real-world decision making problems of engineering, management science, and technology. In the above defined mathematical model, we considered that the cost parameter $c_{ijk}, i = 1,2,...,m; j = 1,2,...,n; k = 1,2,...,\ell$, demand parameters $d_j, j = 1,2,...,n$, external purchasing cost $b_j, j = 1,2,...,n$ and unloaded capacity $g_j, j = 1,2,...,n$ are uncertain and follow some random variables. Due to uncertainty, we are unable to solve the problem. Thus, we need to reformulate into deterministic model of the problem. In order to remove the randomness, the proposed problem is stated as a chance constraint programming model with certain probability level. Since the uncertainty is characterized in the objective function. Thus, it has been transformed into set of probabilistic constraint by introduce new auxiliary variables $u_k, t_k, k = 1,2,...,\ell$ and w. Hence, chance constraint formulation of the mathematical model (2.1)-(2.7) as:

$$\min \quad q\sum_{k=1}^{\ell} u_k + qw - \alpha q\sum_{k=1}^{\ell} t_k, \tag{3.1}$$

subject to

$$Pr\left(\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ijk}x_{ijk} \leq u_k\right) \geq 1-\epsilon_k, \quad k=1,2,...,\ell, \tag{3.2}$$

$$Pr\left(\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ijk}(x_{ijk}-z_{ijk}) \leq t_k\right) \geq 1-\epsilon_k^1, \quad k=1,2,...,\ell, \tag{3.3}$$

$$Pr\left(\sum_{j=1}^{n} b_j y_j \leq w\right) \geq 1-\epsilon^2, \tag{3.4}$$

$$Pr\left(q\sum_{i=1}^{m}\sum_{k=1}^{\ell}x_{ijk}\le g_j\right) \;\ge\; 1-\beta_j^{'}, \quad j=1,2,...,n, \tag{3.5}$$

$$Pr\left(L_j^0+q\left(\sum_{i=1}^{m}\sum_{k=1}^{\ell}z_{ijk}+y_j\right)\ge d_j\right) \;\ge\; 1-\beta_j, \quad j=1,2,...,n, \tag{3.6}$$

$$r_k\le q\sum_{i=1}^{m}\sum_{j=1}^{n}z_{ijk} \;\le\; v_k, \qquad k=1,2,...,\ell, \tag{3.7}$$

$$z_{ijk} \;\le\; x_{ijk}, \quad i=1,2,...,m; j=1,2,...,n; \; k=1,2,...,\ell, \tag{3.8}$$

$$x_{ijk}, z_{ijk} \;\in\; \mathbb{N}, \qquad i=1,2,...,m; j=1,2,...,n; k=1,2,...,\ell, \tag{3.9}$$

$$y_j\in\mathbb{R}^+, u_k, t_k, w \;\ge\; 0, \tag{3.10}$$

where $\epsilon_k, \epsilon_k^1, k=1,2,...,\ell$, ϵ^2, $\beta_j^{'}$ and $\beta_j, j=1,2,...,n$ are the probability levels. In the next Section, we discuss Essen inequality and its use in the chance constraint programming model.

3.1. Deterministic model formulation

In this section, we obtained the deterministic model of the proposed problem defined in the above section with the help of two different inequalities, Essen inequality and Chebyshev inequality under some different circumstances. The Chebyshev inequality used in constraints where only one random variable is presented like constraint (3.5) and (3.6). On the other hand, Essen inequality uses where the parameter is given a linear combination of n independent random variables. To find the deterministic formulation equivalent to the chance constraint, we need to define the corresponding probability distribution function, which includes the chance constraint. If the constraint has only one random variable, it is easy to obtain an equivalent deterministic form using the Chebyshev inequality. But, if the constraint has more than one random variable and which are a form of linear combination then we need to define the corresponding joint probation density function. Here, we can also use the Chebyshev inequality but the big data defining the joint probability density function becomes too complex. To avoid this situation, we proposed an alternative technique, in which we can easily obtain an equivalent deterministic model.

To obtain the deterministic form, we consider the absolute distance between the sum of the independent random variables and the standard normal distribution. The theorem related to this approximation is known as the Essen inequality, given as follows:

Theorem 3.1. *(Petrov [18]). If $X_1, X_2, \ldots, X_n$ are independent random variable with*

$E(X_j) = 0$ and $E|X_j|^3 < \infty, (\ j = 1, 2, \ldots, n)$, where if it is follows as:

$$\sigma_j^2 = E(X_j^2), \quad B_n = \sum_{j=1}^{n} \sigma_j^2, \quad f_n(x) = Pr\left(\frac{\sum_{j=1}^{n} X_j}{\sqrt{B_n}} < x\right),$$
$$L_n = B_n^{-3/2} \sum_{j=1}^{n} E(|X_j|^3), \quad then \quad \sup_x |f_n(x) - \Phi(x)| \leq S L_n, \tag{3.11}$$

where $S = 0.7975$ suggested by [19] is a positive constant and $f_n(x)$ be the distribution function corresponding the random variables and $\Phi(x)$ is a standard normal distribution function.

Proof. Proof of this theorem is given into [18]. □

Now with the help of above Theorem 3.1 we find the deterministic formulation of the constraint (3.2). Assume that $d_k^{'} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ijk} x_{ijk}, k = 1, 2, ..., \ell$ then constraint (3.2) becomes:

$$Pr(d_k^{'} \leq u_k) \geq (1 - \epsilon_k), \quad k = 1, 2, ..., \ell, \tag{3.12}$$

If $E[d_k^{'}] \neq 0$ then the Essen inequality is not applicable precisely. Thus, we redefine new random variable taking with fixed value of i and j, $r_k = c_{ijk} x_{ijk} - E[c_{ijk} x_{ijk}], \ \forall \ k$. Then we obtain

$$\begin{aligned} E[r_{ij}] &= E\left[c_{ijk} x_{ijk} - E(c_{ijk} x_{ijk})\right] \\ &= 0, \\ \text{and} \qquad \mathrm{Var}[r_{ij}] &= \mathrm{Var}[c_{ijk} x_{ijk} - E(c_{ijk} x_{ijk})] \\ &= x_{ijk}^2 \mathrm{Var}(c_{ijk}). \end{aligned}$$

Now, form the Essen inequality B_k becomes:

$$B_k = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk}^2 \mathrm{Var}(c_{ijk}), \quad k = 1, 2, \cdots \ell.$$

The third absolute moment of r_{ij} is given by:

$$\begin{aligned} E\left(|r_{ij}|^3\right) &= E\left(|c_{ijk} x_{ijk} - E(c_{ijk} x_{ijk})|^3\right) \\ &= x_{ijk}^3 E\left(|c_{ijk} - E(c_{ijk})|^3\right). \end{aligned}$$

Thus, we obtain L_k as:

$$L_k \quad = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk}^3 E(|c_{ijk} - E(c_{ijk})|^3)}{\left[\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk}^2 \mathrm{Var}(c_{ijk})\right]^{3/2}}, \quad k = 1, 2, \cdots \ell. \tag{3.13}$$

The distribution function f_k is defined as:

$$\begin{aligned} F_k(x) =& Pr\left(\frac{\sum_{i=1}^m \sum_{j=1}^n r_{ij}}{\sqrt{B_k}} \leq x\right) \\ =& Pr\left(\frac{\sum_{i=1}^m \sum_{j=1}^n c_{ijk}x_{ijk} - \sum_{i=1}^m \sum_{j=1}^n E(c_{ijk})x_{ijk}}{\sqrt{\sum_{i=1}^m \sum_{j=1}^n x_{ijk}^2 \text{Var}(c_{ijk})}} \leq x\right) \end{aligned} \tag{3.14}$$

Now, we normalize the chance constraint given by (3.12) as:

$$\begin{aligned} Pr\Bigg(& \frac{\sum_{i=1}^m \sum_{j=1}^n c_{ijk}x_{ijk} - \sum_{i=1}^m \sum_{j=1}^n E(c_{ijk})x_{ijk}}{\sqrt{\sum_{i=1}^m \sum_{j=1}^n x_{ijk}^2 \text{Var}(c_{ijk})}} \\ & \leq \frac{u_k - \sum_{i=1}^m \sum_{j=1}^n E(c_{ijk})x_{ijk}}{\sqrt{\sum_{i=1}^m \sum_{j=1}^n \text{Var}(c_{ijk}){x_{ijk}}^2}}\Bigg) \geq (1-\epsilon_k), \end{aligned} \tag{3.15}$$

and now we compare (3.14) and (3.15), we get

$$\Rightarrow \quad F_k\left(\frac{u_k - \sum_{i=1}^m \sum_{j=1}^n E(c_{ijk})x_{ijk}}{\sqrt{\sum_{i=1}^m \sum_{j=1}^n \text{Var}(c_{ijk}){x_{ijk}}^2}}\right) \geq (1-\epsilon_k). \tag{3.16}$$

From the Theorem (3.1) and (3.16), we get

$$\Phi\left[\frac{u_k - \sum_{i=1}^m \sum_{j=1}^n E(c_{ijk})x_{ijk}}{\sqrt{\sum_{i=1}^m \sum_{j=1}^n \text{Var}(c_{ijk}){x_{ijk}}^2}}\right] \geq (1-\epsilon_k - SL_k). \tag{3.17}$$

Due to functional value of L_k, we can not determine $\Phi^{-1}(1-(\epsilon_k + SL_k))$. Thus, we have used approximation function of standard normal distribution $\Phi(x), x \geq 0$ suggested by [20] as:

$$\begin{aligned} \Rightarrow \Phi & \left[\frac{u_k - \sum_{i=1}^m \sum_{j=1}^n E(c_{ijk})x_{ijk}}{\sqrt{\sum_{i=1}^m \sum_{j=1}^n \text{Var}(c_{ijk}){x_{ijk}}^2}}\right] \\ &= \frac{1}{2}\left(1 + \{1 - \exp\left(-\frac{2}{\pi}\left(\frac{u_k - \sum_{i=1}^m \sum_{j=1}^n E(c_{ijk})x_{ijk}}{\sqrt{\sum_{i=1}^m \sum_{j=1}^n \text{Var}(c_{ijk})x_{ijk}^2}}\right)^2\right)\}^{1/2}\right). \end{aligned}$$

Therefore, (3.17) becomes:

$$\begin{aligned} & \frac{1}{2}\left(1 + \left\{1 - \exp\left(-\frac{2}{\pi}\left(\frac{u_k - \sum_{i=1}^m \sum_{j=1}^n E(c_{ijk})x_{ijk}}{\sqrt{\sum_{i=1}^m \sum_{j=1}^n \text{Var}(c_{ijk})x_{ijk}^2}}\right)^2\right)\right\}^{1/2}\right) \\ & \qquad \geq (1-\epsilon_k - SL_k), \quad k = 1, 2, ..., \ell, \end{aligned} \tag{3.18}$$

where L_k is defined by (3.13).Thus, equation (3.18) is the deterministic form of equation (3.2). Similarly, we obtained deterministic form of equation (3.3) as:

$$\frac{1}{2}\left(1+\left\{1-\exp\left(-\frac{2}{\pi}\left(\frac{t_k-\sum_{i=1}^{m}\sum_{j=1}^{n}E(c_{ijk})(x_{ijk}-z_{ijk})}{\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}\text{Var}(c_{ijk})(x_{ijk}-z_{ijk})^2}}\right)^2\right)\right\}^{1/2}\right)$$
$$\geq\left(1-\epsilon_k^1-SL_k^{'}\right),\quad k=1,2,...,\ell, \tag{3.19}$$

where ϵ_k^1 is the probability level and

$$L_k^{'}=\frac{\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}(x_{ijk}-z_{ijk})^3E(|c_{ijk}-E(c_{ijk})|^3)}{\left[\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}(x_{ijk}-z_{ijk})^2\text{Var}(c_{ijk})\right]^{3/2}}.$$

Constraint (3.4) is single constraint. Therefore, in a similar way, we obtained a deterministic form of equation (3.4) which is given as,

$$\frac{1}{2}\left(1+\left\{1-\exp\left(-\frac{2}{\pi}\left(\frac{w-\sum_{j=1}^{n}E(b_j)y_j}{\sqrt{\sum_{j=1}^{n}\text{Var}(b_j)y_j^2}}\right)^2\right)\right\}^{1/2}\right)\geq\left(1-\epsilon^2-SL_0\right), \tag{3.20}$$

where ϵ^2 is the probability level and $L_0=\dfrac{\sum\limits_{j=1}^{n}y_j^3E(|b_j-E(b_j)|^3)}{\left[\sum\limits_{j=1}^{n}y^2\text{Var}(b_j)\right]^{3/2}}$.

3.2. Deterministic form when the demand $d_j,(j=1,2,...,n)$ and unloaded capacity $g_j,(j=1,2,...,n)$ follows some random variable

Now we consider the chance constraint given by the equation (3.6), demand parameters $d_j(j=1,2,....,n)$ follow some random variable. Since the constraint has only one random variable d_j then we consider the corresponding probability density function (pdf) of d_j is given by F_{d_j}. The chance constraint in the equation (3.6) can be rewritten as:

$$Pr\left(d_j\leq L_j^0+q\left(\sum_{i=1}^{m}\sum_{k=1}^{\ell}z_{ijk}+y_j\right)\right)\geq 1-\beta_j. \tag{3.21}$$

Now with the help of Chebyshev inequality the deterministic form of the chance constraint can be presented as:

$$L_j^0+q\left(\sum_{i=1}^{m}\sum_{k=1}^{\ell}z_{ijk}+y_j\right)\geq F_{d_j}^{-1}\left(1-\beta_j\right). \tag{3.22}$$

Similarly, we can obtain the deterministic form for other chance constraint.

Thus, deterministic form of the mathematical model (3.1)-(3.9) is given by,

$$\min \quad q\sum_{k=1}^{\ell} u_k + qw - \alpha q \sum_{k=1}^{\ell} t_k, \tag{3.23}$$

subject to

$$\frac{1}{2}\left(1+\left\{1-\exp\left(-\frac{2}{\pi}\left(\frac{u_k-\sum_{i=1}^{m}\sum_{j=1}^{n}E(c_{ijk})x_{ijk}}{\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}\mathrm{Var}(c_{ijk})x_{ijk}^2}}\right)^2\right)\right\}^{1/2}\right) \geq \left(1-\epsilon_k-SL_k\right), \quad k=1,2,...,\ell, \tag{3.24}$$

$$\frac{1}{2}\left(1+\left\{1-\exp\left(-\frac{2}{\pi}\left(\frac{t_k-\sum_{i=1}^{m}\sum_{j=1}^{n}E(c_{ijk})(x_{ijk}-z_{ijk})}{\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}\mathrm{Var}(c_{ijk})(x_{ijk}-z_{ijk})^2}}\right)^2\right)\right\}^{1/2}\right) \geq \left(1-\epsilon_k^1-SL_k^{'}\right), \quad k=1,2,...,\ell, \tag{3.25}$$

$$\frac{1}{2}\left(1+\left\{1-\exp\left(-\frac{2}{\pi}\left(\frac{w-\sum_{j=1}^{n}E(b_j)y_j}{\sqrt{\sum_{j=1}^{n}\mathrm{Var}(b_j)y_j^2}}\right)^2\right)\right\}^{1/2}\right) \geq \left(1-\epsilon^2-SL_0\right), \tag{3.26}$$

$$q\sum_{i=1}^{m}\sum_{k=1}^{\ell} x_{ijk} \leq F_{g_j}^{-1}\left(\beta_j^{'}\right), \qquad j=1,2...,n, \tag{3.27}$$

$$L_j^0 + q\left(\sum_{i=1}^{m}\sum_{k=1}^{\ell} z_{ijk} + y_j\right) \geq F_{d_j}^{-1}\left(1-\beta_j\right), \quad j=1,2...,n, \tag{3.28}$$

$$r_k \leq q\sum_{i=1}^{m}\sum_{j=1}^{n} z_{ijk} \leq v_k, \qquad k=1,2,...,\ell, \tag{3.29}$$

$$z_{ijk} \leq x_{ijk}, \qquad i=1,2,...,m; j=1,2,...,n; k=1,2,...,\ell, \tag{3.30}$$

$$x_{ijk}, z_{ijk} \in \mathbb{N}, \qquad i=1,2,...,m; j=1,2,...,n; k=1,2,...,\ell, \tag{3.31}$$

$$y_j \in \mathbb{R}^+, u_k, t_k, w \geq 0. \tag{3.32}$$

Note that, the deterministic model obtained form (3.23) - (3.32) is nonlinear. In general, it is not necessary that the above deterministic model gives a feasible solution. Since, in the proposed problem, the constraint-wise uncertainty has to be considered. Thus, for each set of constraints, these two conditions must be satisfied to determine that the deterministic model has a feasible solution.

Theorem 3.2. *The deterministic model* (3.23) - (3.32) *gives a feasible solution if, for each set of constraints, where the approximation to the standard normal distribution is used, one must be satisfied with the following properties:*

a. $u_k > \sum_{i=1}^{m}\sum_{j=1}^{n} E(c_{ijk})x_{ijk}, \qquad k = 1, 2, ..., \ell.$

b. $S\dfrac{\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^3 E(|c_{ijk} - E(c_{ijk})|^3)}{\left[\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^2 Var(c_{ijk})\right]^{3/2}} \le 1 - \epsilon_k, \qquad k = 1, 2, ..., \ell.$

Proof. a. Since, we have consider new variable $r_{ij} > 0 \;\; (\forall\, i, j)$ is rewritten as:

$$
\begin{aligned}
r_{ij} &= c_{ijk}x_{ijk} - E(c_{ijk}x_{ijk}),\\
\Rightarrow \quad \sum_{i=1}^{m}\sum_{j=1}^{n} E(c_{ijk}x_{ijk}) &= \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ijk}x_{ijk} - \sum_{i=1}^{m}\sum_{j=1}^{n} r_{ij},\\
&\le u_k - \sum_{i=1}^{m}\sum_{j=1}^{n} r_{ij},\\
&< u_k.
\end{aligned}
$$

b. From the approximation of the standard normal distribution

$$
\begin{aligned}
&\Phi(x) > 0, \quad \text{if} \quad x > 0\\
&\Rightarrow \quad \Phi^{-1}(x) > 0,
\end{aligned}
$$

Thus $\quad \Phi^{-1}(1 - \epsilon_k - SL_k) > 0 \quad \text{if} \quad 1 - \epsilon_k - SL_k > 0$

$$
\Rightarrow \qquad S\frac{\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^3 E(|c_{ijk} - E(c_{ijk})|^3)}{\left[\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^2 \mathrm{Var}(c_{ijk})\right]^{3/2}} < 1 - \epsilon_k.
$$

□

Same conditions must be satisfied for the others set of constraint such as (3.25) and (3.26).

4. Uniform distribution approach

In the above Section, we have established an equivalent deterministic model for the proposed stochastic model by considering the parameters namely transportation cost, external purchasing cost and demand parameters follow independent uniform random variables with given mean and variance. The mean and variance of cost parameter c_{ijk} is $\frac{(\alpha_{ijk}+\beta_{ijk})}{2}$ and $\frac{(\beta_{ijk}-\alpha_{ijk})^2}{12}$ respectively. Thus we compute B_k as:

$$B_k = \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{(\beta_{ijk}-\alpha_{ijk})^2}{12} x_{ij}^2, \quad k = 1, 2, ..., \ell.$$

Now, we compute the absolute value of third moment, which is given by:

$$\begin{aligned} E\left(\left|c_{ijk} - \frac{(\alpha_{ijk}+\beta_{ijk})}{2}\right|^3\right) &= \int_{\alpha_{ijk}}^{\beta_{ijk}} \left|c_{ijk} - \frac{(\alpha_{ijk}+\beta_{ijk})}{2}\right|^3 f(c_{ijk}) dc_{ijk}, \\ &= -\int_{\alpha_{ijk}}^{\frac{(\alpha_{ijk}+\beta_{ijk})}{2}} \left(c_{ijk} - \frac{(\alpha_{ijk}+\beta_{ijk})}{2}\right)^3 f(c_{ijk}) dc_{ijk} \\ &\quad + \int_{\frac{(\alpha_{ijk}+\beta_{ijk})}{2}}^{\beta_{ijk}} \left(c_{ijk} - \frac{(\alpha_{ijk}+\beta_{ijk})}{2}\right)^3 f(c_{ijk}) dc_{ijk}, \\ &= \frac{(\beta_{ijk}-\alpha_{ijk})^3}{32}. \end{aligned}$$

Again we compute the L_k as:

$$L_k = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} \frac{(\beta_{ijk}-\alpha_{ijk})^3}{32} x_{ijk}^3}{\left[\sum_{i=1}^{m}\sum_{j=1}^{n} \frac{(\beta_{ijk}-\alpha_{ijk})^2}{12} x_{ijk}^2\right]^{3/2}}, \quad k = 1, 2, \cdots \ell. \tag{4.1}$$

Similarly, we obtained the value of $L_k^{'}$ and L_0 for remaining uncertain constraints as:

$$L_k^{'} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} \frac{(\beta_{ijk}-\alpha_{ijk})^3}{32} (x_{ijk}-z_{ijk})^3}{\left[\sum_{i=1}^{m}\sum_{j=1}^{n} \frac{(\beta_{ijk}-\alpha_{ijk})^2}{12} (x_{ijk}-z_{ijk})^2\right]^{3/2}}, \quad L_0 = \frac{\sum_{j=1}^{n} \frac{(\beta_j-\alpha_j)^3}{32} y_j^3}{\left[\sum_{j=1}^{n} \frac{(\beta_j-\alpha_j)^2}{12} y_j^2\right]^{3/2}}. \tag{4.2}$$

Putting the value of $L_0, L_k, L_k^{'}, k = 1, 2, , , ., \ell$ in the deterministic model (3.23)-(3.32) is given by:

$$\min \quad q\sum_{k=1}^{\ell} u_k + qw - \alpha q \sum_{k=1}^{\ell} t_k, \tag{4.3}$$

subject to

$$\frac{1}{2}\left(1 + \left\{1 - \exp\left(-\frac{2}{\pi}\left(\frac{u_k - \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{(\alpha_{ijk}+\beta_{ijk})}{2} x_{ijk}}{\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n} \frac{(\beta_{ijk}-\alpha_{ijk})^2}{12} x_{ijk}^2}}\right)^2\right)\right\}^{1/2}\right) \geq 1 - (\epsilon_k + SL_k), \; k = 1, 2, ..., \ell, \tag{4.4}$$

$$\frac{1}{2}\left(1+\left\{1-\exp\left(-\frac{2}{\pi}\left(\frac{t_k-\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{(\alpha_{ijk}+\beta_{ijk})}{2}(x_{ijk}-z_{ijk})}{\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{(\beta_{ijk}-\alpha_{ijk})^2}{12}(x_{ijk}-z_{ijk})^2}}\right)^2\right)\right\}^{1/2}\right) \geq 1-(\epsilon_k^1+SL_k^{'}), k=1,2,...,\ell, \tag{4.5}$$

$$\frac{1}{2}\left(1+\left\{1-\exp\left(-\frac{2}{\pi}\left(\frac{w-\sum_{j=1}^{n}\frac{(\beta_j+\alpha_j)}{2}y_j}{\sqrt{\sum_{j=1}^{n}\frac{(\beta_j-\alpha_j)^2}{12}y_j^2}}\right)^2\right)\right\}^{1/2}\right) \geq 1-(\epsilon^2+SL_0), \tag{4.6}$$

$$q\sum_{i=1}^{m}\sum_{k=1}^{\ell}x_{ijk} \leq U_j^{'}-(U_j^{'}-L_j^{'})(1-\beta_j^{'}), \quad j=1,2...,n, \tag{4.7}$$

$$L_j^0+q\left(\sum_{i=1}^{m}\sum_{k=1}^{\ell}z_{ijk}+y_j\right) \geq L_j+(1-\beta_j)(U_j-L_j), \quad j=1,2...,n, \tag{4.8}$$

$$L_j^0+q\left(\sum_{i=1}^{m}\sum_{k=1}^{\ell}z_{ijk}+y_j\right) \geq L_j+(1-\beta_j)(U_j-L_j), \quad j=1,2...,n, \tag{4.9}$$

$$u_k > \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{(\alpha_{ijk}+\beta_{ijk})}{2}x_{ijk}, \qquad k=1,2,...,\ell, \tag{4.10}$$

$$t_k > \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{(\alpha_{ijk}+\beta_{ijk})}{2}(x_{ijk}-z_{ijk}), \quad k=1,2,...,\ell, \tag{4.11}$$

$$w > \sum_{j=1}^{n}\frac{(\beta_j+\alpha_j)}{2}y_j, \qquad k=1,2,...,\ell, \tag{4.12}$$

$$SL_k < 1-\epsilon_k, \qquad k=1,2,...,\ell, \tag{4.13}$$

$$SL_k^{'} < 1-\epsilon_k^1, \qquad k=1,2,...,\ell, \tag{4.14}$$

$$SL_0 < 1-\epsilon^2, \tag{4.15}$$

$$r_k \leq q\sum_{i=1}^{m}\sum_{j=1}^{n}z_{ijk} \leq v_k, \qquad k=1,2,...,\ell, \tag{4.16}$$

$$z_{ijk} \leq x_{ijk}, \qquad i=1,2,...,m; j=1,2,...,n; k=1,2,...,\ell, \tag{4.17}$$

$$x_{ijk}, z_{ijk} \in \mathbb{N}, \qquad i=1,2,...,m; j=1,2,...,n; k=1,2,...,\ell, \tag{4.18}$$

$$y_j \in \mathbb{R}^+, \qquad j=1,2,...,n, \tag{4.19}$$

$$u_k, t_k, w \geq 0 \qquad k=1,2,...,\ell. \tag{4.20}$$

5. Numerical Example

In this example we have considered three suppliers (A, B, C). Supplier (A) has three numbers of plants, i.e. $i=3$, supplier (B) has two number of plants i.e, $i=2$ and supplier (C) has four number of plants i.e, $i=4$. Consider that no one conveyance has

been canceled i.e., $\alpha = 0$. From these three suppliers (origins), factory manager wants to transport sugarcane into three different sugar factories (Destination). Transportation cost is random variable which is given into Table-1. Suppose capacity of each conveyance is $q = 50$ quintals. Initial inventory at each destination is $20, 18,$ and 15 quintals. The maximum and minimum requirement of the capacity of suppliers is $1500, 2000, 1800$ quintals and $400, 800, 500$ quintals respectively.
Moreover demand, external purchasing cost, unloaded capacity follows uniform distribution i.e, $d_1 = U(1400, 1600), d_2 = U(2000, 2100), d_3 = U(1600, 1800), b_1 = U(190, 220), b_2 = U(210, 220), b_3 = U(230, 250), g_1 = U(1100, 1400), g_2 = U(1700, 1900), g_3 = U(1800, 2000)$ and probability levels are $\beta_1 = 99\%, \beta_2 = 94\%, \beta_3 = 96\%, \epsilon^2 = 95\%, \beta_1^{'} = 95\%, \beta_2^{'} = 94\%$, and $\beta_3^{'} = 94\%$ respectively. Then $E(b_1) = 205, E(b_2) = 215, E(b_3) = 240$, $E(d_1) = 1500, E(d_2) = 2050, E(d_3) = 1700, E(g_1) = 1250, E(g_2) = 1800$ and $E(g_3) = 1900$.

Table 1
Transportation cost in rupees

Supplier (A)	D_1	D_2	D_3
	$U(18, 24)$	$U(20, 25)$	$U(24, 28)$
$i = 3$	$U(20, 24)$	$U(20, 24)$	$U(25, 30)$
$k = 1$	$U(16, 20)$	$U(22, 26)$	$U(25, 28)$

Supplier (B)	D_1	D_2	D_3
$i = 2$	$U(20, 24)$	$U(20, 26)$	$U(25, 30)$
$k = 2$	$U(26, 30)$	$U(27, 33)$	$U(30, 35)$

Supplier (C)	D_1	D_2	D_3
	$U(26, 30)$	$U(16, 20)$	$U(20, 28)$
$i = 4$	$U(26, 30)$	$U(20, 25)$	$U(24, 30)$
$k = 3$	$U(25, 28)$	$U(24, 30)$	$U(24, 28)$
	$U(22, 27)$	$U(20, 24)$	$U(18, 23)$

For solving the problem, we have used Essen inequality which is given by Theorem (3.1) is define as follow:

$$r_{ij} = c_{ijk}x_{ijk} - E(c_{ijk}x_{ijk}), \qquad \forall \ k,$$
$$\text{then} \quad \text{Var}(r_{ij}) = \frac{(\beta_{ijk} - \alpha_{ijk})^2}{12} x_{ijk}^2, \qquad \forall \ k.$$

Now, we compute:

$$\begin{aligned}
B_1 &= 3x_{111}^2 + 2.08x_{121}^2 + 1.33x_{131}^2 + 1.33x_{211}^2 + 1.33x_{221}^2 + 2.08x_{231}^2 + 1.33x_{311}^2 + \\
&\quad 1.33x_{321}^2 + 0.75x_{331}^2, \\
B_2 &= 1.33x_{112}^2 + x_{122}^2 + 2.08x_{132}^2 + 1.33x_{212}^2 + x_{222}^2 + 2.08x_{232}^2, \\
B_3 &= 1.33x_{113}^2 + 1.33x_{123}^2 + 5.33x_{133}^2 + 1.33x_{213}^2 + 0.75x_{223}^2 + x_{233}^2 + 0.75x_{313}^2 + \\
&\quad x_{323}^2 + 1.33x_{333}^2 + 2.08x_{413}^2 + 1.33x_{423}^2 + 0.75x_{433}^2, \\
B_0 &= 75y_1^2 + 8.33y_2^2 + 33.33y_3^2.
\end{aligned}$$

Now, we computed L_1, L_2, L_3 and L_0 by using (4.1) and (4.2) as follows:

$$\begin{aligned}
L_1 &= \frac{6.75x_{111}^3 + 3.9x_{121}^3 + 2x_{131}^3 + 2x211^3 + 2x_{221}^3 + 3.9x_{231}^3 + 2x_{311}^3 + 2x_{321}^3 + 0.84x_{331}^3}{B_1^{3/2}}, \\
L_2 &= \frac{2x_{112}^3 + 6.75x_{122}^3 + 3.9x_{132}^3 + 2x_{212}^3 + 6.75x_{222}^3 + 3.9x_{232}^3}{B_2^{3/2}}, \\
L_3 &= (2x_{113}^3 + 2x_{123}^3 + 16x_{133}^3 + 2x_{213}^3 + 3.9x_{223}^3 + 6.75x_{233}^3 + 0.84x_{313}^3 + 6.75x_{323}^3 + \\
&\quad 2x_{333}^3 + 3.9x_{413}^3 + 2x_{423}^3 + 3.9x_{433}^3)/(B_3^{3/2}), \\
L_0 &= \frac{843.75y_1^3 + 31.25y_2^3 + 205y_3^3}{B_0^{3/2}},
\end{aligned}$$

and also find the expected value of for $k = 1$

$$\sum_{i=1}^{3}\sum_{j=1}^{3} \frac{(\beta_{ij1} + \alpha_{ij1})}{2} x_{ij1} = 21x_{111} + 22.5x_{121} + 26x_{131} + 22x_{211} + 22x_{221}$$
$$+22.5x_{231} + 18x_{311} + 24x_{321} + 26.5x_{331},$$

similarly, for $k = 2$ and $k = 3$

$$\begin{aligned}
\sum_{i=1}^{2}\sum_{j=1}^{3} \frac{(\beta_{ij2} + \alpha_{ij2})}{2} x_{ij2} &= 22x_{112} + 23x_{122} + 27.5x_{132} + 28x_{212} + 30x_{222} + 32.5x_{232}, \\
\sum_{i=1}^{4}\sum_{j=1}^{3} \frac{(\beta_{ij3} + \alpha_{ij3})}{2} x_{ij3} &= 28x_{113} + 18x_{123} + 24x_{133} + 28x_{213} + 22.5x_{223} + 27x_{233} + \\
&\quad 21.5x_{313} + 27x_{323} + 26x_{333} + 24.5x_{413} + 22x_{423} + 20.5x_{433}.
\end{aligned}$$

Thus, the deterministic form of mathematical model (4.3)-(4.20) becomes:

$$\min = 50(u_1 + u_2 + u_3) + 50w,$$

Subject to

$$\frac{1}{2}\left(1+\left\{1-\exp\left(\left(-\frac{2}{\pi}\right)\left(\frac{u_1-(\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{(\beta_{ij1}+\alpha_{ij1})}{2}x_{ij1})}{\sqrt{B_1}}\right)^2\right)\right\}^{\frac{1}{2}}\right)\geq 1-(0.01+0.7975L_1),$$

$$\frac{1}{2}\left(1+\left\{1-\exp\left(\left(-\frac{2}{\pi}\right)\left(\frac{u_2-(\sum_{i=1}^{2}\sum_{j=1}^{3}\frac{(\beta_{ij2}+\alpha_{ij2})}{2}x_{ij2})}{\sqrt{B_2}}\right)^2\right)\right\}^{\frac{1}{2}}\right)\geq 1-(0.05+0.7975L_2),$$

$$\frac{1}{2}\left(1+\left\{1-\exp\left(\left(-\frac{2}{\pi}\right)\left(\frac{u_3-(\sum_{i=1}^{4}\sum_{j=1}^{3}\frac{(\beta_{ij3}+\alpha_{ij3})}{2}x_{ij3})}{\sqrt{B_3}}\right)^2\right)\right\}^{\frac{1}{2}}\right)\geq 1-(0.05+0.7975L_3),$$

$$21x_{111}+22.5x_{121}+26x_{131}+22x_{211}+22x_{221}+22.5x_{231}+18x_{311}+24x_{321}+26.5x_{331}<u_1,$$

$$22x_{112}+23x_{122}+27.5x_{132}+28x_{212}+30x_{222}+32.5x_{232}<u_2,$$

$$28x_{113}+18x_{123}+24x_{133}+28x_{213}+22.5x_{223}+27x_{233}+21.5x_{313}+27x_{323}+26x_{333}+24.5x_{413}+22x_{423}+20.5x_{433}<u_3,$$

$$205y_1+215y_2+240y_3<w,$$

$$0.7975L_1<0.9,$$

$$0.7975L_2<0.95,$$

$$0.7975L_3<0.95,$$

$$0.7975L_0<0.9,$$

$$50(x_{111}+x_{211}+x_{311}+x_{112}+x_{212}+x_{113}+x_{213}+x_{313}+x_{413})\leq 1205,$$

$$50(x_{121}+x_{221}+x_{321}+x_{122}+x_{222}+x_{123}+x_{223}+x_{323}+x_{423})\leq 1608,$$

$$50(x_{131}+x_{231}+x_{331}+x_{132}+x_{232}+x_{133}+x_{233}+x_{333}+x_{433})\leq 1908,$$

$$20+50(x_{111}+x_{211}+x_{311}+x_{112}+x_{212}+x_{113}+x_{213}+x_{313}+x_{413}+y_1)\geq 1598,$$

$$18+50(x_{121}+x_{221}+x_{321}+x_{122}+x_{222}+x_{123}+x_{223}+x_{323}+x_{423}+y_2)\geq 2096,$$

$$25+50(x_{131}+x_{231}+x_{331}+x_{132}+x_{232}+x_{133}+x_{233}+x_{333}+x_{433}+y_3)\geq 1788,$$

$$50(x_{111}+x_{121}+x_{131}+x_{211}+x_{221}+x_{231}+x_{311}+x_{321}+x_{331})\leq 1800,$$

$$50(x_{112}+x_{122}+x_{132}+x_{212}+x_{222}+x_{232})\leq 2500,$$

$$50(x_{113}+x_{123}+x_{133}+x_{213}+x_{223}+x_{233}+x_{313}+x_{323}+x_{333}+x_{413}+x_{423}+x_{433})\leq 2000,$$

$$50(x_{111}+x_{121}+x_{131}+x_{211}+x_{221}+x_{231}+x_{311}+x_{321}+x_{331})\geq 400,$$

$$50(x_{112}+x_{122}+x_{132}+x_{212}+x_{222}+x_{232})\geq 800,$$

$$50(x_{113}+x_{123}+x_{133}+x_{213}+x_{223}+x_{233}+x_{313}+x_{323}+x_{333}+x_{413}+x_{423}+x_{433})\geq 500,$$

$$x_{ijk}\in\mathbb{N},\quad \forall\ i,j,k\quad \text{and}\ y_j\geq 0\quad \forall\ j,$$

$$u_1,u_2,u_3,w\geq 0.$$

Putting the value of $B_1, B_2, B_3, L_0, L_1, L_2,$ and L_3 in the above problem and solve the problem by using Lingo software. Thus, we obtain optimal solution is: 275783.1 and $x_{231} = 18, x_{311} = 17, x_{112} = 7, x_{122} = 5, x_{132} = 5, x_{123} = 27, x_{433} = 13, y_1 = 7.56, y_2 = 9.56, y_3 = 0, u_1 = 751.462, u_2 = 406.5, u_3 = 752.5, w = 3605.2$ and rest of the variables are zero.

5.1. Result analysis

In the previous Section, we have considered a mathematical model by considering parameters related to cost, demand, external purchasing cost, and unloaded capacity and follows a uniform random variable. To convert to deterministic form, a chance constraint programming technique is used which is based on Essen inequality. Therefore, the deterministic form of the proposed model is a nonlinear programming problem. The problem is solved on a personal computer with a 1.9GHz CPU and 4GB of memory space. Lingo 11.0 is used to solve the nonlinear programming model. After solving the example, we obtained the optimal value 275783.1 i.e. which needs to book 92 conveyance for the transport of the sugarcane to the destination point and 17.12 quintals sugarcane needs to purchase from the external sources. Furthermore, we have also solved the problem with the existing method like the Expected model and Variance model. When we used the Expected model, then obtained the optimal value is 272760 which needs to book 92 conveyance for the transport of the sugarcane to the destination point and 17.12 quintals sugarcane needs to purchase from the external sources. If we used variance model, then the optimal solution is 274708.6 which needs to book 91 conveyance for the transport of sugarcane to the destination point and 17.38 quintals sugarcane needs to purchase from the external sources.

6. Conclusion

In this paper, we a proposed mathematical model for stochastic sugarcane transport problems, where the parameters namely transport cost, external purchasing cost, demand and unload capacity, are follows some random variables. An alternative technique based on Essen inequality is proposed to derive the equivalent deterministic form of the problem. The obtained deterministic model is nonlinear. We have also provided a theorem for feasibility conditions. In addition, two existing methods, namely the expected value model and the variance model, are used to compare the solutions obtained by the existing method, by considering the parameters as uniform random variables. Although the number of bookings required in the current method is almost the same as in the case of other methods available in the literature, our proposed method is simpler and simpler than the existing methods. In the future, it would be interesting to extend the multiple stages of stochastic transportation planning, assuming that the information of the uncertain data is given in the set of the family of distribution.

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