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Neutrosophic fuzzy number and its application in solving linear programming problem by simplex algorithm: An alternative approach

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Abstract The fuzzy logic and fuzzy numbers have been applied in many fields such as operation research, differential equations, fuzzy system reliability etc. Now days in engineering applications the fuzzy logic and fuzzy numbers are widely used. In this paper we first describe Neutrosophic Fuzzy Number with arithmetic operations and solve a linear programming problem by Neutrosophic Fuzzy Number using Simplex algorithm.

Keywords Fuzzy set; Neutrosophic fuzzy number; Simplex algorithm

1. Introduction

Necessity of solving fuzzy LPP

In practical field we are sometimes faced with a type of problem which consists in assigning men to offices, jobs to machines, classes in a school to rooms, drivers to trucks, delivery trucks to different routes or problems to different research teams etc in which the assignees possess varying degree of efficiency, called cost or effectiveness. The basic assumption of this type of problem is that one person can perform one job at a time. An assignment plan is optimal if it minimizes the total cost or effectiveness or maximizes the profit of performing all the jobs. Assignment problem is a special type of transportation problem in which the objectives is to optimize the effect of allocating a number of jobs to an equal number of facilities. It is appropriate to investigate the assignment problem by using fuzzy optimization methodologies. The applicable theoretical methods can be referred to as fuzzy set theory and triangular fuzzy number. In our daily life a big problem is that, the data which we are collect for our mathematical or statistical use are full of vagueness or sometimes incomplete or insufficient, and the

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probabilistic approach to the conventional reliability analysis is inadequate to account for such built-in uncertainties in data. On the basis of this uncertainty concept of fuzzy introduced which is fuzzy reliability. It has been introduced and formulated either in the context of the possibility measures or as a transition from fuzzy success state to fuzzy failure state.

Review on Neutrosophic fuzzy mathematics

Bhowmik and Pal (2010) defined intuitionistic neutrosophic set (INSs). They show by means of example that the definition for neutrosophic sets the complement and union are not true for INSs also give new definition of complement, union and intersection of INSs. S.Mondal *et al*. (2018) introduced in this paper we the concept of neutrosophic number from different viewpoints. They defined different types of linear and non-linear generalized triangular neutrosophic numbers which are very important for uncertainty theory. They introduced the de-neutrosophication concept for neutrosophic number for triangular neutrosophic numbers. A.Chakraborty *et al.* (2019) added to the theory of the generalized neutrosophic number from a distinctive frame of reference. It is universally known that the concept of a neutrosophic number is generally associated with and strongly related to the concept of positive, indeterminacy and non-belongingness membership functions. S.Mondal *et al.* (2020) were envisaged the neutrosophic number from various distinct rational perspectives and viewpoints to give it a look of a conundrum. They focused and analysed neutrosophic fuzzy numbers when indeterminacy and falsity functions are dependent on each other, which serves an indispensable role for the uncertainty concept. S.A.Hussain *et al.* (2018) were discussed in this chapter the VIKOR method for solving MCDM problem with interval valued neutrosophic number. First they defined interval-valued neutrosophic number. The different properties of that type of numbers were also discussed. S.Maity *et al.* (2020) were envisaged the hexagonal number from various distinct rational perspectives and viewpoints to give it a look of a conundrum. Hexagonal fuzzy number is used as an authoritative logic to ease understanding of vagueness information. This article portrays an impression of different representation, ranking, defuzzification and application of hexagonal fuzzy number. S.Mondal *et al.* (2020) tried to classify the dependency and independency of the membership functions for the pentagonal neutrosophic number. Theory of pentagonal neutrosophic number had been studied in a disjunctive frame of reference. B.Sen *et al.* (2018) were introduced the concept of neutrosophic number from different point of views. They defined different types of linear and non linear generalized neutrosophic numbers which is very important for uncertainty theory. The different properties of that type of numbers were also discussed. S.Alam *et al.* (2020) focused on the brief study of finding the solution to and analyzingthe homogeneous linear difference equation in a neutrosophic environment, i.e., they interpreted the solution of the homogeneous difference equation with initial information, coefficient and both as a neutrosophic number. The idea for solving and analyzing the above using the characterization theorem is demonstrated. S.K.De *et al.* (2020) deal with a backorder inventory problem under intuitionistic dense fuzzy environment. In fuzzy set theory, the concept of dense fuzzy set is quite new that depends upon the number of

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negotiations/turnovers made by the decision makers (DMs) of any kind of industrial setup. Moreover, they have discussed the preliminary concept on intuitionistic dense fuzzy set (IDFS) with their corresponding (non)membership functions and defuzzification methods. The graphical overview resembles the graphs obtained from a cloud aggregation model developed by Mao et al. in 2018.

Motivation

Fuzzy sets theory plays an important role in uncertainty modelling. Now the question is if we wish to take a fuzzy number then how its geometrical representations are. So, if decision maker takes a fuzzy number which can be graphically looks like a Neutrosophic then how its membership function can be defined. From this point of view, we try to define different type of Neutrosophic fuzzy number which can be a better choice of a decision maker in different situation.

Novelties

There is various articles where pentagonal fuzzy sets and number are introduced and apply to different fields. But there are so many scopes to work on that topic. Some new interest and new work have done by our self which is mentioned below:

i. Try to utilise the properties of pentagonal fuzzy number to solve an optimization problem.
ii. Described simplex method.
iii. Solved an optimization problem by Neutrosophic fuzzy number.
iv. We used all the allocation of number in the simplex problem by Neutrosophic fuzzy number.

2. Preliminaries

Neutrosophic Number

Definition: Neutrosophic Set: A set in the universal discourse X, which is denoted generically by x, is said to be a neutrosophic set if $\psi = \{\langle x, [\pi_\psi(x), \mu_\psi(x); \upsilon_\psi(x)]\rangle : x\epsilon X\}$ where $\pi_\psi(x): X \to [0,1]$ is called the truth membership function which represent the degree of confidence, $\pi_\psi(x): X \to [0,1]$ is called the indeterminacy membership function which represent the degree of uncertainty and $\upsilon_\psi(x): X \to [0,1]$ is called the falsity membership function which represent the degree of scepticism on decision given by the decision maker.

Definition: Single valued neutrosophic number: It is defined as $\tau = \langle [(a,b,c,d):\alpha], [(e,f,g,h);\beta], [i,j,k,l);\gamma]\rangle$ where $,\beta.\gamma\epsilon[0,1]$,the truth membership function $\varpi: \Re \to [0,\alpha]$, the indeterminacy membership function $\rho: \Re \to [\beta, 1]$ and the falsity membership function $\varsigma: \Re \to [\gamma, 1]$ is given as:

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$$\varpi=\begin{cases}\varpi_l & a\le x\le b\\ \alpha & b\le x\le c\\ \varpi_u & c\le x\le d\\ 0 & otherwise\end{cases}$$

$$\rho=\begin{cases}\rho_l & e\le x\le f\\ \alpha & f\le x\le g\\ \rho_u & g\le x\le h\\ 0 & otherwise\end{cases}$$

$$\varsigma=\begin{cases}\varsigma_l & i\le x\le j\\ \alpha & i\le x\le j\\ \varsigma_u & j\le x\le k\\ 0 & otherwise\end{cases}$$

Definition: Triangular single valued neutrosophic number: The unity of the truth, indeterminacy and falsity are not dependent. A triangular single valued neutrosophic number is defined as $M=(a,b,c;d,e,f;g,h,i)$whose truth membership, indeterminacy and falsity membership are defined as follows:

$$T=\begin{cases}\frac{x-a}{b-a} & a\le x\le b\\ 1 & x=b\\ \frac{c-x}{c-b} & b\le x\le c\\ 0 & otherwise\end{cases}$$

$$I=\begin{cases}\frac{e-x}{e-d} & d\le x\le e\\ 1 & x=e\\ \frac{x-d}{f-d} & e\le x\le f\\ 0 & otherwise\end{cases}$$

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$$F = \begin{cases} \dfrac{h-x}{h-g} & g \le x \le h \\ 1 & x = h \\ \dfrac{x-g}{i-g} & h \le x \le i \\ 0 & otherwise \end{cases}$$

Where $0 \le T(x) + I(x) + F(x) \le 3$

3. Theoretical part

3.1. Neutrosophic Simplex Algorithm

The iterative procedure for finding an optimal basic feasible solution to an L.P.P.is as follows:

Step-1: Check whether the given L.P.P is a maximization problem. If it is a minimization Problem then convert it into a problem of maximization, using
Minimize z=-Minimize (-z)
Where z be the objective function of the given L.P.P.

Step-2: Check whether all $b_i(i = 1,2, \dots, m)$ are non-negative. If any one of b $_j$ is negative then multiply the corresponding inequation/ equation of the constraint by (-1) so as to get all $b_i(i = 1,2, \dots, m)$ as positive.

Step-3: Convert all the inequality constraints into equation by introducing slackand/ or surplus variables in the constraints. Put the coefficient of these variables in the objective function as zero.

Step-4: Obtain and initial B.F.S. (Basic feasible solution) x $_B$usingx $B = B^{-1}$b where B be the initial unit basis matrix. Alternatively, by taking non- basic variables as zero, obtain the initial B.F.S.

Step-5: Construct the simplex table.

Step-6: Compute the net evolutions $\Delta_j = z_j - c_j (j = 1,2, \dots, n)$ by using the formula $\Delta_j = z_j - c_j(cB.y_j) - c_j$ and examine their sign .Two case may arise:
(i)if all $\Delta_j \geq 0$ and either no artificial variable is in the basis or artificial variables present in the basis at zero level. Then the current B. F.S. Is optimal and stop the process.
(ii) if at least one$\Delta_j \geq 0$, go to next step

Step-7: If at least one $\Delta_j < 0$ choose the most negative of Δ_j . Let it be $\Delta_j$some for j = r .Two cases may arise:

(i) if all $y_{ir} \le 0 (i = 1,2, \dots, m)$ then there is an unbounded solution to the given L.P.P and stop the process.

(ii) If at least one $y_{ir} \le 0$, then the corresponding vector y $_r$ enters the basis. The vector is known as incoming or entering vector.

		C_j	C_1	C_2	…	C_j	…	C_n
C_B	y_B	x_B	y_1	y_2	…	y_j	…	y_n
C_{B1}	y_{B1}	x_{B1}	y_{11}	y_{12}	…	y_{1j}	…	y_{1n}
…	…	…	…	…	…	…	…	…
C_{Bm}	y_{Bm}	x_{Bm}	y_{m1}	y_{m2}	…	y_{mj}	…	y_{mn}
$z = (c^T{}_B, X_B$			$Z_1 - C_1$	$Z_2 - C_2$	…	$Z_j - C_j$	…	$Z_n - C_n$

Step-8: To select the departing for outgoing vector, computer $min\{x\,B_r/y_{ir}\,, y_{ir} > 0\}$ j, Late minimum value be xB_k/y_{kr}. Then the factor y_k will leave the basis. this better

known as departing or outgoing vector. The common element y_{kr} which is in the k-throw and r- the column is known as key or leading element.
Step-9: Convert the key element to unity by dividing it's row by the element itself and allother elements in its column to zero by which the relations
$\hat{y}_{ij} = y_{ij} - y_{kj}/y_{kr}y_{ir}\,, i = 1,2, \dots, m \;\; i \neq k, j = 1,2, \dots, n$
$\hat{y}_{ij} = y_{kj}/y_{kr}\,, j = 1,2, \dots, n$

Then modify the components of x_B by using the relations $\hat{x}_{Bi} = x_{Bi} - y_{kj}/y_{kr}x_{Bk}\,, i = 1,2, \dots, m \;\; i \neq k$ and $\hat{x}_{Bk} = x_{Bk}/y_{kr}$

Using these values, construct the next simplex table and go to step-6

3.2. Simplex table

Let consider the following standard linear programming problem
Maximum $z = (C^T, x)$
Subject to $Ax = b$ and $x \geq 0$
Where $c = (c_1, c_2, \dots, c_n) =$ profit vector
$x = (x_1, x_2, \dots, x_n)^T =$ decisionvector
$A = (a_1, a_2, \dots, a_n)$ =coefficient matrix
$b = (b_1, b_2, \dots, b_m)^T =$ requirement vector
$c_B = (C_{B1}, C_{B2}, \dots, C_{Bm})^T$ =The m- component column vector having the element corresponding to the basic variables.
$B = (B_1, B_2, \dots, B_M) =$ basis matrix
$X_B = B^{-1}b =$ basic feasible solution
$v_j = B^{-1}{}_{a_j} \;\; j = 1,2, \dots, n$
Initially, $B = I_m =$ unit matrix of order m
For the initial simplex table, $X_B = B^{-1}b = I_m b = b.$ [‘.’ B $= I_m$’$. B^{-1} = I_m$]
$y_1 = B^{-1}{}_{a_j} = a_j$

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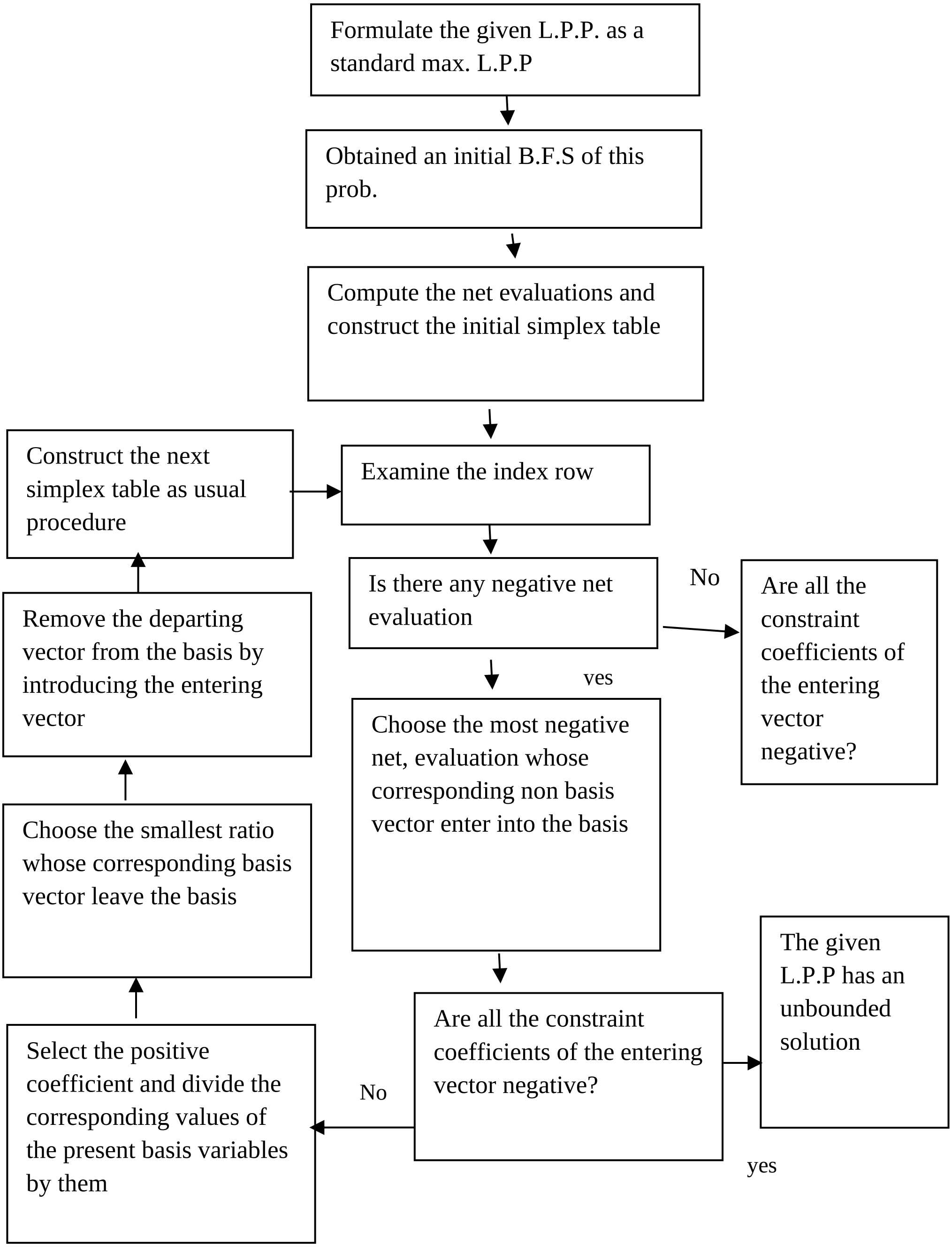
Formulate the given L.P.P. as a standard max. L.P.P
Obtained an initial B.F.S of this prob.
Compute the net evaluations and construct the initial simplex table
Construct the next simplex table as usual procedure
Examine the index row
Is there any negative net evaluation
No
Are all the constraint coefficients of the entering vector negative?
Remove the departing vector from the basis by introducing the entering vector
yes
Choose the most negative net, evaluation whose corresponding non basis vector enter into the basis
Choose the smallest ratio whose corresponding basis vector leave the basis
The given L.P.P has an unbounded solution
Select the positive coefficient and divide the corresponding values of the present basis variables by them
No
Are all the constraint coefficients of the entering vector negative?
yes

4. Application

In this paper we are going to solve a linear programming problem by Neutrosophic fuzzy number using simplex algorithm. Our problem is described below:

$$\max z = (4,6,7;8,9,10;11,12,13)x_1 + (8,10,11;12,13,14;15,16,17)x_2$$
$$(2,4,5;6,7,8;9,10,11)x_1 + (1,3,4;5,6,7;8,9,10)x_2 = 50$$
$$(2,4,5;6,7,8;9,10,11)x_1 + (4,6,7;8,9,10,11,12,13)x_2 = 100$$
$$(2,4,5;6,7,8;9,10,11)x_1 + (3,5,6;7,8,9;10,11,12)x_2 = 90$$

Now this problem rewrite by introducing the slack variables $x_3, x_4$and x_5 as,

$$\begin{aligned}\max z = {} & (4,6,7;8,9,10;11,12,13)x_1 + (8,10,11;12,13,14;15,16,17)x_2 \\ & + (0,0,0;0,0,0;0,0,0)x_3 + (0,0,0;0,0,0;0,0,0)x_4 \\ & + (0,0,0;0,0,0;0,0,0)x_5\end{aligned}$$

Subject to constraint

$$(2,4,5;6,7,8;9,10,11)x_1 + (1,3,4;5,6,7;8,9,10)x_2 + (1,1,1;1,1,1;1,1,1)x_3 = 50$$
$$(2,4,5;6,7,8;9,10,11)x_1 + (4,6,7;8,9,10,11,12,13)x_2 + (1,1,1;1,1,1;1,1,1)x_4 = 100$$
$$(2,4,5;6,7,8;9,10,11)x_1 + (3,5,6;7,8,9;10,11,12)x_2 + (1,1,1;1,1,1;1,1,1)x_5 = 90$$

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	c_i	(4,6,7;8,9,10;11,12,13)	(8,10,11;12,13,14;15,16,17)	(0,0,0;0,0,0;0,0,0)	(0,0,0;0,0,0;0,0,0)	(0,0,0;0,0,0;0,0,0)
	BV	x_1	x_2	x_3	x_4	x_5
(0,0,0;0,0,0;0,0,0)	x_3	(2,4,5;6,7,8;9;10,11)	(1,3,4;5,6,7;8,9,10)	(1,1,1;1,1,1;1,1,1)	(0,0,0;0,0,0;0,0,0)	(0,0,0;0,0,0;0,0,0)
(0,0,0;0,0,0;0,0,0)	x_4	(2,4,5;6,7,8;9;10,11)	(4,6,7;8,9,10;11,12,13)	(0,0,0;0,0,0;0,0,0)	(1,1,1;1,1,1;1,1,1)	(0,0,0;0,0,0;0,0,0)
(0,0,0;0,0,0;0,0,0)	x_5	(2,4,5;6,7,8;9;10,11)	(3,5,6;7,8,9;10,111,12)	(0,0,0;0,0,0;0,0,0)	(0,0,0;0,0,0;0,0,0)	(1,1,1;1,1,1;1,1,1)
	zj	(0,0,0;0,0,0;0,0,0)	(0,0,0;0,0,0;0,0,0)	(0,0,0;0,0,0;0,0,0)	(0,0,0;0,0,0;0,0,0)	(0,0,0;0,0,0;0,0,0)
	Cj-z	(4,6,7;8,9,10;11,12,13)	(8,10,11;12,13,14;15,16,17)	(0,0,0;0,0,0;0,0,0)	(0,0,0;0,0,0;0,0,0)	(0,0,0;0,0,0;0,0,0)
(0,0,0;0,0,0;0,0,0)	x_3	(3/2,2,15/7;16/7,17/7,18/7;19/7,20/7,3)	(0,0,0;0,0,0;0,0,0)	(1,1,1;1,1,1;1,1,1)	(-1/4,-1/2,-4/7;-5/7,-6/7,-1;-8/7,-9/7,-10/7)	(0,0,0;0,0,0;0,0,0)
(8,10,11;12,13,14;15,16,17)	x_2	(1/2,2/3,5/7;6/7,1,8/7;9/7,10/7,11/7)	(1,1,1;1,1,1;1,1,1)	(0,0,0;0,0,0;0,0,0)	(1/7,1/5,1/4;1/5,1/6,1/7;1/8,1/9,1/10)	(0,0,0;0,0,0;0,0,0)
(0,0,0;0,0,0;0,0,0)	x_5	(1/2,2/3,5/7;2/7,3/7,4/7;5/7,6/7,1)	(0,0,0;0,0,0;0,0,0)	(0,0,0;0,0,0;0,0,0)	(8/7,2,11/4;7/4,8/4,9/4;10/4,11/4,3)	(1,1,1;1,1,1;1,1,1)
	zj	(4,20/3,55/7;56/7,1,2;3,4,5)	(8,10,11;12,13,14;15,16,17)	(0,0,0;0,0,0;0,0,0)	(8/7,2,11/4;7/4,8/4,9/4;10/4,11/4,3)	(0,0,0;0,0,0;0,0,0)
	Cj-z	(0,-2/3,-6/7;-7/8,-1,-2;-3,-4,-5)	(0,0,0;0,0,0;0,0,0)	(0,0,0;0,0,0;0,0,0)	(-2,-5/3,-11/7;-6,-7,-8;-9,-10,-11)	(0,0,0;0,0,0;0,0,0)

It is noted that in the last row all the coefficients are ≤ 0so the stop condition is fulfilled. The solution is optimal as $C_j - Z_j \leq 0$ for all j. Hence the required solution is; $x_1 = (0,0,0; 0,0,0; 0,0,0)$ and $x_2 = (25, 50/3\,, 20; 22,23,24; 25,26,27)$

5. Conclusion

In this paper the concept on Neutrosophic fuzzy number is defined. Arithmetic operations of a particular Neutrosophic fuzzy number are also addressed. The procedure of solving simplex problem using Neutrosophic fuzzy number may help us to solve many optimization problems. Our approaches and computational procedures may be efficient and simple to implement for calculation in a fuzzy environment for all fields of engineering and science where impreciseness occur.

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