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Comparative analysis between network systems with standby components

Ibrahim Yusuf[1]✉ • Surajo Mahmud Umar[2] • Kabiru Suleiman[3]

[1] Department of Mathematical Sciences, Bayero University, Kano, Nigeria

[2] Department of Mathematical Sciences, Bayero University, Kano

[3] Department of Mathematics, Yusuf Maitama Sule University, Kano

✉ iyusuf.mth@buk.edu.ng

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Abstract This paper analyzes the reliability of serial networking systems. This paper explores two probabilistic models. Model I is a serial network computer consisting of three A, B and C subsystems, with a C subsystem consisting of three out of six parts. Model II is a serial network system consisting of three subsystems A, B and C, with subsystem C consisting of 2-out-of-6 components. The failure and repair time of the running and standby components is believed to be exponentially distributed. The system of first-order linear differential equations is developed and resolved to obtain explicit expressions for steady-state availability and mean failure time for each model. In addition, both numerical and theoretical comparisons were conducted. The results showed that Model II was the best in terms of availability, mean time failure and cost benefit ratio.

Keywords Availability; Mean time to failure; Network; Reliability

1. Introduction

Several modern network architectures consist of components that are known to be nodes, interconnected in a network system by linking arcs. One of the most significant competitive factors on the network system market is the reliability of the system, provided that the simplest component failure can stop the system as a whole. Network networks are used in all aspects of human life, such as water supply, telecommunications, oil and gas supply, power generation and transmission, rail and road transport, etc. The increasing diffusion and reliance of such systems requires their rational design and operation in terms of risk and reliability.

Reliability and availability are the most important factors for any successful networking device. Reliability and availability of network networks can be enhanced by using higher reliability of the highly stable structural architecture of the system or subsystem.

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Improving the reliability and availability of the system/subsystem, production and related revenues will also increase. This can be done by maintaining the highest degree of reliability and availability.

Due to their importance in industries and economy, many researchers have done tremendous works in the field of reliability theory by constructing different types of mathematical models to evaluate their performances under different types of failure rates. For instance, Amari *et al.*, (2012) studied reliability characteristics of k-out-of-n warm system. Ashok *et al.*, (2020) recently analyzed the analysis of a redundant System with 'FCFS' repair policy subject to weather conditions. Chauhan and malik, (2017) focused on reliability and mean time to system failure evaluation of parallel system with Weibull failure laws. Hirata *et al.*, (2020) analyzed the reliability in priority standby redundant systems based on maximum entropy principle. GarG and Sharma, (2012) analyzed the behavior of synthesis unit in fertilizer plant. Garg *et al.*, (2014) dealt with an approach for analyzing the reliability of industrial systems using soft-computing based technique. Garg, (2014) presented RAM analysis of industrial systems using PSO and fuzzy methodology. Garg and Sharma, (2012) focused on two-phase approach for reliability and maintainability analysis of an industrial system. Garg, (2015) analyzed the reliability of industrial system using fuzzy Kolmogorov's differential equations. Lado and Singh, (2019) presented cost assessment of complex repairable system consisting two subsystems in series configuration using Gumbel Hougaard family copula. Lado *et al.*, (2018) focused on performance and cost assessment of repairable complex system. Niwas and Garg, (2018) analyzed the reliability and profit of an industrial system based on the cost-free warranty policy. Singh and Ayagi, (2018) discussed the stochastic behavior of a complex system under preemptive resume repair policy using Gumbel-Hougaard family copula. Singh *et al.*, (2020a) studied the performance of complex repairable system with two subsystems in series configuration with an imperfect switch. Singh *et al.*, (2020b) analysed the reliability of repairable network system of three computer labs connected with a server under 2- out- of- 3 G configuration. Singh and Poonia, (2019) dealt with probabilistic assessment of two unit's parallel system with correlated lifetime under inspection using regenerative point technique.

Network networks must also be planned to operate continuously for years without interruption. Usually, the reliability of network connections can be increased by adding a number of redundant paths/units. As network flow systems are prevalent in power plants as well as in manufacturing and industrial systems, several researchers have investigated their reliability characteristics and implemented a large number of models to illustrate their efficiency and performance. Yusuf, (2020) analyzed the reliability of communication network system with redundant relay station under partial and complete failure. The reliability of network flows with stochastic capacity and cost constraints was studied by Fathabadi and Khodaei, (2012). Markov models for analyzing the reliability of faults in wireless sensor networks was proposed by Vasar *et al.*, (2009). Performance evaluation of the reliability of a network with respect to the simplest system satisfying the capacity constraints was studied by Hassan, (2012). An investigation of reliability of wireless body area networks which are used for monitoring the movement and health of individuals was carried out by Ali, (2009). A study of the system reliability of a multi-commodity limited-flow network was presented by Lin, (2007). Rocco and Zio, (2005) presented cellular automata and Monte Carlo sampling method adapted to solving problems involving the reliability of advanced networks. An approach based on cellular automata for the assessment of network reliability was studied by Rocco, (2002).

This paper analyzes some of the reliability characteristics of repairable network networks with built-in redundancy. Study is used to build mathematical models to test the effectiveness of the method. The objectives of this analysis are twofold: first to determine the effect on mean time to failure (MTTF) and the steady-state availability of individual unit failures and repair rates. Second, to perform a comparative study of the implementations in order to configure the best one.

The organization of the paper shall be as follows. Definition of some reliability terms are presented in section 2. Section 3 offers a summary of the network flow system under analysis. Section 4 describes the formulations of the models of reliability. Comparison and numerical examples are shown in section 5. Finally, in Section 6, we make a few closing remarks.

2. DEFINITION

Definition 1: Reliability is defined as measure of the performance of the system under the specified conditions.

$$R(t) = e^{-\int_0^t r(t)}dt \tag{1}$$

Where r(t) is called the failure rate

Definition 2: Mean time to failure (MTTF) is a reliability metric that measures the average amount of time a non-repairable asset operates before it fails. It is the time interval in which the system experiences the first failure.

$$MTTF = \int_0^{\infty} R(t)dt \tag{2}$$

Definition 3: Availability is defined as the probability that the system is operating properly when it is required for use.

$$A = \lim_{t\to\infty} A(t) \tag{3}$$

Definition 4: Standby Redundancy: This is a type of redundancy that can be introduced in a system.

3. DESCRIPTION OF THE NETWORK SYSTEMS

The system consists of three subsystems A, B and C, in series with subsystem B consisting of linear consecutive components arranged in parallel series in different cold standby pathways, as shown in Table 1 below:

Table 1.Models with their corresponding Paths

Model	Path
I	$AB_{11}B_{12}B_{13}C$
	$AB_{21}B_{12}B_{13}C$
	$AB_{21}B_{22}B_{13}C$
	$AB_{21}B_{22}B_{23}C$
II	$AB_{11}B_{12}C$
	$AB_{11}B_{22}C$
	$AB_{21}B_{12}C$
	$AB_{21}B_{22}C$
	$AB_{21}B_{32}C$
	$AB_{31}B_{22}C$
	$AB_{31}B_{32}C$

When the primary component B_{11} in the path P_1 fails, which occurs with failure rate β_0, it is sent for repair with the service rate equal to α_0 and the standby component B_{21} in the path P_2 is switched into operation. At the failure of all components in the same path, next consecutive path will switch to operation. The system works whenever subsystem A, B and C are working. It is assumed that switching from standby to operation is perfect and instantaneous. Signals from subsystem A are received by subsystem B through the operating path and are conveyed to subsystem C for usage. System failure results from the failure of any of the subsystem A, B or C. It is also assumed that both subsystems A and C fail with failure rates β_0 and are returned from repair at service rates α_0.

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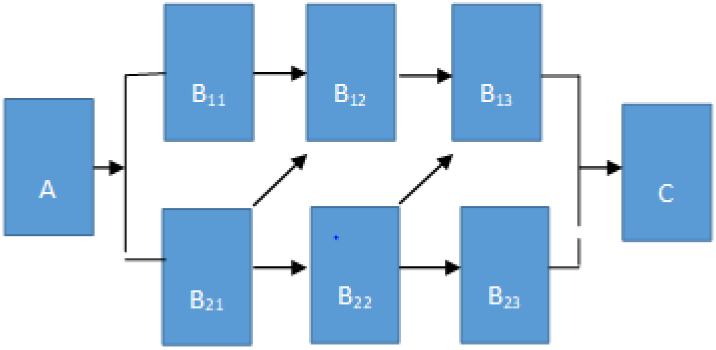

Figure 1.Reliability block diagram of Model I

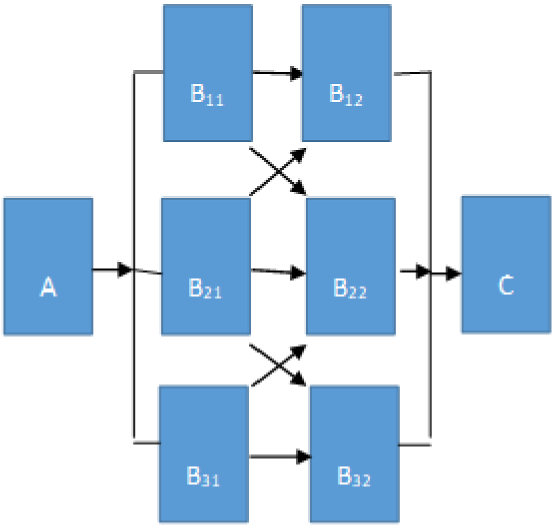

Figure 2. Reliability block diagram of Model II

4. AVAILABILITY MODELS FORMULATION

4.1 AVAILABILITY ANALYSIS OF MODEL I

In order to analyze the system availability of the network $p(t)$ be the row vector of these probabilities at time t.

The initial condition for this problem is:

$$p(0) = [p_0(0), p_1(0), p_2(0), \dots, p_{12}(0)] = [1,0,0,0,0,0,0,0,0,0,0,0,0,0]$$

We obtain the following differential equations:

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$$\frac{d}{dt}p_0(t) = -3\beta_0 p_0(t) + \alpha_0 p_1(t) + \alpha_0 p_2(t) + \alpha_0 p_3(t)$$

$$\frac{d}{dt}p_1(t) = -\alpha_0 p_1(t) + \beta_0 p_0(t)$$

$$\frac{d}{dt}p_2(t) = -\alpha_0 p_2(t) + \beta_0 p_0(t)$$

$$\frac{d}{dt}p_3(t) = -(\alpha_0 + 3\beta_0)\, p_3(t) + \beta_0 p_0(t) + \alpha_0 p_4(t) + \alpha_0 p_5(t) + \alpha_0 p_6(t)$$

$$\frac{d}{dt}p_4(t) = -\alpha_0 p_4(t) + \beta_0 p_3(t)$$

$$\frac{d}{dt}p_5(t) = -\alpha_0 p_5(t) + \beta_0 p_3(t)$$

$$\frac{d}{dt}p_6(t) = -(\alpha_0 + 3\beta_0)\, p_6(t) + \beta_0 p_3(t) + \alpha_0 p_7(t) + \alpha_0 p_8(t) + \alpha_0 p_9(t)$$

$$\frac{d}{dt}p_7(t) = -\alpha_0 p_7(t) + \beta_0 p_6(t)$$

$$\frac{d}{dt}p_8(t) = -\alpha_0 p_8(t) + \beta_0 p_6(t)$$

$$\frac{d}{dt}p_9(t) = -(\alpha_0 + 3\beta_0)\, p_9(t) + \beta_0 p_6(t) + \alpha_0 p_{10}(t) + \alpha_0 p_{11}(t) + \alpha_0 p_{12}(t)$$

$$\frac{d}{dt}p_{10}(t) = -\alpha_0 p_{10}(t) + \beta_0 p_9(t)$$

$$\frac{d}{dt}p_{11}(t) = -\alpha_0 p_{11}(t) + \beta_0 p_9(t)$$

$$\frac{d}{dt}p_{12}(t) = -\alpha_0 p_{12}(t) + \beta_0 p_9(t) \qquad (1)$$

This can be written in the matrix form as

$$\frac{d}{dt}p(t) = Q_1 p \qquad (4)$$

where

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$$Q_1=\begin{bmatrix}-3\beta_0 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \beta_0 & 0 & 0 & -h_1 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \beta_0 & 0 & 0 & -h_0 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & -h_1 & \alpha_0 & \alpha_0 & \alpha_0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0\end{bmatrix}$$

Equation (2) is expressed explicitly in the form

$$\begin{bmatrix}p_0'\\p_1'\\p_2'\\p_3'\\p_4'\\p_5'\\p_6'\\p_7'\\p_8'\\p_9'\\p_{10}'\\p_{11}'\\p_{12}'\end{bmatrix}=\begin{bmatrix}-3\beta_0 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \beta_0 & 0 & 0 & -h_1 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \beta_0 & 0 & 0 & -h_0 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & -h_1 & \alpha_0 & \alpha_0 & \alpha_0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0\end{bmatrix}\begin{bmatrix}p_0(t)\\p_1(t)\\p_2(t)\\p_3(t)\\p_4(t)\\p_5(t)\\p_6(t)\\p_7(t)\\p_8(t)\\p_9(t)\\p_{10}(t)\\p_{11}(t)\\p_{12}(t)\end{bmatrix}$$

Let represent $T_1$the time-to-failure of the Model I. The steady-state availability (the proportion of time the system is in a functioning condition or equivalently, the sum of the probabilities of operational states) is given by

$$A_{T_1}(\infty)=p_0(\infty)+p_3(\infty)+p_6(\infty)+p_9(\infty) \quad (5)$$

In the steady state, the derivatives of the state probabilities become zero and therefore equation (4) become

$$Q_1p=0 \quad (6)$$

which is in matrix form

$$\begin{bmatrix} p_0' \\ p_1' \\ p_2' \\ p_3' \\ p_4' \\ p_5' \\ p_6' \\ p_7' \\ p_8' \\ p_9' \\ p_{10}' \\ p_{11}' \\ p_{12}' \end{bmatrix} = \begin{bmatrix} -3\beta_0 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_0 & 0 & 0 & -h_1 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_0 & 0 & 0 & -h_0 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & -h_1 & \alpha_0 & \alpha_0 & \alpha_0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0 \end{bmatrix} \begin{bmatrix} p_0(t) \\ p_1(t) \\ p_2(t) \\ p_3(t) \\ p_4(t) \\ p_5(t) \\ p_6(t) \\ p_7(t) \\ p_8(t) \\ p_9(t) \\ p_{10}(t) \\ p_{11}(t) \\ p_{12}(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Subject to following normalizing conditions:

$$p_0(\infty) + p_1(\infty) + p_2(\infty) + \ldots + p_{12}(\infty) = 1 \tag{7}$$

Equation (7) is substituted in the last row of (6) to give the system of linear equation in matrix form below:

$$\begin{bmatrix} -3\beta_0 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_0 & 0 & 0 & -h_1 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_0 & 0 & 0 & -h_0 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & -h_1 & \alpha_0 & \alpha_0 & \alpha_0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \\ p_5(\infty) \\ p_6(\infty) \\ p_7(\infty) \\ p_8(\infty) \\ p_9(\infty) \\ p_{10}(\infty) \\ p_{11}(\infty) \\ p_{12}(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The system equations above is solved using MATLAB package to give the following steady – state probabilities:

Table 2. State probabilities for Model I

$p_o(\infty)=\dfrac{\alpha_0^4}{\alpha_0^4+3\alpha_0^3\beta_0+3\alpha_0^2\beta_0^2+3\alpha_0\beta_0^3+3\beta_0^4}$	$p_1(\infty)=\dfrac{\alpha_0^3\beta_0}{\alpha_0^4+3\alpha_0^3\beta_0+3\alpha_0^2\beta_0^2+3\alpha_0\beta_0^3+3\beta_0^4}$
$p_2(\infty)=\dfrac{\alpha_0^3\beta_0}{\alpha_0^4+3\alpha_0^3\beta_0+3\alpha_0^2\beta_0^2+3\alpha_0\beta_0^3+3\beta_0^4}$	$p_3(\infty)=\dfrac{\alpha_0^3\beta_0}{\alpha_0^4+3\alpha_0^3\beta_0+3\alpha_0^2\beta_0^2+3\alpha_0\beta_0^3+3\beta_0^4}$
$p_4(\infty)=\dfrac{\alpha_0^2\beta_0^2}{\alpha_0^4+3\alpha_0^3\beta_0+3\alpha_0^2\beta_0^2+3\alpha_0\beta_0^3+3\beta_0^4}$	$p_5(\infty)=\dfrac{\alpha_0^2\beta_0^2}{\alpha_0^4+3\alpha_0^3\beta_0+3\alpha_0^2\beta_0^2+3\alpha_0\beta_0^3+3\beta_0^4}$
$p_6(\infty)=\dfrac{\alpha_0^2\beta_0^2}{\alpha_0^4+3\alpha_0^3\beta_0+3\alpha_0^2\beta_0^2+3\alpha_0\beta_0^3+3\beta_0^4}$	$p_7(\infty)=\dfrac{\alpha_0\beta_0^3}{\alpha_0^4+3\alpha_0^3\beta_0+3\alpha_0^2\beta_0^2+3\alpha_0\beta_0^3+3\beta_0^4}$
$p_8(\infty)=\dfrac{\alpha_0\beta_0^3}{\alpha_0^4+3\alpha_0^3\beta_0+3\alpha_0^2\beta_0^2+3\alpha_0\beta_0^3+3\beta_0^4}$	$p_9(\infty)=\dfrac{\alpha_0\beta_0^3}{\alpha_0^4+3\alpha_0^3\beta_0+3\alpha_0^2\beta_0^2+3\alpha_0\beta_0^3+3\beta_0^4}$
$p_{10}(\infty)=\dfrac{\beta_0^4}{\alpha_0^4+3\alpha_0^3\beta_0+3\alpha_0^2\beta_0^2+3\alpha_0\beta_0^3+3\beta_0^4}$	$p_{10}(\infty)=\dfrac{\beta_0^4}{\alpha_0^4+3\alpha_0^3\beta_0+3\alpha_0^2\beta_0^2+3\alpha_0\beta_0^3+3\beta_0^4}$
$p_{12}(\infty)=\dfrac{\beta_0^4}{\alpha_0^4+3\alpha_0^3\beta_0+3\alpha_0^2\beta_0^2+3\alpha_0\beta_0^3+3\beta_0^4}$	

The expression for the steady-state availability given in (5) is

$$A_{V1}(\infty)=\frac{\alpha_0^4+3\alpha_0^3\beta_0+3\alpha_0^2\beta_0^2+\alpha_0\beta_0^3}{\alpha_0^4+3\alpha_0^3\beta_0+3\alpha_0^2\beta_0^2+3\alpha_0\beta_0^3+3\beta_0^4} \tag{8}$$

4.2 AVAILABILITY ANALYSIS OF MODEL II

For the availability case of Model II, the differential-difference equations are given by

$$\begin{bmatrix} p_0' \\ p_1' \\ p_2' \\ p_3' \\ p_4' \\ p_5' \\ p_6' \\ p_7' \\ p_8' \\ p_9' \\ p_{10}' \\ p_{11}' \\ p_{12}' \\ p_{13}' \\ p_{14}' \\ p_{15}' \end{bmatrix} = \begin{bmatrix}
-3\beta_0 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\beta_0 & 0 & 0 & -h_1 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \beta_0 & 0 & 0 & -h_1 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & -h_1 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & -h_1 & 0 & 0 & \alpha_0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0
\end{bmatrix} \begin{bmatrix} p_0(t) \\ p_1(t) \\ p_2(t) \\ p_3(t) \\ p_4(t) \\ p_5(t) \\ p_6(t) \\ p_7(t) \\ p_8(t) \\ p_9(t) \\ p_{10}(t) \\ p_{11}(t) \\ p_{12}(t) \\ p_{13}(t) \\ p_{14}(t) \\ p_{15}(t) \end{bmatrix}$$

Let represent T_2 the time-to-failure of Model II. The steady-state availability (the proportion of time the system is in a functioning condition or equivalently, the sum of the probabilities of operational states) is given by

$$A_{T_2}(\infty) = p_0(\infty) + p_3(\infty) + p_6(\infty) + p_9(\infty) + p_{12}(\infty) \tag{9}$$

In the steady state, the derivatives of the state probabilities become zero and therefore

$$\begin{bmatrix}
-3\beta_0 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\beta_0 & 0 & 0 & -h_1 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \beta_0 & 0 & 0 & -h_1 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & -h_1 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & -h_1 & 0 & 0 & \alpha_0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0
\end{bmatrix}
\begin{bmatrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \\ p_5(\infty) \\ p_6(\infty) \\ p_7(\infty) \\ p_8(\infty) \\ p_9(\infty) \\ p_{10}(\infty) \\ p_{11}(\infty) \\ p_{12}(\infty) \\ p_{13}(\infty) \\ p_{14}(\infty) \\ p_{15}(\infty) \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Subject to following normalizing conditions:

$$p_0(\infty) + p_1(\infty) + p_2(\infty) + \cdots + p_{15}(\infty) = 1 \tag{10}$$

Equation (21) is substituted in the last of the matrix above to give the following:

$$\begin{bmatrix}
-3\beta_0 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\beta_0 & 0 & 0 & -h_1 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \beta_0 & 0 & 0 & -h_1 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & -h_1 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & -h_1 & 0 & 0 & \alpha_0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{bmatrix}
\begin{bmatrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \\ p_5(\infty) \\ p_6(\infty) \\ p_7(\infty) \\ p_8(\infty) \\ p_9(\infty) \\ p_{10}(\infty) \\ p_{11}(\infty) \\ p_{12}(\infty) \\ p_{13}(\infty) \\ p_{14}(\infty) \\ p_{15}(\infty) \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The system equations above is solved using MATLAB package to give the following steady – state probabilities

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Table 3: State probabilities for Model II

$p_o(\infty)=\dfrac{\alpha_0^5}{\alpha_0^3\delta_0+3\beta_0^3(\alpha_0^2+\alpha_0\beta_0+\beta_0^2)}$	$p_1(\infty)=\dfrac{\alpha_0^4\beta_0}{\alpha_0^3\delta_0+3\beta_0^3(\alpha_0^2+\alpha_0\beta_0+\beta_0^2)}$
$p_2(\infty)=\dfrac{\alpha_0^4\beta_0}{\alpha_0^3\delta_0+3\beta_0^3(\alpha_0^2+\alpha_0\beta_0+\beta_0^2)}$	$p_3(\infty)=\dfrac{\alpha_0^4\beta_0}{\alpha_0^3\delta_0+3\beta_0^3(\alpha_0^2+\alpha_0\beta_0+\beta_0^2)}$
$p_4(\infty)=\dfrac{\alpha_0^3\beta_0^2}{\alpha_0^3\delta_0+3\beta_0^3(\alpha_0^2+\alpha_0\beta_0+\beta_0^2)}$	$p_5(\infty)=\dfrac{\alpha_0^3\beta_0^2}{\alpha_0^3\delta_0+3\beta_0^3(\alpha_0^2+\alpha_0\beta_0+\beta_0^2)}$
$p_6(\infty)=\dfrac{\alpha_0^3\beta_0^2}{\alpha_0^3\delta_0+3\beta_0^3(\alpha_0^2+\alpha_0\beta_0+\beta_0^2)}$	$p_7(\infty)=\dfrac{\alpha_0^2\beta_0^3}{\alpha_0^3\delta_0+3\beta_0^3(\alpha_0^2+\alpha_0\beta_0+\beta_0^2)}$
$p_8(\infty)=\dfrac{\alpha_0^2\beta_0^3}{\alpha_0^3\delta_0+3\beta_0^3(\alpha_0^2+\alpha_0\beta_0+\beta_0^2)}$	$p_9(\infty)=\dfrac{\alpha_0^2\beta_0^3}{\alpha_0^3\delta_0+3\beta_0^3(\alpha_0^2+\alpha_0\beta_0+\beta_0^2)}$
$p_{10}(\infty)=\dfrac{\alpha_0\beta_0^4}{\alpha_0^3\delta_0+3\beta_0^3(\alpha_0^2+\alpha_0\beta_0+\beta_0^2)}$	$p_{11}(\infty)=\dfrac{\alpha_0\beta_0^4}{\alpha_0^3\delta_0+3\beta_0^3(\alpha_0^2+\alpha_0\beta_0+\beta_0^2)}$
$p_{12}(\infty)=\dfrac{\alpha_0\beta_0^4}{\alpha_0^3\delta_0+3\beta_0^3(\alpha_0^2+\alpha_0\beta_0+\beta_0^2)}$	$p_{13}(\infty)=\dfrac{\beta_0^5}{\alpha_0^3\delta_0+3\beta_0^3(\alpha_0^2+\alpha_0\beta_0+\beta_0^2)}$
$p_{14}(\infty)=\dfrac{\beta_0^5}{\alpha_0^3\delta_0+3\beta_0^3(\alpha_0^2+\alpha_0\beta_0+\beta_0^2)}$	$p_{15}(\infty)=\dfrac{\beta_0^5}{\alpha_0^3\delta_0+3\beta_0^3(\alpha_0^2+\alpha_0\beta_0+\beta_0^2)}$

The expression for the steady-state availability given in (9) is

$$A_{T2}(\infty)=\frac{\alpha_0^5+\alpha_0^4\beta_0+\alpha_0^3\beta_0^2+\alpha_0^2\beta_0^3+\alpha_0\beta_0^4}{\alpha_0^3\delta_0+3\beta_0^3(\alpha_0^2+\alpha_0\beta_0+\beta_0^2)} \tag{11}$$

4.3 MEAN TIME TO FAILURE OF MODEL I

The time dependent analytic solution is difficult to obtain. So that we calculate the MTTF by taking the transpose matrix of $Q_1$and delete the rows and columns for the absorbing state and designation the new matrix by M_1 following Wang and Kuo, (2000), Wang *et al*., (2006) and El-Said and El-Sherbeny, (2005). The expected time to reach an absorbing state is evaluated from

$$\mathrm{E}[T_{P(0)\to P(absorbing)}]=P_1(0)(-M_1^{-1})[1,1,1,1]^T \tag{12}$$

Using the relation

$$\mathrm{E}[T_{P(0)\to P(absorbing)}]=P_K(0)\int_0^\infty e^{M_1^t}dt \tag{13}$$

and

$$\int_0^\infty e^{M_1^t} dt = -M_1^{-1} \tag{14}$$

The explicit expression for the MTTF for Model I is obtained as

$$MTTF_1 = [T_{P(0)\to P(absorbing)}] = P_1(0)(-M_1^{-1})[1,1,1,1]^T \tag{15}$$

$$= \frac{\alpha_0^3 + 8\alpha_0^2\beta_0 + 26\alpha_0\beta_0^2 + 40\beta_0^3}{\beta_0(2\alpha_0^3 + 16\alpha_0^2\beta_0 + 54\alpha_0\beta_0^2 + 81\beta_0^3)}$$

Where

$P_1(0) = [1,0,0,0]$

and

$$M_1 = \begin{bmatrix} -3\beta_0 & \beta_0 & 0 & 0 \\ \alpha_0 & -(\alpha_0+3\beta_0) & \beta_0 & 0 \\ 0 & \alpha_0 & -(\alpha_0+3\beta_0) & \beta_0 \\ 0 & 0 & \alpha_0 & -(\alpha_0+3\beta_0) \end{bmatrix}$$

4.4 MEAN TIME TO FAILURE OF MODEL II

The time dependent analytic solution is difficult to obtain. So that we calculate the MTTF by taking the transpose matrix of Q_2 and delete the rows and columns for the absorbing state and designation the new matrix by M_2 following Wang and Kuo (2000), Wang *et al.*, (2006) and El-Said and El-Sherbeny, (2005). The expected time to reach an absorbing state is evaluated from

$$MTTF_2 = [T_{P(0)\to P(absorbing)}] = P_2(0)(-M_2^{-1})[1,1,1,1]^T \tag{16}$$

$$= \frac{\alpha_0^3 + 8\alpha_0^2\beta_0 + 26\alpha_0\beta_0^2 + 40\beta_0^3}{\beta_0(2\alpha_0^3 + 16\alpha_0^2\beta_0 + 54\alpha_0\beta_0^2 + 81\beta_0^3)}$$

$$= \frac{\alpha_0^4 + 10\alpha_0^3\beta_0 + 45\alpha_0^2\beta_0^2 + 108\alpha_0\beta_0^2 + 121\beta_0^4}{\beta_0(2\alpha_0^4 + 20\alpha_0^3\beta_0 + 90\alpha_0^2\beta_0^2 + 216\alpha_0\beta_0^2 + 243\beta_0^4)} \tag{17}$$

where

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$$Q_2 = \begin{bmatrix}
-3\beta_0 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\beta_0 & 0 & 0 & -h_1 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \beta_0 & 0 & 0 & -h_1 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & -h_1 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & -h_1 & 0 & 0 & \alpha_0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & -\alpha_0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0
\end{bmatrix}$$

$P_2(0) = [1,0.0.0.0$

and

$$M_2 = \begin{bmatrix}
-3\beta_0 & \beta_0 & 0 & 0 & 0 \\
\alpha_0 & -(\alpha_0+3\beta_0) & \beta_0 & 0 & 0 \\
0 & \alpha_0 & -(\alpha_0+3\beta_0) & \beta_0 & 0 \\
0 & 0 & \alpha_0 & -(\alpha_0+3\beta_0) & \beta_0 \\
0 & 0 & 0 & \alpha_0 & -(\alpha_0+3\beta_0)
\end{bmatrix}$$

5. RESULTS AND DISCUSSION

5.1 ANALYTICAL COMPARATIVE ANALYSIS

The main purpose of this section is to present analytical comparisons between the models to determine the optimal model with respect to steady-state availability and mean time to failure using MAPLE software.

$$A_{V1} - A_{V2} = \frac{\alpha_0\beta_0^4(2\alpha_0^3\beta_0 + 2\alpha_0^2\beta_0^2 + 2\alpha_0\beta_0^3 + 2\beta_0^4 - \alpha_0^4)}{\alpha_0^5 + 3\alpha_0^4\beta_0 + 3\alpha_0^3\beta_0^2 + 3\alpha_0^2\beta_0^3 + 3\alpha_0\beta_0^4 + 5\beta_0^5)(\alpha_0^4 + 3\alpha_0^3\beta_0 + 3\alpha_0^2\beta_0^2 + 3\alpha_0\beta_0^3 + 3\beta_0^4)} > 0 \quad (18)$$

$A_{V1} > A_{V2}$ for some $\alpha_0, \beta_0 > 0$

$$MTTF_2 - MTTF_1 = \frac{\beta_0^2(\alpha_0^4 + 9\alpha_0^3\beta_0 + 37\alpha_0^2\beta_0^2 + 81\alpha_0\beta_0^3 + 81\beta_0^4)}{(2\alpha_0^4 + 20\alpha_0^3\beta_0 + 90\alpha_0^2\beta_0^2 + 216\alpha_0\beta_0^3 + 243\beta_0^4)(2\alpha_0^3 + 16\alpha_0^2\beta_0 + 54\alpha_0\beta_0^2 + 81\beta_0^3)} \tag{19}$$

$MTTF_2 - MTTF_1 \quad \forall \alpha_0, \beta_0 > 0$

5.2 NUMERICAL EXAMPLES

In this section, surface plot of performance of each configuration using availability and mean time to failure is evaluated via MATLAB software.

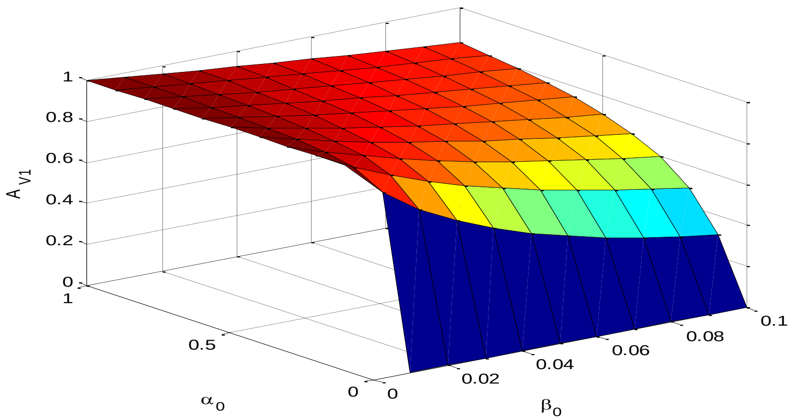

Figure 3.Surface plot of Availability of Model I against β_0 and α_0

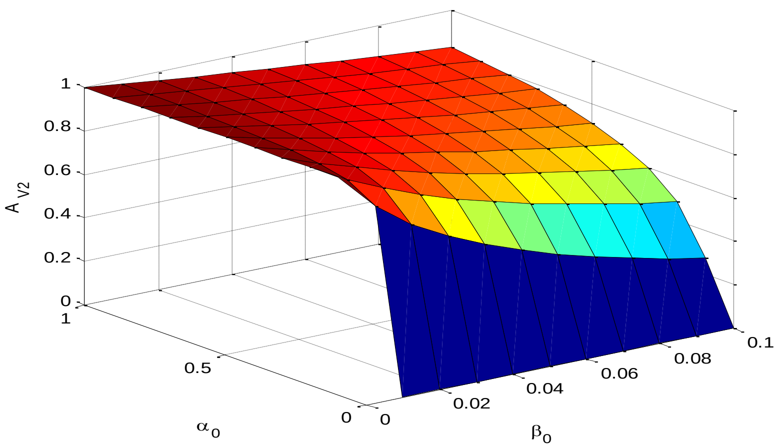

Figure 4.Surface plot of Availability of Model II against β_0 and α_0

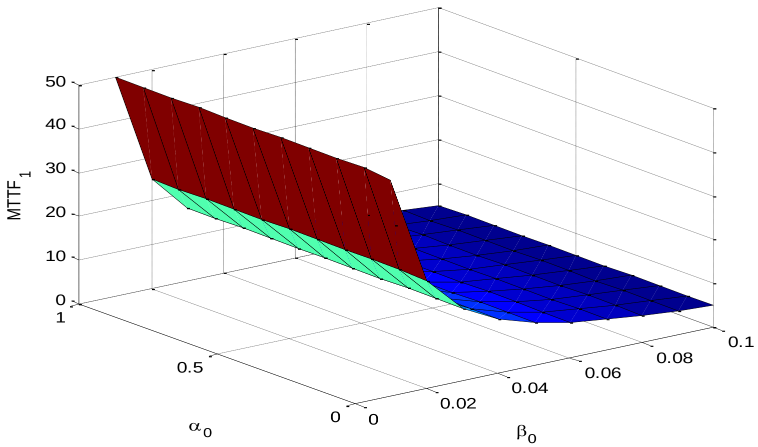

Figure 5. Surface plot of MTTF of Model I against β_0 and α_0

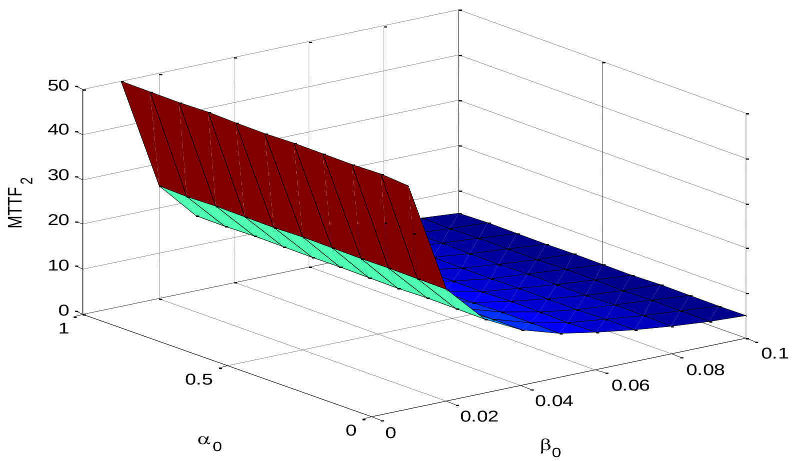

Figure 6. Surface plot of MTTF of Model II against β_0 and α_0

Figures 3 to 6 depict the surface plot of availability and main time to failure of configurations with respect to β_0 and α_0. It is evident from the figures that availability and main time to failure decrease as $\beta_0$increases and increases and $\alpha_0$increase. It can be seen that as $\beta_0$increase availability and main time to failure decreases leading to less production output and generated revenue. Major maintenance, inspection, online and offline preventive maintenance should be introduced to avoid system failure. On the other hand, availability and mean time to failure tend to increase as $\alpha_0$increases. This led to high production output, quality of the product as well as generative revenue. More

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systems with higher reliability, fault tolerant system should be introduced avoid catastrophic break down.

5.3 RANKING OF THE MODELS

The purpose of this section is to rank the configurations for their availability and mean time to failure using MATLAB software package. The results are summarized in Tables below.

Table 4. Ranking between the configurations in terms of their mean time to failure for different values of β_0.

Parameter Range	Result				
	$\beta_0 = 0.2$	$\beta_0 = 0.3$	$\beta_0 = 0.4$	$\beta_0 = 0.5$	$\beta_0 = 0.6$
$0 < \alpha_0 < 0.1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$
$0.1 < \alpha_0 < 0.2$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$
$0.2 < \alpha_0 < 0.3$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$
$0.3 < \alpha_0 < 0.4$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$
$0.4 < \alpha_0 < 0.5$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$
$0.5 < \alpha_0 < 0.6$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$
$0.6 < \alpha_0 < 0.7$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$
$0.7 < \alpha_0 < 0.8$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$
$0.8 < \alpha_0 < 0.9$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$
$0.9 < \alpha_0 < 1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$

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Table 5. Ranking between the configurations in terms of their mean time to failure for different values of α_0

Parameter Range	Result				
	$\alpha_0 = 0.2$	$\alpha_0 = 0.3$	$\alpha_0 = 0.4$	$\alpha_0 = 0.5$	$\alpha_0 = 0.6$
$0 < \beta_0 < 0.1$	$MTTF_2 = MTTF_1$	$MTTF_2 = MTTF_1$	$MTTF_2 = MTTF_1$	$MTTF_2 = MTTF_1$	$MTTF_2 = MTTF_1$
$0.1 < \beta_0 < 0.2$	$MTTF_2 = MTTF_1$	$MTTF_2 = MTTF_1$	$MTTF_2 = MTTF_1$	$MTTF_2 = MTTF_1$	$MTTF_2 = MTTF_1$
$0.2 < \beta_0 < 0.3$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$
$0.3 < \beta_0 < 0.4$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$
$0.4 < \beta_0 < 0.5$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$
$0.5 < \beta_0 < 0.6$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$
$0.6 < \beta_0 < 0.7$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$
$0.7 < \beta_0 < 0.8$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$
$0.8 < \beta_0 < 0.9$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$
$0.9 < \beta_0 < 1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$	$MTTF_2 > MTTF_1$

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Table 6. Ranking between the configurations in terms of their availability for different values of β_0.

Parameter Range	Result				
	$\beta_0 = 0.2$	$\beta_0 = 0.3$	$\beta_0 = 0.4$	$\beta_0 = 0.5$	$\beta_0 = 0.6$
$0 < \alpha_0 < 0.1$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$
$0.1 < \alpha_0 < 0.2$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$
$0.2 < \alpha_0 < 0.3$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$
$0.3 < \alpha_0 < 0.4$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$
$0.4 < \alpha_0 < 0.5$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$
$0.5 < \alpha_0 < 0.6$	$A_{V1} = A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$
$0.6 < \alpha_0 < 0.7$	$A_{V1} = A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$
$0.7 < \alpha_0 < 0.8$	$A_{V1} = A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$
$0.8 < \alpha_0 < 0.9$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$
$0.9 < \alpha_0 < 1$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$

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Table 7.Ranking between the configurations in terms of their availability for different values of α_0.

Parameter Range	Result				
	$\alpha_0 = 0.2$	$\alpha_0 = 0.3$	$\alpha_0 = 0.4$	$\alpha_0 = 0.5$	$\alpha_0 = 0.6$
$0 < \beta_0 < 0.1$	$A_{V1} = A_{V2}$	$A_{V1} = A_{V2}$	$A_{V1} = A_{V2}$	$A_{V1} = A_{V2}$	$A_{V1} = A_{V2}$
$0.1 < \beta_0 < 0.2$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} = A_{V2}$	$A_{V1} = A_{V2}$	$A_{V1} = A_{V2}$
$0.2 < \beta_0 < 0.3$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} = A_{V2}$
$0.3 < \beta_0 < 0.4$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$
$0.4 < \beta_0 < 0.5$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$
$0.5 < \beta_0 < 0.6$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$
$0.6 < \beta_0 < 0.7$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$
$0.7 < \beta_0 < 0.8$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$
$0.8 < \beta_0 < 0.9$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$
$0.9 < \beta_0 < 1$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$	$A_{V1} > A_{V2}$

Tables 4 to 7 depict the ranking of configuration base on their availability and mean time to failure. It clear from Table 4 that configuration II is the optimal configuration in terms of mean time to failure whenever $0 \leq \alpha_0 \leq 1$ for $0 \leq \beta_0 \leq 0.6$. Also, from Table 5 the optimal configuration in terms of mean time to failure is again Configuration II whenever $0.2 \leq \beta_0 \leq 1$.for$0 \leq \alpha_0 \leq 0.6$ .Thus, from Table2 and 3 $MTTF_2 > MTTF_1$

On the other hand, Tables 6 and 7 depict the ranking of configuration base on their availability. It clear from Table 4 that configuration I is the optimal configuration in terms of availability whenever $0 \leq \alpha_0 \leq 1$ and $0.4 \leq \beta_0 \leq 0.6$. It is observed from Table 5 the optimal configuration in terms of availability is again Configuration I whenever $0.3 \leq \beta_0 \leq 0.4$ for$0 \leq \alpha_0 \leq 0.6$. Thus, from Table 4 and 5 $A_{V1} > A_{V2}$.

Table 8. Variation of **availability and MTTF** with respect to β_0 and α_0

β_0	$\alpha_0 = 0.6$				α_0	$\beta_0 = 0.2$			
	A_{V1}	A_{V2}	$MTTF_1$	$MTTF_2$		A_{V1}	A_{V2}	$MTTF_1$	$MTTF_2$
0	1.0000	1.0000	∞	∞	0	0	0	2.4691	2.4897
0.1	0.7496	0.7498	4.9965	4.9996	0.1	0.1648	0.1235	2.4777	2.4934
0.2	0.5970	0.5970	2.4943	2.4990	0.2	0.3077	0.2778	2.4837	2.4956
0.3	0.4918	0.4882	1.6605	1.6653	0.3	0.4140	0.4024	2.4878	2.4970
0.4	0.4140	0.4024	1.2439	1.2485	0.4	0.4918	0.4882	2.4907	2.4980
0.5	0.3544	0.3330	0.9942	0.9985	0.5	0.5507	0.5499	2.4928	2.4986
0.6	0.3077	0.2778	0.8279	0.8319	0.6	0.5970	0.5970	2.4943	2.4990
0.7	0.2706	0.2345	0.7092	0.7129	0.7	0.6344	0.6347	2.4955	2.4992
0.8	0.2407	0.2009	0.6202	0.6237	0.8	0.6654	0.6657	2.4963	2.4994
0.9	0.2163	0.1746	0.5511	0.5543	0.9	0.6914	0.6917	2.4970	2.4996

Table 7 displayed the impact of failure and repair rates on availability and mean time to failure respectively for each configuration. It is evident from the table that availability and mean time to failure increases as repair rates increase and decreases with increase in failure rates. The variation in availability and mean time to failure corresponding to different failure (repair) rates evidently indicates that incremental change in values of parameter decreases (increases) the availability and mean time to failure of the system. It is interesting to note that in this case, availability and mean time to failure of the system decreases (increases) smoothly with each failure (repair) rate.

Comparison of two models based on their cost/benefit C_i/B where B is either $MTTF$or $A_T(\infty)$

Table 9. Cost of the Models

Model	Cost $C_i, i = I, II$
I	48000000
II	39000000

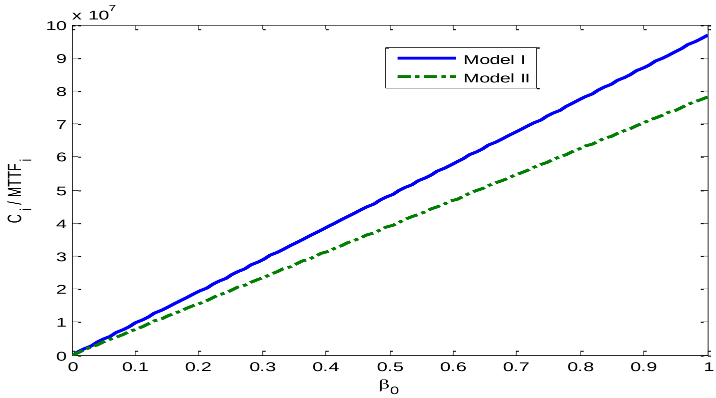

Figure 7. $C_i/MTTF$ versus failure rate β_0 for $\alpha_0 \geq 0.2$

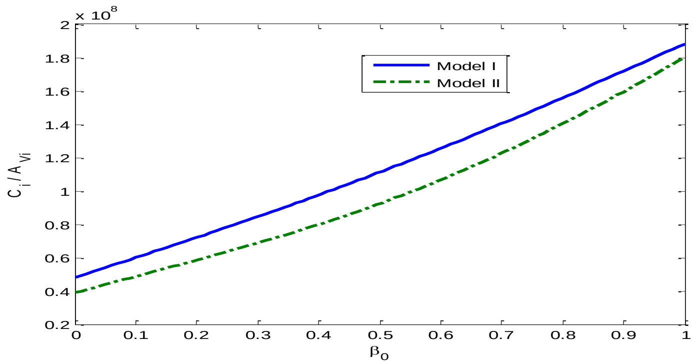

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Figure 8. C_i/A_{Vi} versus failure rateβ_0 for $\alpha_0 \geq 0.7$

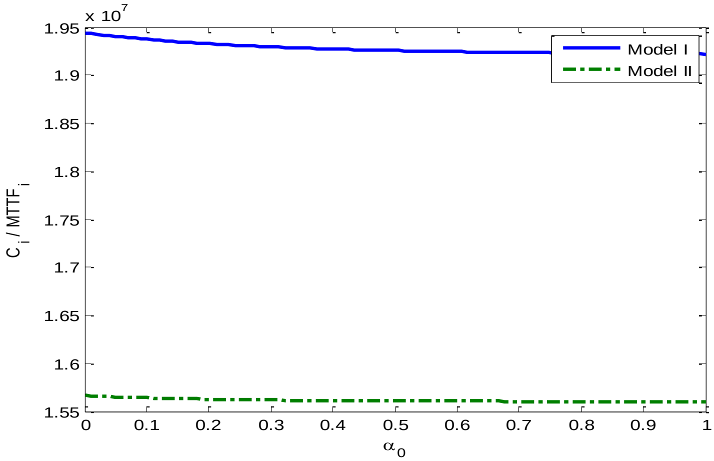

Figure 9. $C_i/MTTF_i$ versus failure rate α_0 for $\beta_0 \geq 0.2$

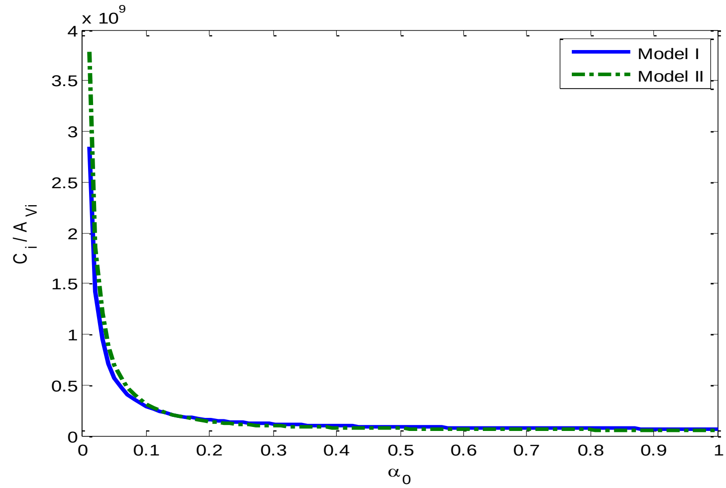

Figure 10. C_i/A_{Vi} versus repair rate α_0 for $\beta_0 \geq 0.2$

Figures 7 and 8 depict the results of $C_k/MTTF_k$and C_k/A_{Vk} for each model with respect toβ_0. From these figures, it is evident that $C_k/MTTF_k$and C_k/A_{Vk} increase as $\beta_0$increases for each model. From these Figures that the optimal configuration using $C_k/MTTF_k$ and C_k/A_{Vk} is Model II for $\alpha_0 \geq 0.2$ and $\alpha_0 \geq 0.7$ respectively. Similar observations can be seen from Figures 9 and 10 in which $C_k/MTTF_k$and C_k/A_{Vk} decrease as $\alpha_0$increases for each model. The optimal model is again Model II using $C_k/MTTF_k$ and C_k/A_{Vk} for$\beta_0 \geq 0.2$.

6.CONCLUSION

In this paper, we studied the steady-state availability and mean time to failure of a network with built in redundancy. Explicit expressions for the mean time to system failure and steady-state availability are derived. The numerical examples and simulations presented in Tables 2 to 5 and Figures 3 to 6 provide a description of the effect of the failure rate β_0 and repair rate α_0 on steady-state availability and mean time to failure. It is clear from both analytical and numerical comparison, that the optimal model is model I. On the basis of the numerical and analytical results obtained for a particular case, it is suggested that the system reliability can be improved significantly by:

(i) Adding more paths and components in cold standby
(ii) Adequate preventive maintenance
(iii) Increasing the repair rate.
(iv) Reducing the failure rate of the system by hot duplication

The present work can be extended further for a reliability optimization of multi component network communication system and solve approaches such as soft computing, fuzzy optimization technique.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

References

1. Ali, P.(2009). "Reliability of Wireless Body Area Networks used for Ambulatory Monitoring and Health Care, Life Science Journal,2009. Vol. 6, No. 4, pp. 5-14.
2. Amari, S.V., Pham, H. and Misra, R.B. (2012), "Reliability characteristics of k-out-of-n warm standby systems", *IEEE Trans. Reliab*., Vol. 61, pp. 1007–1018
3. Ashok Kumar, D. Pawar1 & S. C. Malik. (2020). Reliability analysis of a redundant System with 'FCFS' repair policy subject to weather conditions, International Journal of Advanced Science and Technology, 29(3),7568 – 7578
4. Chauhan, S.K. and Malik, S.C. (2017) 'Evaluation of reliability and MTSF of a parallel system with Weibull failure laws', *Journal of Reliability and Statistics*, Vol. 10, No. 1, pp.137–146
5. El-Said, K.M., and M.S. El-Sherbeny .Evaluation of reliability and availability characteristics of two different systems by using linear first order differential equations. J. Maths and Stat. 2005,1 (2) 119-123
6. Fathabadi H.S and Khodaei, M. Reliability evaluation of network flows with stochastic capacity and cost constraint, Int. J. of Mathematics in Operational Research, 2012, Vol.4, No.4, pp. 439 – 452.
7. Garg, H and Sharma, S.P. (2012). Behavior analysis of synthesis unit in fertilizer plant, International Journal of Quality & Reliability Management, Vol. 29 No. 2, 2012, pp. 217-232
8. Garg, H., Rani, M and Sharma .S.P. . (2014). An approach for analyzing the reliability of industrial systems using soft-computing based technique, Expert Systems with Applications, Volume 41, Issue 2, Pages 489-501

9. Garg, H. (2014) 'Reliability, availability and maintainability analysis of industrial systems using PSO and fuzzy methodology', *MAPAN*, Vol. 29, No. 2, pp.115–129.
10. Garg, H and Sharma, S.P. (2012).A two-phase approach for reliability and maintainability analysis of an industrial system, International Journal of Reliability Quality and Safety Engineering 19(3), DOI: 10.1142/S0218539312500131
11. Garg, H. (2015) 'An approach for analyzing the reliability of industrial system using fuzzy
Kolmogorov's differential equations', *Arabian Journal for Science and Engineering*, Vol. 40, No. 3, pp.975–987
12. Hassan, M. Reliability evaluation of stochastic-flow network under quickest path and system capacity constraints, Int. J. Comput. Netw,2012, vol. 4, pp. 98–103.
13. Lado,A and Singh, V. V. (2019),"Cost Assessment of complex repairable system consisting two subsystems in Series configuration using Gumbel Hougaard family copula", *International Journal of Quality Reliability and Management,* Vol. 36(10), pp1683-1698.
14. Lado, A., Singh, V.V., Ismail, H, K and Yusuf, I. (2018). Performance and cost assessment of repairable complex system. International Journal of Reliability and Applications, 19,(1). 27-42.
15. Niwas, R. and Garg, H. (2018), "An approach for analyzing the reliability and profit of an industrial system based on the cost-free warranty policy", Journal of the Brazilian Society of Mechanics and Engineering, Vol. 40 No. 5, pp. 1-9.
16. Rocco S.C.M., Zio E., Solving advanced network reliability problems by means of cellular automata and Monte Carlo sampling, Reliability Engineering and System Safety 2005; 89(2): 219-226
17. Rocco S.C.M., Moreno J.A., Network reliability assessment using a cellular automata approach, Reliability Engineering and System Safety 2002; **78**: 289-295
18. Singh, V. V and Ayagi, H.I. (2018), "Stochastic analysis of a complex system under preemptive resume repair policy using Gumbel- Hougaard family copula," *International Journal of Mathematics and Operation Research (IJMOR)*. Vol 12 Issue 2, pp 273-291, 2018
19. Singh, V. V., Poonia, P. K. and Abdullahi,A.H.(2020a), "Performance analysis of a complex repairable system with two subsystems in series configuration with an imperfect switch". Journal of Mathematics & Computational Science. (JMCS). Vol. 10(2). pp. 359-383, 2020.
20. Singh, V. V., Poonia, P. K. and Rawal, D. K. (2020b), Reliability analysis of repairable network system of three computer labs connected with a server under 2-out- of- 3 G configuration", Published, May 2020, Life Cycle Reliability and Safety Engineering. (DOI: 10.1007/s41872-020-00129-w)
21. Singh, V. V. and Poonia, P. K. (2019),"Probabilistic assessment of two unit's parallel system with correlated lifetime under inspection using regenerative point technique". International Journal of Reliability, Risk& Safety, Theory and applications. Vol. 2(1), pp 5-18, 2019
22. Vasar, C., Prostean, O., Filip, I., Robu, R., Popescu, D. "Markov models for wireless sensor network reliability," Proceedings of IEEE ICCP'9, 2009,pp.323-328
23. Wang, K.-H. and Kuo, C.-C. Cost and probabilistic analysis of series systems with mixed standby components. Applied Mathematical Modelling, 2000,24, 957-967.

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24. Wang, K, -H., Hsieh, C,-H, Liou, C, -H. Cost benefit analysis of series systems with cold standby components and a repairable, QTQM, 3(1), 77-9https://doi.org/10.1080/16843703.2006.11673101
25. Yusuf, I., Lado, A.K. and Ali, U.A. (2020) 'Reliability analysis of communication network system with redundant relay station under partial and complete failure', J. Math. Comput. Sci., Vol. 10, No. 4, pp.863–880