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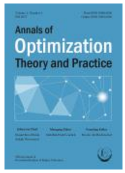

Fuzzy multicriteria decision making in medical diagnosis using an advanced distance measure on linguistic Pythagorean fuzzy sets

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Abstract Decision making is becoming one of the most vast research fields in medical science. Since last few years, the role of distance measure utilizing fuzzy sets in decision science has been significantly increased. A lot of studies have been carried out form the field of fuzzy decision making using distance measures. In this paper, Linguistic Pythagorean fuzzy sets have been utilized medical diagnosis problem. In addition, a novel distance measure on Linguistic Pythagorean fuzzy sets has been introduced and utilized in a fuzzy MCDM approach. Also, a case study is carried out in real data sets to show the usability and reliability of our proposed distance measure.

Keywords Intuitionistic fuzzy set; Decision making; Distance measure; Medical diagnosis

1. Introduction

Uncertainty is an integral part of decision-making process which arises due to the lack of knowledge, data or information. Initially, fuzzy set theory (FST) was introduced by L.A. Zadeh (1965) to handle this type of uncertainty. Later, intuitionistic fuzzy set (IFS), which is the generalization of FST was developed by Faizulkhakov,(1986) to encounter uncertainty in a more specific manner. In IFS, membership degree and non-membership degree are in such a way that the sum of the two degrees must not exceeds to one. But, in some real life situation, the sum of the membership degree and the non-membership degree provided by some expert may be great than one. For example, if an expert provides his preference towards an object is 0.7 and his disappointment is 0.5 then clearly sum becomes greater than one. Thus, IFS cannot handle such type of situation. To overcome this barrier, Yager, (2013)extends the IFS condition to its square sum that is not greater than one and introduced it as Pythagorean fuzzy set (PFS). After the introduction of this novel notion, Yager and Abbasov, (2013) studied the relations between Pythagorean fuzzy numbers (PFNs) and complex numbers. Further, Yager, (2013) developed some aggregation operator for PFS, whereas Zhang and Xu, (2014) discussed the TOPSIS under the environment of PFS. Garg,(2016), Garg,(2016), Garg,(2016) introduced some generalized averaging and geometric aggregation operators based on the Einstein t-norm operations and confidence-based Pythagorean fuzzy weighted averaging and geometric aggregation operators for PFNs. Garg,(2016) also presented the notion of correlation and correlation coefficients of PFSs and applied them in decision making. Ma and Xu ,(2016) provided some symmetric PF weighted geometric and averaging operators. Zeng, (2017) presented a MCDM approach by introducing the probabilistic and ordered weighted averaging information under PFS environment. Recently, Garg,(2018), Garg,(2018) discussed the MCDM problems with immediate probabilities approach and linear programming model–based approach. One of the most important tool is distance measure which has been successfully used in many fields such decision making, pattern recognition, machine learning, market prediction, medical diagnosis and so on. Many distance measures of FST have been extended and defined under the environment of IFS and interval valued intuitionistic fuzzy sets (IVIFS). Zhang,(2016), Zhang,(2016)paid attention to introduced the distance measure of PFNs. Peng *et al.*,(2017) studied the axiomatic approach of distance measure, similarity measure, inclusion and entropy measure for PFNs. Li and Zeng,(2017) , proposed a series of distance measures of PFNs by taking into account the four fundamental parameter of PFNs. Peng and Dai, (2019) developed a new distance measure for PFNs. Recently, Zeng *et. al.*,(2018) have defined a variety of distance measures for PFNs taking into account five parameters, namely membership degree, non-membership degree, hesitancy degree, strength of commitment about membership, and direction of commitment.Qin *et al.,(*2017) developed a new distance measure for PFSs. Garg,(2018) first introduced the notion of linguistic Pythagorean fuzzy set (LPFS) by combining the theory of PFS and linguistic term sets. Recently, Lin *et al.*, (2018) proposed some novel operational laws for linguistic Pythagorean fuzzy numbers (LPFNs) and based on them they put forwarded the interaction partitioned Bonferroni mean (LPFIPBM) operator, the weighted interaction PBM (LPFWIPBM) operator, the interaction PGBM (LPFIPGBM) operator, and the weighted interaction PGBM (LPFWIPGBM) operator under LPFNs environment and applied them to multi attribute group decision making problems.

In this study, an attempt is made to define a novel distance measure for LPFS, which is introduced by Garge. Furthermore, the novel distance measure is applied to study medical decision making and a case study is carried out in this settings. The rest of the paper is condensed as follows.

Section 2 contains the relevant definitions and basic results. In section 3, some existing distance measures for PFSs are reviewed. In section 4, a novel distance measure for PFSs is proposed and it is also verified that the distance satisfies all the axioms of distance measures. Section 5 describes a methodology of multi criteria medical decision making method and a case study is carried out by using the proposed distance measure. Finally, a concrete conclusion is presented in section 6.

2. Preliminaries:

2.1 Fuzzy Set: (Zadeh, 1965) Fuzzy set is a set in which every element has degree of membership of belonging in it. Mathematically, let X be a universal set. Then the fuzzy subset A of X is defined by its membership function $\mu_A; X \rightarrow [0,1]$, which assign a real number $\mu_A(x)$ in the interval [0, 1], to each element $x \in A$, where the value of $\mu_A(x)$ at x shows the grade of membership of x in A.

2.2 Intuitionistic Fuzzy set: (Faizulkhakov, 1986) A Intuitionistic fuzzy set A on a universe of discourse X is of the form $A = \{(X, \mu_A(x), \nu_A(x)); x \in X\}$, Where $\mu_A(x) \in [0,1]$ is called the "degree of membership of x in A", $\nu_A(x) \in [0,1]$ is called the "degree of non-membership of x in A", and $\mu_A(x)$ and $\nu_A(x)$ satisfy the condition that $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.The amount $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called hesitancy of x which is reflection of lack of commitment or uncertainty associated with the membership or non-membership or both in A.

Definition: 2.3 (De *et.al*. 2001) Let $A \in IFS(X)$ and $B \in IFS(X)$ then:

(a) $A \subseteq B$ iff $\forall x \in X$, $\mu_A(x) \leq \mu_B(x)$, $\nu_A(x) \geq \nu_B(x)$

(b) $A = B$ iff $\forall x \in X$, $\mu_A(x) = \mu_B(x)$, $\nu_A(x) = \nu_B(x)$

(c) $A^C = \{(x, \nu_A(x), \mu_A(x)) : x \in X\}$,

Where $A^C$is the complement of A.

(d) $\bigcap A_i = \{ (x, \wedge \mu_{A_i}(x), \vee \nu_{A_i}(x)) : x \in X \}$

(e) $\bigcup A_i = \{ (x, \vee \mu_{A_i}(x), \wedge \nu_{A_i}(x)) : x \in X \}$

Definition: 2.4 (Song *et al.*, 2014) Let d denote a mapping $d: IFS \times IFS \rightarrow [0,1]$, if $d(A, B)$ satisfies the following properties, $d(A, B)$ is called the distance between

$A \in IFSs(X)$ and $B \in IFS(X)$

(a) $0 \leq d \leq 1$

(b) $d(A, B) = 0$,if and only if $A = B$

(c) $d(A, B) = D(B, A)$

(d) $d(A, C) \leq d(A, B) + d(B, C)$

2.5 Pythagorean Fuzzy Sets: (Zeng *et al.*, 2018) Let X be a universe of discourse, a PFS in X is given by $P = \{\langle x, \mu_p(x), \nu_p(x)\rangle; x \in X\}$, where $\mu_P: X \to [0,1]$ denotes the degree of membership and $\nu_P: X \to [0,1]$ denotes the degree of non-membership of the element $x \in X$ to the set P respectively with the condition that $0 \le (\mu_P(x))^2 + (\nu_P(x))^2 \le 1$. The degree of indeterminacy $\pi_P = \sqrt{1 - (\mu_P(x))^2 + (\nu_P(x))^2}$. For the convenience, Zhang and Xu, (2018)called $(\mu_P(x), \nu_P(x))$ a PFN denoted by $p = (\mu_P, \nu_P)$. Yager and Abbasov, (2013) represented a PFN as $= (r_p, d_p)$, where r_p is called strength of p and d_p is called the direction of the strength r_p. Basically, r_p and d_p are associated with μ_P and ν_P, indicating the support for membership of x in P and support against for membership of x respectively. Larger the r_p, stronger the commitment, less the uncertainty. d_p is essentially indicating on a scale of 0 to 1 how fully the strength r_p is pointing towards membership. If $d_p = 1$, the direction of r_p is completely towards membership, whereas $d_p = 0$ indicates the direction of strength is completely towards non-membership. Intermediate values of d_p indicate partial support to membership and non-membership. Zhang and Xu, (2018) established the relation between $p = (\mu_P, \nu_P)$ and $p = (r_p, d_p)$ as $\mu_P = r_p \cos(\theta_p), \nu_P = r_p \sin(\theta_p)$,where $d_p = 1 - {}^{2\theta_p}/_{\pi}$.

2.6 Linguistic Fuzzy sets: (Garg,2018) Let $S = \{s_h: h = 0,1,2, \dots, t\}$ be finite linguistic term set with odd cardinality, where s_i represents a possible linguistic term for a linguistic variable. For a set $S = \{s_0, s_1, s_2, s_3, s_4, s_{5,} s_6\} = \{none, very\ low, medium, high, very\ high, perfect\}$, which satisfies the following properties:

(i) The set is ordered :$s_i > s_j \Leftrightarrow i > j$

(ii) negation operator: Neg $(s_i) = s_j$ where $j = t - i$;

(iii) max operator:$\max(s_i, s_j) = s_i \Leftrightarrow i \ge j$

(iv) min operator: $\min(s_i, s_j) = s_j \Leftrightarrow i \le j$

Further, Qin ,(2017) extended this discrete linguistic term S to a continuous linguistic term $\tilde{S} = \{s_\alpha: s_0 \le s_\alpha \le s_t: \alpha \in [0, t]\}$ whose all elements satisfy the above defined characteristic. If $s_\alpha \in \tilde{S}$, then we call s_α the original term; otherwise, it is called virtual term.

2.7 Linguistic Pythagorean Fuzzy Set (LPFS): (Garg, 2018) Let X be a universal set and $\tilde{S} = \{s_\alpha: s_0 \le s_\alpha \le s_t: \alpha \in [0, t]\}$ be a continuous linguistic set. A LPFS is defined in the finite universe of discourse X mathematically with the form $A = \{x, s_\theta(x), s_\sigma(x): x \in X\}$, where $s_\theta(x), s_\sigma(x) \in \tilde{S}$ stand for linguistic membership degree and linguistic non-membership degree of the element x to A. The pair $(s_\theta(x), s_\sigma(x))$ is denoted as $A = (s_\theta, s_\sigma)$ and called a linguistic Pythagorean fuzzy value (LPFV) or Linguistic Pythagorean fuzzy number (LPFN).

For $x \in X$ the condition $\theta^2 + \sigma^2 \leq t^2$ is always satisfied. $\pi(x)$ is called indeterminacy degree of x to $A: \pi(x) = S_{\sqrt{t^2-\theta^2-\sigma^2}}$

3. Existing distance measures:

Let $p_1 = (\mu_{p1}, \nu_{p1})$ and $p_2 = (\mu_{p2}, \nu_{p2})$ be two PFNs, then the distance between p_1 and p_2 is defined as follows:

Zhang and Xu's (2014) distance measure:

1. $$D(p_1,p_2) = \frac{1}{2}\left(\left|(\mu_{p_1})^2 - (\mu_{p_2})^2\right| + \left|(\nu_{p_1})^2 - (\nu_{p_2})^2\right| + \left|(\pi_{p_1})^2 - (\pi_{p_2})^2\right|\right)$$

Zeng *et al*., (2018) proposed the distance measures incorporating the five parameters μ_p ν_p, π_p, r_p and d_p:

2. The normalised Hamming distance between p_1 and p_2 is defined as follows: Zeng *et al*., (2018)

$$D_H(p_1,p_2) = \frac{1}{5}\left(\left|\mu_{p_1} - \mu_{p_2}\right| + \left|\nu_{p_1} - \nu_{p_2}\right| + \left|\pi_{p_1} - \pi_{p_2}\right| + \left|r_{p_1} - r_{p_2}\right| + \left|d_{p_1} - d_{p_2}\right|\right)$$

3. The normalised Euclidean distance between p_1 and p_2 is defined as follows: Zeng *et al*., (2018)

$$D_E(p_1,p_2) = \left[\frac{1}{5}\left(\mu_{p_1} - \mu_{p_2}\right)^2 + \left(\nu_{p_1} - \nu_{p_2}\right)^2 + \left(\pi_{p_1} - \pi_{p_2}\right)^2 + \left(r_{p_1} - r_{p_2}\right)^2 + \left(d_{p_1} - d_{p_2}\right)^2\right]^{1/2}$$

4. The normalised generalised distance between p_1 and p_2 is defined as follows: Zeng *et al*., (2018)

$$D_G(p_1,p_2) = \left[\frac{1}{5}\left(\left|\mu_{p_1} - \mu_{p_2}\right|^\lambda + \left|\nu_{p_1} - \nu_{p_2}\right|^\lambda + \left|\pi_{p_1} - \pi_{p_2}\right|^\lambda + \left|r_{p_1} - r_{p_2}\right|^\lambda + \left|d_{p_1} - d_{p_2}\right|^\lambda\right)\right]^{\frac{1}{\lambda}}$$

Where $\lambda \geq 1$.

5. Let the universe $X = \{x_1, x_2, \dots, x_n\}$ and p_1 and p_2 be two PFSs in X. For convenience, denote

$$\Delta_\mu(x_i) = \mu_{p_1}(x_i) - \mu_{p_2}(x_i), \Delta_\nu(x_i) = \mu_{p_1}(x_i) - \mu_{p_2}(x_i),$$

$$\Delta_\pi(x_i) = \mu_{p_1}(x_i) - \mu_{p_2}(x_i), \Delta_r(x_i) = \mu_{p_1}(x_i) - \mu_{p_2}(x_i),$$

$$\Delta_d(x_i) = \mu_{p_1}(x_i) - \mu_{p_2}(x_i)$$

Then the normalised Hamming distance between p_1 and p_2 is defined as follows: Zeng *et al*., (2018)

$$D_H(p_1, p_2) = \frac{1}{5n}\sum_{i=1}^{n}\left(\left|\Delta_\mu(x_i)\right| + \left|\Delta_\nu(x_i)\right| + \left|\Delta_\pi(x_i)\right| + \left|\Delta_r(x_i)\right| + \left|\Delta_d(x_i)\right|\right)$$

6. The normalised Euclidean distance between p_1 and p_2 is defined as follows: Zeng *et al*., (2018)

$$D_E(p_1, p_2) = \left[\frac{1}{5n}\sum_{i=1}^{n}\left(\left(\Delta_\mu(x_i)\right)^2 + \left(\Delta_\nu(x_i)\right)^2 + \left(\Delta_\pi(x_i)\right)^2 + \left(\Delta_r(x_i)\right)^2 + \left(\Delta_d(x_i)\right)^2\right)\right]^{1/2}$$

7. The normalised generalised distance between p_1 and p_2 is defined as follows: Zeng *et al*., (2018)

$$D_G(p_1, p_2) = \left[\frac{1}{5n}\sum_{i=1}^{n}\left(\left|\Delta_\mu(x_i)\right|^\lambda + \left|\Delta_\nu(x_i)\right|^\lambda + \left|\Delta_\pi(x_i)\right|^\lambda + \left|\Delta_r(x_i)\right|^\lambda + \left|\Delta_d(x_i)\right|^\lambda\right)\right]^{\frac{1}{\lambda}}$$

Where $\lambda \geq 1$.

8. The weight w_i of each element $x_i \in X$ is taken into account and then the weighted distance measures D_{WH}, D_{WE} and D_{WG} are defined as follows:

The weighted normalised Hamming distance measure between p_1 and p_2 is defined as follows: Zeng *et al*., (2018)

$$D_{WH}(p_1, p_2) = \frac{1}{5n}\sum_{i=1}^{n}\left(\left|\Delta_\mu(x_i)\right| + \left|\Delta_\nu(x_i)\right| + \left|\Delta_\pi(x_i)\right| + \left|\Delta_r(x_i)\right| + \left|\Delta_d(x_i)\right|\right)$$

9. The weighted normalised Euclidean distance measure between p_1 and p_2 is defined as follows: Zeng *et al*., (2018)

$$D_{WE}(p_1, p_2) = \left[\frac{1}{5n}\sum_{i=1}^{n}w_i\left(\left(\Delta_\mu(x_i)\right)^2 + \left(\Delta_\nu(x_i)\right)^2 + \left(\Delta_\pi(x_i)\right)^2 + \left(\Delta_r(x_i)\right)^2 + \left(\Delta_d(x_i)\right)^2\right)\right]^{1/2}$$

10. The weighted normalised generalised distance measure between p_1 and p_2 is defined as follows: Zeng *et al*., (2018)

$$D_{WG}(p_1, p_2) = \left[\frac{1}{5n}\sum_{i=1}^{n}w_i\left(\left|\Delta_\mu(x_i)\right|^\lambda + \left|\Delta_\nu(x_i)\right|^\lambda + \left|\Delta_\pi(x_i)\right|^\lambda + \left|\Delta_r(x_i)\right|^\lambda + \left|\Delta_d(x_i)\right|^\lambda\right)\right]^{\frac{1}{\lambda}}$$

11. Hong *et.al's* (2017) distance measure is defined as follows:

$$d^p(p_1,p_2)=\frac{1}{2}\left\{\left|\left(\mu_{p_1}\right)^2-\left(\mu_{p_2}\right)^2\right|^p+\left|\left(\nu_{p_1}\right)^2-\left(\nu_{p_2}\right)^2\right|^p+\left|\left(\nu_{p_1}\right)^2-\left(\nu_{p_2}\right)^2\right|^p+\left|\left(\pi_{p_1}\right)^2-\left(\pi_{p_2}\right)^2\right|^p\right\}$$

4. A novel distance measure on linguistic Pythagorean fuzzy sets:

Let $X=\{x_1,x_2,\dots,x_n\}$ be the universal set and $S=\{s_\alpha|s_0\le s_\alpha\le s_t:\alpha\in[0,t]\}$ be the linguistic term sets.

Let$A=\left\{\left(x_i,s_{\theta_{1i}}(x_i),s_{\sigma i}(x_i)\right):x_i\in X\right\}$ and $B=\left\{\left(x_i,s_{\theta_{2i}}(x_i),s_{\sigma_{2i}}(x_i)\right):x_i\in X\right\}$ be two LPFSs. Then, the new distance measure can be defined as:

$d(A,B)=$

$$\frac{1}{n}\sum_{i=1}^{n}\left\{\left|\frac{\sqrt{t^2+{\theta_{1i}}^2-{\sigma_{1i}}^2/2}-\sqrt{t^2+{\theta_{2i}}^2-{\sigma_{2i}}^2/2}}{t^2}\right|^\lambda+\left|\frac{\sqrt{{\theta_{1i}}^2+{\sigma_{1i}}^2}-\sqrt{{\theta_{2i}}^2+{\sigma_{2i}}^2}}{t^2}\right|^\lambda\right.$$

$$\left.-\left|\frac{\sqrt{t^2+{\theta_{1i}}^2-{\sigma_{1i}}^2/2}-\sqrt{t^2+{\theta_{2i}}^2-{\sigma_{2i}}^2/2}}{t^2}\right|^\lambda\left|\frac{\sqrt{{\theta_{1i}}^2+{\sigma_{1i}}^2}-\sqrt{{\theta_{2i}}^2+{\sigma_{2i}}^2}}{t^2}\right|^\lambda\right\}^{\frac{1}{\lambda}}$$

The proposed distance measure satisfies all the necessary four conditions and the proof is given below:

(a) $\mathbf{0\le d(A,B)\le 1}$

Proof (a): For a LPFS $A=\left\{\left(x_i,s_{\theta_{1i}}(x_i),s_{\sigma i}(x_i)\right):x_i\in X\right\}$ we allready know that

$0\le(t^2+{\theta_{1i}}^2-{\sigma_{1i}}^2)/2\le t^2$ and $0\le{\theta_{1i}}^2+{\sigma_{1i}}^2\le t$. Therefore ,

$0\le\frac{\sqrt{t^2+{\theta_{1i}}^2-{\sigma_{1i}}^2}}{t^2}\le 1$ and $0\le\frac{\sqrt{{\theta_{1i}}^2+{\sigma_{1i}}^2}}{t^2}\le 1$.

In a similar way we can show that for a LPFS $B=\left\{\left(x_i,s_{\theta_{2i}}(x_i),s_{\sigma_{2i}}(x_i)\right):x_i\in X\right\}$

Thus, $0\le\frac{\sqrt{t^2+{\theta_{1i}}^2-{\sigma_{1i}}^2}}{t^2}\le 1$ and $0\le\frac{\sqrt{{\theta_{1i}}^2+{\sigma_{1i}}^2}}{t^2}\le 1$

So, we can write $0\le\left|\frac{\sqrt{t^2+{\theta_{1i}}^2-{\sigma_{1i}}^2/2}}{t^2}-\frac{\sqrt{t^2+{\theta_{2i}}^2-{\sigma_{2i}}^2/2}}{t^2}\right|\le 1$ and $0\le\left|\frac{\sqrt{{\theta_{1i}}^2+{\sigma_{1i}}^2}}{t^2}-\frac{\sqrt{{\theta_{1i}}^2+{\sigma_{1i}}^2}}{t^2}\right|\le 1$. Therfore, $0\le\left|\frac{\sqrt{t^2+{\theta_{1i}}^2-{\sigma_{1i}}^2/2}-\sqrt{t^2+{\theta_{2i}}^2-{\sigma_{2i}}^2/2}}{t^2}\right|^\lambda\le 1$

and $0 \leq \left|\frac{\sqrt{{\theta_{1i}}^2+{\sigma_{1i}}^2}-\sqrt{{\theta_{2i}}^2+{\sigma_{2i}}^2}}{t^2}\right|^{\lambda} \leq 1$

$$\Rightarrow 0 \leq \left\{\left|\frac{\sqrt{t^2+{\theta_{1i}}^2-{\sigma_{1i}}^2/2}}{t^2}-\frac{\sqrt{t^2+{\theta_{2i}}^2-{\sigma_{2i}}^2/2}}{t^2}\right|^{\lambda}+\left|\frac{\sqrt{{\theta_{1i}}^2+{\sigma_{1i}}^2}}{t^2}-\frac{\sqrt{{\theta_{1i}}^2+{\sigma_{1i}}^2}}{t^2}\right|^{\lambda}\right.$$

$$\left.\left|\frac{\sqrt{t^2+{\theta_{1i}}^2-{\sigma_{1i}}^2/2}-\sqrt{t^2+{\theta_{2i}}^2-{\sigma_{2i}}^2/2}}{t^2}\right|^{\lambda}\left|\frac{\sqrt{{\theta_{1i}}^2+{\sigma_{1i}}^2}-\sqrt{{\theta_{2i}}^2+{\sigma_{2i}}^2}}{t^2}\right|^{\lambda}\right\}^{\frac{1}{\lambda}} \leq 1$$

$$\Rightarrow 0 \leq \frac{1}{n}\sum_{i=1}^{n}\left\{\left|\frac{\sqrt{t^2+{\theta_{1i}}^2-{\sigma_{1i}}^2/2}-\sqrt{t^2+{\theta_{2i}}^2-{\sigma_{2i}}^2/2}}{t^2}\right|^{\lambda}\right.$$

$$+\left|\frac{\sqrt{{\theta_{1i}}^2+{\sigma_{1i}}^2}-\sqrt{{\theta_{2i}}^2+{\sigma_{2i}}^2}}{t^2}\right|^{\lambda}$$

$$\left.-\left|\frac{\sqrt{t^2+{\theta_{1i}}^2-{\sigma_{1i}}^2/2}-\sqrt{t^2+{\theta_{2i}}^2-{\sigma_{2i}}^2/2}}{t^2}\right|^{\lambda}\left|\frac{\sqrt{{\theta_{1i}}^2+{\sigma_{1i}}^2}-\sqrt{{\theta_{2i}}^2+{\sigma_{2i}}^2}}{t^2}\right|^{\lambda}\right\}^{\frac{1}{\lambda}} \leq 1$$

$$\Rightarrow 0 \leq d(A,B) \leq 1$$

(b) $d(A,B)=0$ if and only if $A=B$

Proof: (b) $d(A,B)=0$

$$\Leftrightarrow \frac{1}{n}\sum_{i=1}^{n}\left\{\left|\frac{\sqrt{t^2+{\theta_{1i}}^2-{\sigma_{1i}}^2/2}-\sqrt{t^2+{\theta_{2i}}^2-{\sigma_{2i}}^2/2}}{t^2}\right|^{\lambda}\right.$$

$$+\left|\frac{\sqrt{{\theta_{1i}}^2+{\sigma_{1i}}^2}-\sqrt{{\theta_{2i}}^2+{\sigma_{2i}}^2}}{t^2}\right|^{\lambda}$$

$$\left.-\left|\frac{\sqrt{t^2+{\theta_{1i}}^2-{\sigma_{1i}}^2/2}-\sqrt{t^2+{\theta_{2i}}^2-{\sigma_{2i}}^2/2}}{t^2}\right|^{\lambda}\left|\frac{\sqrt{{\theta_{1i}}^2+{\sigma_{1i}}^2}-\sqrt{{\theta_{2i}}^2+{\sigma_{2i}}^2}}{t^2}\right|^{\lambda}\right\}^{\frac{1}{\lambda}}=0$$

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$$\Rightarrow \left\{ \left| \frac{\sqrt{t^2 + {\theta_{1i}}^2 - {\sigma_{1i}}^2/2}}{t^2} - \frac{\sqrt{t^2 + {\theta_{2i}}^2 - {\sigma_{2i}}^2/2}}{t^2} \right|^\lambda + \left| \frac{\sqrt{{\theta_{1i}}^2 + {\sigma_{1i}}^2}}{t^2} - \frac{\sqrt{{\theta_{1i}}^2 + {\sigma_{1i}}^2}}{t^2} \right|^\lambda \right.$$

$$\left. \left| \frac{\sqrt{t^2 + {\theta_{1i}}^2 - {\sigma_{1i}}^2/2} - \sqrt{t^2 + {\theta_{2i}}^2 - {\sigma_{2i}}^2/2}}{t^2} \right|^\lambda \left| \frac{\sqrt{{\theta_{1i}}^2 + {\sigma_{1i}}^2} - \sqrt{{\theta_{2i}}^2 + {\sigma_{2i}}^2}}{t^2} \right|^\lambda \right\}^{\frac{1}{\lambda}} = 0$$

$$\Leftrightarrow \left| \frac{\sqrt{t^2 + {\theta_{1i}}^2 - {\sigma_{1i}}^2/2}}{t^2} - \frac{\sqrt{t^2 + {\theta_{2i}}^2 - {\sigma_{2i}}^2/2}}{t^2} \right| = 0, \left| \frac{\sqrt{{\theta_{1i}}^2 + {\sigma_{1i}}^2}}{t^2} - \frac{\sqrt{{\theta_{1i}}^2 + {\sigma_{1i}}^2}}{t^2} \right| = 0$$

$$\Leftrightarrow \sqrt{t^2 + {\theta_{1i}}^2 - {\sigma_{1i}}^2/2} = \sqrt{t^2 + {\theta_{2i}}^2 - {\sigma_{2i}}^2/2} \text{ and } \sqrt{{\theta_{1i}}^2 + {\sigma_{1i}}^2} = \sqrt{{\theta_{2i}}^2 + {\sigma_{2i}}^2}$$

$\Leftrightarrow S(A) = S(B)$ and $H(A) = H(B)$ by definition of score function and accuracy function

$\Leftrightarrow A = B$

Hence, $d(A, B) = 0$ if and only if $A = B$.

(c) $d(A, B) = d(B, A)$

Proof (c) :

$d(A, B)$

$$= \frac{1}{n} \sum_{i=1}^{n} \left\{ \left| \frac{\sqrt{t^2 + {\theta_{1i}}^2 - {\sigma_{1i}}^2/2} - \sqrt{t^2 + {\theta_{2i}}^2 - {\sigma_{2i}}^2/2}}{t^2} \right|^\lambda + \left| \frac{\sqrt{{\theta_{1i}}^2 + {\sigma_{1i}}^2} - \sqrt{{\theta_{2i}}^2 + {\sigma_{2i}}^2}}{t^2} \right|^\lambda \right.$$

$$\left. - \left| \frac{\sqrt{t^2 + {\theta_{1i}}^2 - {\sigma_{1i}}^2/2} - \sqrt{t^2 + {\theta_{2i}}^2 - {\sigma_{2i}}^2/2}}{t^2} \right|^\lambda \left| \frac{\sqrt{{\theta_{1i}}^2 + {\sigma_{1i}}^2} - \sqrt{{\theta_{2i}}^2 + {\sigma_{2i}}^2}}{t^2} \right|^\lambda \right\}^\lambda$$

$$\frac{1}{n} \sum_{i=1}^{n} \left\{ \left| \frac{\sqrt{t^2 + {\theta_{2i}}^2 - {\sigma_{2i}}^2/2} - \sqrt{t^2 + {\theta_{1i}}^2 - {\sigma_{1i}}^2/2}}{t^2} \right|^\lambda + \left| \frac{\sqrt{{\theta_{2i}}^2 + {\sigma_{2i}}^2} - \sqrt{{\theta_{1i}}^2 + {\sigma_{1i}}^2}}{t^2} \right|^\lambda \right.$$

$$-\left|\frac{\sqrt{t^2+{\theta_{2i}}^2-{\sigma_{2i}}^2/2}-\sqrt{t^2+{\theta_{1i}}^2-{\sigma_{1i}}^2/2}}{t^2}\right|^{\lambda}\left|\frac{\sqrt{{\theta_{2i}}^2+{\sigma_{2i}}^2}-\sqrt{{\theta_{1i}}^2+{\sigma_{1i}}^2}}{t^2}\right|^{\lambda}\Bigg\}^{\lambda}$$

$$= d(A,B)$$

(d) Let us consider A,B and D be three intuitionistic fuzzy sets, then the distance measure satisfies the triangular inequality. i.e.,$d(A,C) \le d(A,B) + d(B,C)$.

Proof(d): Consider, $A = \{(x_i, s_{\theta_{1i}}(x_i), s_{\sigma i}(x_i)): x_i \in X\}$,

$B = \{(x_i, s_{\theta_{2i}}(x_i), s_{\sigma_{2i}}(x_i)): x_i \in X\}$, and $C = \{(x_i, s_{\theta_{3i}}(x_i), s_{\sigma 3i}(x_i)): x_i \in X\}$ be Linguistic Pythagorean fuzzy sets.

Now, from the basic inequality of real numbers we can have:

$$\left|\frac{\sqrt{t^2+{\theta_{1i}}^2-{\sigma_{1i}}^2/2}}{t^2}-\frac{\sqrt{t^2+{\theta_{3i}}^2-{\sigma_{3i}}^2/2}}{t^2}\right| \le$$

$$\left|\frac{\sqrt{t^2+{\theta_{1i}}^2-{\sigma_{1i}}^2/2}}{t^2}-\frac{\sqrt{t^2+{\theta_{2i}}^2-{\sigma_{2i}}^2/2}}{t^2}\right| + \left|\frac{\sqrt{t^2+{\theta_{2i}}^2-{\sigma_{2i}}^2/2}}{t^2}-\frac{\sqrt{t^2+{\theta_{3i}}^2-{\sigma_{3i}}^2/2}}{t^2}\right|$$

$$\Rightarrow \left|\frac{\sqrt{t^2+{\theta_{1i}}^2-{\sigma_{1i}}^2/2}}{t^2}-\frac{\sqrt{t^2+{\theta_{3i}}^2-{\sigma_{3i}}^2/2}}{t^2}\right|^{\lambda} \le$$

$$\left|\frac{\sqrt{t^2+{\theta_{1i}}^2-{\sigma_{1i}}^2/2}}{t^2}-\frac{\sqrt{t^2+{\theta_{2i}}^2-{\sigma_{2i}}^2/2}}{t^2}\right|^{\lambda} + \left|\frac{\sqrt{t^2+{\theta_{2i}}^2-{\sigma_{2i}}^2/2}}{t^2}-\frac{\sqrt{t^2+{\theta_{3i}}^2-{\sigma_{3i}}^2/2}}{t^2}\right|^{\lambda}$$

Also

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$$\left|\frac{\sqrt{{\theta_{1i}}^2+{\sigma_{1i}}^2}}{t^2}-\frac{\sqrt{{\theta_{3i}}^2+{\sigma_{3i}}^2}}{t^2}\right| \le \left|\frac{\sqrt{{\theta_{1i}}^2+{\sigma_{1i}}^2}}{t^2}-\frac{\sqrt{{\theta_{2i}}^2+{\sigma_{2i}}^2}}{t^2}\right|+\left|\frac{\sqrt{{\theta_{2i}}^2+{\sigma_{2i}}^2}}{t^2}-\frac{\sqrt{{\theta_{3i}}^2+{\sigma_{3i}}^2}}{t^2}\right|$$

$$\Rightarrow \left|\frac{\sqrt{{\theta_{1i}}^2+{\sigma_{1i}}^2}}{t^2}-\frac{\sqrt{{\theta_{3i}}^2+{\sigma_{3i}}^2}}{t^2}\right|^\lambda \le \left|\frac{\sqrt{{\theta_{1i}}^2+{\sigma_{1i}}^2}}{t^2}-\frac{\sqrt{{\theta_{2i}}^2+{\sigma_{2i}}^2}}{t^2}\right|^\lambda+\left|\frac{\sqrt{{\theta_{2i}}^2+{\sigma_{2i}}^2}}{t^2}-\frac{\sqrt{{\theta_{3i}}^2+{\sigma_{3i}}^2}}{t^2}\right|^\lambda$$

5. Methodology

Consider $\{D_1, D_2, \dots, D_m\}$be the set of m number of diseases and $\{P_1, P_2, \dots, P_n\}$ be the set of n number of patients. Depending upon the symptoms we have to decide what disease may be suffered by the patients.

Let $\left\{S_1\left(s^{D}_{\theta^{D_i}_{11}}, \nu^{D}_{\sigma^{D_i}_{11}}\right), S_2\left(s^{D}_{\theta^{D_i}_{12}}, \nu^{D}_{\sigma^{D_i}_{12}}\right), \dots, S_l(s^{D}_{\theta^{D_i}_{1l}}, \nu^{D}_{\sigma^{D_i}_{1l}})\right\}$ be the symptoms the disease D_i and $\left\{S_1\left(s^{P_j}_{\theta^{P_i}_{11}}, \nu^{P_j}_{\sigma^{P_i}_{11}}\right), S_2\left(s^{P_j}_{\theta^{P_i}_{12}}, \nu^{P_j}_{\sigma^{P_i}_{12}}\right), \dots, S_l(s^{P_j}_{\theta^{P_i}_{1l}}, \nu^{P_j}_{\sigma^{P_i}_{1l}})\right\}$ be the symptoms of the patient P_j expressed in LPFSs. Now, the distance between the symptoms the disease D_i and symptoms of patient P_j can be evaluated using the proposed distance measure on LPFSs:

$$d(D_i, P_j) = \frac{1}{l}\sum_{i=1}^{n}\left\{\left|\frac{\sqrt{(t^2-(\theta^{D_i}_{11})^2-(\sigma^{D_i}_{11})^2)/2}-\sqrt{(t^2-(\theta^{P_j}_{11})^2-(\sigma^{P_j}_{11})^2)/2}}{t^2}\right|^\lambda + \left|\frac{\sqrt{(\theta^{D_i}_{11})^2+(\sigma^{D_i}_{11})^2}-\sqrt{(\theta^{P_j}_{11})^2-(\sigma^{P_j}_{11})^2}}{t^2}\right|^\lambda - \left|\frac{\sqrt{(t^2-(\theta^{D_i}_{11})^2-(\sigma^{D_i}_{11})^2)/2}-\sqrt{(t^2-(\theta^{P_j}_{11})^2-(\sigma^{P_j}_{11})^2)/2}}{t^2}\right|^\lambda \left|\frac{\sqrt{(\theta^{D_i}_{11})^2+(\sigma^{D_i}_{11})^2}-\sqrt{(\theta^{P_j}_{11})^2-(\sigma^{P_j}_{11})^2}}{t^2}\right|^\lambda\right\}^{\frac{1}{\lambda}}$$

Where $i = 1,2, \dots, m$, $j = 1,2, \dots, n$ and $\lambda > 0$.

The distances between each pair of disease and patients can be represented with the help of the following matrix:

$$\begin{array}{c} \\ D_1 \\ D_2 \\ \vdots \\ D_m \end{array} \begin{array}{c} \begin{array}{cccc} P_1 & P_2 & & P_n \end{array} \\ \begin{bmatrix} d(D_1,P_1) & d(D_1,P_2) & \cdots & d(D_1,P_n) \\ d(D_2,P_1) & d(D_2,P_2) & \cdots & d(D_2,P_n) \\ \vdots & \vdots & \ddots & \vdots \\ d(D_m,P_1) & d(D_m,P_2) & \cdots & d(D_m,P_n) \end{bmatrix} \end{array}$$

Now, distance measures can be used to select the disease suffered by patient. Using the fact that, less distance between two LPFSs implies more similarity between them, it can be said that for the patient P_j the disease most possibly suffered by him or her is the disease corresponding to ${}^{min}{}_{i}d(D_i, P_j)$.

5.1 A case study in medical diagnosis:

In this case study, the proposed distance measure has been applied at a decision-making problem of medical diagnosis. The data are taken from the study carried out by De et al., in 2001 and it is transformed to LPFSs. In this case study, we consider a set of symptoms S, a set of diagnosis D and a set of patients P.

Let P = {Abhi, Bikram, Champak, Dibya}, S = {Temperature, Headache, Stomach pain, Cough, Chest pain} and

$D = \{Viral\ fever, Malaria, Typhoid, Stomach\ pain, Chest\ problem\}$.Our objective is to carry out the right decision of diagnosis for each patient, from the set of symptoms, for each disease.

The relation between symptom and disease and relation between patient and symptom is given in Table-1 and Table-2 respectively.

Table 1. Symptoms-diseases intuitionistic fuzzy relation

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	$Viral\ fever$ (vf)	$Malaria$ (Ml)	$Typhoid$ (Ty)	$Stomach\ pain$ (Sp)	$Chest\ problem$ (Cpr)
$Temperatute$ (Tm)	$(s_{0.4}, s_{0.0})$	$(s_{0.7}, s_{0.0})$	$(s_{0.3}, s_{0.3})$	$(s_{0.1}, s_{0.7})$	$(s_{0.1}, s_{0.8})$
$Headach$ (He)	$(s_{0.3}, s_{0.5})$	$(s_{0.2}, s_{0.6})$	$(s_{0.6}, s_{0.1})$	$(s_{0.2}, s_{0.4})$	$(s_{0.0}, s_{0.8})$
$Stomach\ pain$ (Sp)	$(s_{0.1}, s_{0.7})$	$(s_{0.0}, s_{0.9})$	$(s_{0.2}, s_{0.7})$	$(s_{0.8}, s_{0.0})$	$(s_{0.2}, s_{0.8})$
$Cough\ (co)$	$(s_{0.4}, s_{0.3})$	$(s_{0.7}, s_{0.0})$	$(s_{0.2}, s_{0.6})$	$(s_{0.2}, s_{0.7})$	$(s_{0.2}, s_{0.8})$
$Chest\ pain$ (Cp)	$(s_{0.1}, s_{0.7})$	$(s_{0.1}, s_{0.8})$	$(s_{0.1}, s_{0.9})$	$(s_{0.2}, s_{0.7})$	$(s_{0.8}, s_{0.1})$

Table 2: Patients- symptoms intuitionistic fuzzy relation

	$Temperatute$ (Tm)	$Headach$ (He)	$Stomach\ pain$ (Sp)	$Cough$ (co)	$Chest\ pain$ (Cp)
Abhi	$(s_{0.8}, s_{0.1})$	$(s_{0.6}, s_{0.1})$	$(s_{0.2}, s_{0.8})$	$(s_{0.6}, s_{0.1})$	$(s_{0.1}, s_{0.6})$
Bikram	$(s_{0.0}, s_{0.8})$	$(s_{0.4}, s_{0.4})$	$(s_{0.6}, s_{0.1})$	$(s_{0.1}, s_{0.7})$	$(s_{0.1}, s_{0.8})$
Champ ak	$(s_{0.8}, s_{0.1})$	$(s_{0.8}, s_{0.1})$	$(s_{0.0}, s_{0.6})$	$(s_{0.2}, s_{0.7})$	$(s_{0.0}, s_{0.5})$
Dibya	$(s_{0.6}, s_{0.1})$	$(s_{0.5}, s_{0.4})$	$(s_{0.3}, s_{0.4})$	$(s_{0.7}, s_{0.2})$	$(s_{0.3}, s_{0.4})$

Now, we can represent the fevers as intuitionistic fuzzy sets for symptoms.

$$Vf = \{\langle Tm, (s_{0.16}, s_{0.0})\rangle, \langle He, (s_{0.09}, s_{0.25})\rangle, \langle St, (s_{0.01}, s_{0.49})\rangle, \langle Co, (s_{0.16}, s_{0.09})\rangle, \langle Cp, (s_{0.01}, s_{0.49})\rangle\}$$

$$Ml = \{\langle Tm, (s_{0.49}, s_{0.0}),\rangle \langle He, (s_{0.04}, s_{0.36})\rangle, \langle St, (s_{0.0}, s_{0.81})\rangle, \langle Co, (s_{0.49}, s_{0.0})\rangle, \langle Cp, (s_{0.01}, s_{0.64})\rangle\}$$

$$Ty = \{\langle Tm, (s_{0.09}, s_{0.09}), \rangle \langle He, (s_{0.36}, s_{0.01}) \rangle, \langle St, (s_{0.04}, s_{0.49}) \rangle, \langle Co, (s_{0.04}, s_{0.36}) \rangle, \langle Cp, (s_{0.01}, s_{0.81}) \rangle\}$$

$$Sp = \{\langle Tm, (s_{0.01}, s_{0.49}), \rangle \langle He, (s_{0.04}, s_{0.16}) \rangle, \langle St, (s_{0.64}, s_{0.0}) \rangle, \langle Co, (s_{0.04}, s_{0.49}) \rangle, \langle Cp, (s_{0.04}, s_{0.49}) \rangle\}$$

$$Cpr = \{\langle Tm, (s_{0.01}, s_{0.64}), \rangle \langle He, (s_{0.0}, s_{0.64}) \rangle, \langle St, (s_{0.04}, s_{0.64}) \rangle, \langle Co, (s_{0.04}, s_{0.64}) \rangle, \langle Cp, (s_{0.64}, s_{0.01}) \rangle\}$$

Similarly, we can represent the patients as intuitionistic fuzzy sets as follows:

$$Abhi = \{\langle Tm, (s_{0.64}, s_{0.01}), \rangle \langle He, (s_{0.36}, s_{0.01}) \rangle, \langle St, (s_{0.04}, s_{0.64}) \rangle, \langle Co, (s_{0.36}, s_{0.01}) \rangle, \langle Cp, (s_{0.01}, s_{0.36}) \rangle\}$$

$$Bikram = \{\langle Tm, (s_{0.0}, s_{0.64}), \rangle \langle He, (s_{0.16}, s_{0.16}) \rangle, \langle St, (s_{0.36}, s_{0.01}) \rangle, \langle Co, (s_{0.01}, s_{0.49}) \rangle, \langle Cp, (s_{0.01}, s_{0.64}) \rangle\}$$

$$Champak = \{\langle Tm, (s_{0.64}, s_{0.01}), \rangle \langle He, (s_{0.64}, s_{0.01}) \rangle, \langle St, (s_{0.0}, s_{0.36}) \rangle, \langle Co, (s_{0.04}, s_{0.49}) \rangle, \langle Cp, (s_{0.0}, s_{0.25}) \rangle\}$$

$$Dibya = \{\langle Tm, (s_{0.36}, s_{0.01}), \rangle \langle He, (s_{0.25}, s_{0.16}) \rangle, \langle St, (s_{0.09}, s_{0.16}) \rangle, \langle Co, (s_{0.49}, s_{0.04}) \rangle, \langle Cp, (s_{0.09}, s_{0.16}) \rangle\}$$

Now, the LPFSs distance is measured between the disease and the patients in terms of their symptoms.

Table 3. Distances between the disease and patients obtained by the proposed distance measure for $\boldsymbol{\lambda = 1}$

	Abhi	Bikram	Champak	Dibya
Viral fever	0.239791	0.322253636	0.348834	0.26144471
Malaria	0.208425	0.338127838	0.36994	0.269452727
Typhoid	0.280359	0.286019862	0.332038	0.325061754
Stomach pain	0.351965	0.157491344	0.34667	0.333505355
Chest problem	0.392005	0.323627969	0.388987	0.460278328
Disease	Malaria	Stomach pain	Typhoid	Viral fever

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Table 4.Distances between the disease and patients obtained by the proposed distance measure for $\boldsymbol{\lambda = 2}$

	Abhi	Bikram	Champak	Dibya
Viral fever	0.198119	0.263875102	0.283016	0.2061986
Malaria	0.170437	0.302604507	0.310464	0.224200174
Typhoid	0.233848	0.23073514	0.273987	0.262720369
Stomach pain	0.306592	0.121738986	0.291639	0.283397096
Chest problem	0.342681	0.280036731	0.347947	0.384262245
Disease	Malaria	Stomach pain	Typhoid	Viral fever

Table 5. Distances between the disease and patients obtained by the proposed distance measure for $\boldsymbol{\lambda = 3}$

	Abhi	Bikram	Champak	Dibya
Viral fever	0.105795445	0.151855028	0.180645468	0.148385891
Malaria	0.161181171	0.29260375	0.292017891	0.207789305
Typhoid	0.219005698	0.213707318	0.256277379	0.243028957
Stomach pain	0.295897875	0.112517001	0.27558257	0.268943922
Chest problem	0.326640391	0.265420779	0.33428808	0.36024562
Disease	Malaria	Stomach pain	Typhoid	Viral fever

Table 6. Distances between the disease and patients obtained by the proposed distance measure for $\boldsymbol{\lambda = 4}$

	Abhi	Bikram	Champak	Dibya
Viral fever	0.18450264	0.235434881	0.254436	0.18386272
Malaria	0.15782022	0.288862364	0.284253	0.19956852
Typhoid	0.21248994	0.206406276	0.249158	0.23447188
Stomach pain	0.29218573	0.108960249	0.269468	0.26312077
Chest problem	0.32037446	0.25904296	0.328002	0.35033503
Disease	Malaria	Stomach pain	Typhoid	Viral fever

Table 7. Comparison of existing results with the proposed distance measure

	[25]	[26]	[27]	[28]	[29]	[30]	Proposed distance measure
Abhi	Malaria	Malaria	Viral fever	Viral fever	Malaria	Malaria	Malaria
Bikram	Stomach problem	Stomach problem	Stomach problem	Stomach problem	Stomach problem	Stomach problem	Stomach problem
Champak	Malaria	Typhoid	Typhoid	Typhoid	Typhoid	Typhoid	Typhoid
Dibya	Malaria	Viral fever	Malaria	Viral fever	Viral fever	Viral fever	Viral fever

From Table-7, comparison has been made and it is observed that the results obtained by using our proposed distance measure are similar with results obtained by (Ngan *et al.*, (2018) and Szmidt & Kacprzyk,(2000).

6. Conclusion

LPFS, a novel concept which is introduced by Garge, can be used to model many real life problems under uncertain environment. In this paper, a novel distance measure for LPFSs is introduced and the axioms of the distance measure are verified. Also, the proposed novel distance measure is used in multi criteria decision making method to execute medical diagnosis problem. In the above discussed case study, table-3,4,5,6 indicate the distances between the patients and the diseases for different values of $\lambda(\lambda = 1,2,3,4)$. The principle of minimum distance grade states that the lesser distance degree of alternative signifies a proper diagnosis. From the tables-3,4,5,6, it is observed that the distance between the patient Abhi and disease Malaria is minimum. Therefore, it can be concluded that Abhi is suffering from Malaria. Similarly, Bikram is suffering from Stomach problem, Champak is suffering from Typhoid and Dibya is suffering from viral fever. It is observed that the result which is evaluated using the novel distance coincides with the earlier results found by many researchers as shown in table-7. This shows the reliability and validity of the proposed distance measures.

As a future work, it will be beneficial to extend the proposed distance measure in a generalized form and use in the development of fuzzy decision-making problems in medical diagnosis. As an extension of the study, the distance measure proposed in this paper can be extended to the linguistic interval-valued Pythagorean fuzzy set (LIVPFS) or the linguistic interval-valued intuitionistic type-2 fuzzy set [31]. Since, the LIVPFS is proven to be more robust and trustworthy tool than LPFSs to accomplish the imprecise information while solving the decision-making problems. Therefore, it will be beneficial to extend the distance measure in LIVPFS. Also, researcher may extend the distance measure in the

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