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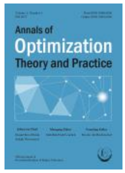

On discounted discrete scheduled replacement model

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Abstract This paper gives a chance of incorporating a discounting rate with discrete scheduled replacement model involving minor repair. An operating unit sometimes cannot be replaced at the exact optimum replacement time for some reasons. The unit may be rather replaced at idle times, such as a day, a week, a month, a year and so on. To address such problem of replacing a unit at idle times, this paper come up with an explicit expression of a discounted discrete scheduled replacement model for a unit subjected to three categories of failures (which are Category I, Category II and Category III failures). It is assumed that the replacement is at scheduled times $NT(N = 1, 2, 3, \ldots)$ for a fixed $T > 0$, is such that the model involves minimal repair and discounting rate $(\propto > 0)$. Finally, a numerical example is given to illustrate the theoretical results of this model.

Keywords Category; Discounted; Discrete; Optimal; Repair

1. Introduction

The failures of an operating units might sometimes be costly or dangerous. It is an important problem to determine when to replace or preventively maintain a unit before failure. Because such failures have negative effect on revenue, production of defective items and causes delay in customer services. A situation may arise, such that an operating unit sometimes cannot be replaced at the exact optimum replacement times due to some reasons like shortage of spare units and lack of workers. At such incidences, units may be rather replaced in idle times, such as weekend, month-end or year-end. For instance, the tires of fighter jets are replaced preventively at a required number of times of flights, which may depends on the type of uses. Briš *et al.* (2017) described a new method for optimal maintenance strategy of a complex system respecting a given reliability constraint. Coria *et al.* (2015) proposed an analytical optimization method for preventive maintenance (PM) policy. Chang (2014) considered a system which suffers one of two types of failures based on a specific random mechanism. Cha and Finkelstein (2018) suggested the new type of minimal repair to be called conditional statistical minimal repair, and their approach goes further and deals with the corresponding minimal repair processes for systems operating in a random environment. Fallahnezhad and Najafian (2016) studied the best time for performing preventive maintenance operations for many systems. Jain and Gupta (2013) studied optimal replacement policy for a repairable system with multiple vacation and imperfect coverage. Enogwe *et al.* (2018) used the knowledge of probability distribution of failure times and proposed a replacement model for items that fail suddenly. Lim *et al.* (2016) studied age replacement policies in which a system is replaced by new one at planned age and when failure occurs before the planned replacement age, it can be either perfectly repaired with random probability p or minimally repaired with random probability $1-p$. Liu *et al.* (2018) established uncertain reliability mathematical models of simple repairable series systems, simple repairable parallel systems, simple repairable series-parallel systems, and simple repairable parallel-series systems, respectively. Malki *et al.* (2015) investigated on age replacement policies for a parallel system with stochastic dependence. Murthy and Hwang (2007) discussed that, the failures can be reduced (in a probabilistic sense) through effective maintenance actions, and such maintenance actions can occur either at discrete time instants or continuously over time. Nakagawa (2005) presented a modified standard age replacement (SAR) model to discrete time age replacement model. Rebaiaia and Ait-kadi (2020) studied the problem of selecting the best among three maintenance strategies for conducting maintenance planning that are the most economical. Safaei *et al.* (2018) investigated the optimal period for preventive maintenance and the best decision for repair or replacement in terms of some measures. Sudheesh *et al.* (15) considered the discrete age-replacement model, and then studied the properties of mean time to failure (MTTF) of a system. Tsoukalas and Agrafiotis (16) presented a new replacement policy for a system with correlated failure and usage time. Waziri *et al.* (17) constructed some discounted age replacement models with minimal repair for a series system, which is subjected to two types of failures. Waziri *et al.* (18) investigated some characteristics of the age replacement model with minimal repair for a series-parallel system with six units, such that the six units are having non-uniform failure rates. Xie *et al.* (2019) analyzed the impacts of cascading failures on the reliability of series-parallel systems, where they studied the effects of safety barriers on preventing cascading failures, and evaluated the importance of the safety barriers. Yaun and Xu (2011) studies a cold standby repairable system with two different components and one repairman taking multiple vacations. Yusuf and Ali (2012) considered two parallel units in which, both the units operate simultaneously. Yusuf *et al.* (2015) modified the standard age replacement model by introducing random working time Y. Zaharaddeen and Bashir (2014) studied on optimal repair replacement policies with two types of failures. Zhao *et al.* (2014) answered the problem which replacement is better between continuous and discrete scheduled replacement times. Zhao *et al.* (2017) collected some recent results on age replacement policies.

This paper tends to extend the proposed discrete age replacement model presented by Nakagawa (11). The objectives of this paper are : (1) to come up with the assumptions that describe the unit; (2) to come up with the expression of failure rate and reliability

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function of the unit based on the assumptions; (3) to come up with the discounted discrete scheduled replacement model of the unit ; (4) to provide simple numerical example, to test the system. The main contributions of this paper are: (1) to provide chance of replacing a unit at ideal time; (2) to investigate characteristics of a discounted discrete age replacement model involving minimal repair; (3) to investigate the effect of discounting rate ($\propto > 0$) in obtaining optimal discrete scheduled replacement time of a unit. Furthermore, the remainder of this paper is organized as follows: Section 2 discussed the methodology of the study; Section 3 discussed the proposed models; Section 4 presents the numerical example; Section 5 presents the discussion of results obtained; Section 6 discussed the conclusion.

2. Methodology

2.1. Notations

- $\propto$: Discounting rate.
- $r_1(t)$: Rate of Category I failure of the unit.
- $r_2(t)$: Rate of Category II failure of the unit.
- $r_3(t)$: Rate of Category III failure of the unit.
- $R_1(t)$: Reliability function of the unit due to Category I failure.
- $F_1(t)$: Category I failure distribution of the unit.
- C_2 : Cost of minimal repair of the unit due to Category II failure.
- C_3 : Cost of minimal repair of the unit due to Category III failure.
- C_p : Cost of planned replacement of the unit at NT, for $N = 1, 2, 3$ ....
- C_r : Cost of un-planned replacement of the unit due to Category I failure.
- N^*: Optimal discrete scheduled replacement time of the unit.

2.2. Description of the unit

Consider a unit which is subjected to three Categories of failures, which are Category I failure, Category II failure and Category III failure. Category I failure is catastrophic failure, which occurs suddenly and it is un-repairable failure. Category II failure and Category III failure occurs gradually with time and usage, but the two failures (Category II and Category III failures). The rate of arrival of Category III failure is higher than that of Category II failure. The rate of arrival of Category II failure is higher than that of Category I failure. The unit is subjected to total and instant replacement action, whenever it reaches planned replacement time NT ($N = 1, 2, 3, \dots$) for a fixed T. To include the discounting factor (α) in the replacement model, $e^{-\alpha(NT)}$ will be included for each of an associated cost of replace/repair of a type of failure of the unit, where α is fixed.

2.3. Assumptions

1. Category I failure is un-repairable one, while Category II and Category III failure are both repairable failures.
2. The cost of replacement/minimal repair follows the order : $C_r > C_p > C_2 > C_3$.
3. All the three failures are detected instantaneously.

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4. All required resources are available when needed, which means that replacement/minimal repair.
5. If the unit failed due to Category I failure, the unit will be replaced completely with new one.
6. If the unit failed due to Category II failure, then the unit is minimally repair, and allow the unit to continue operating from where it stopped.
7. If the unit failed due to Category III failure, then the unit is minimally repair, and allow the unit to continue operating from where it stopped.
8. The unit is replaced at scheduled time $NT(N = 1, 2, 3 \dots)$ for a fixed T after its installation or Category I failure of the unit, whichever occurs first.
9. The repair time for both Category I and Category II failures is negligible.

3. The discounted replacement model

Based on the assumptions, the reliability function of the unit with respect to Category I failure is

$$R_1(NT) = e^{-\int_0^{NT} r_1(t)dt} \tag{1}$$

Based on the assumptions, the cost of un-scheduled replacement of the unit in one replacement cycle is

$$\int_0^{NT} C_r e^{-\alpha(t)} dF_1(t) \tag{2}$$

Based on the assumptions, the cost of scheduled replacement of the unit at time NT in one replacement cycle is

$$C_p e^{-\alpha(NT)} R_1(NT) \tag{3}$$

Based on the assumptions, the cost of minimal repair of the unit due to Category II failure in one replacement cycle is

$$\int_0^{NT} C_2 r_2(t) e^{-\alpha(t)} R_1(t)dt \tag{4}$$

Based on the assumptions, the cost of minimal repair of the unit due to Category III failure in one replacement cycle is

$$\int_0^{NT} C_3 r_3(t) e^{-\alpha(t)} R_1(t)dt \tag{5}$$

Based on the assumptions, the mean of one replacement cycle is

$$\int_0^{NT} \alpha e^{-\alpha(t)} R_1(t)dt \tag{6}$$

Adding up equation (1) to equation (6), the discounted discrete scheduled replacement model of the unit in one replacement cycle is

$$C(N) = \frac{C_p e^{-\alpha(NT)} R_1(NT) + \int_0^{NT} C_r e^{-\alpha(t)} dF_1(t) + \int_0^{NT} C_2 e^{-\alpha(t)} R_1(t) r_2(t) dt + \int_0^{NT} C_3 e^{-\alpha(t)} R_1(t) r_3(t) dt}{\int_0^{NT} \alpha e^{-\alpha(t)} R_1(t) dt} \quad (7)$$

Using the expressions: $F_1(t) = 1 - R_1(t)$ and $dF_1(t) = r_1(t)R_1(t)dt$, equation (7) can be written as

$$C(N) = \frac{C_p e^{-\alpha(NT)} R_1(NT) + \int_0^{NT} C_r e^{-\alpha(t)} r_1(t) R_1(t) dt + \int_0^{NT} C_2 e^{-\alpha(t)} R_1(t) r_2(t) dt + \int_0^{NT} C_3 e^{-\alpha(t)} R_1(t) r_3(t) dt}{\int_0^{NT} \alpha e^{-\alpha(t)} R_1(t) dt} \quad (8)$$

The discounted discrete scheduled replacement model (equation (8)) is simplify to

$$C(N) = \frac{C_p e^{-\alpha(NT)} R_1(NT) + \int_0^{NT} K(t) R_1(t)\, e^{-\alpha(t)} dt}{\int_0^{NT} \alpha e^{-\alpha(t)} R_1(t) dt} \quad (9)$$

Where

$$k(t) = C_r r_1(t) + C_2 r_2(t) + C_3 r_3(t) \quad (10)$$

Noting that, C(N) is adopted as the objective function of an optimization problem, and the aim is to determine an optimal replacement number N^*that minimizes $C(N)$.

4. Numerical example

Let the rate of Category I, Category II and Category III failures of the unit follows Weibull distribution:

$$r_i(t) = \lambda_i \propto_i t^{\propto_i - 1} \quad \text{for } i = 1, 2, 3, \quad (11)$$

where $\propto_i > 1$ and $t \geq 0$.

Let the set of parameters and cost of repair/replacement be used throughout this particular example:

1. $\propto_1 = 2$, $\propto_2 = 3$, $\propto_3 = 3$.
2. $\lambda_1 = 0.0002$, $\lambda_1 = 0.04$, $\lambda_3 = 0.02$.
3. $C_r = 50$, $C_p = 40$, $C_2 = 3$, $C_3 = 1.5$.

By substituting the parameters in equation (11), the failure rates of Category I, Category II and Category III as follows :

$$r_1(t) = 0.0004t \quad (12)$$

and

$$r_2(t) = 0.12t^2 \quad (13)$$

and

$$r_3(t) = 0.06t^3 \quad (14)$$

Table 1 below is obtained by substituting the assumed cost of replacement/repair ($C_r = 50, C_p = 40,\ C_2 = 3,\ C_3 = 1.5$) and rates of Category I, Category II and Category III failures (equations (12), (13) and (14)) in the total replacement cost rate of the unit (equation (10)) where T=1 and different values of discounting rate ($\alpha = 0.01, \alpha = 0.05, \alpha = 0.1, \alpha = 0.15$ and $\alpha = 0.2$). While table 2 is obtained by substituting the assumed cost of replacement/repair ($C_r = 50,\ C_p = 40,\ C_2 = 3,\ C_3 = 1.5$) and rates of Category I, Category II and Category III failures in the total replacement cost rate of the unit, where α =1 and different values of T ($T = 1, T = 2, T = 3, T = 4$ and $T = 5$).

Table 1. The values of C(N) versus $N\ (1, 2, 3 \dots)$ with different values of discounting factor (α), when $T = 1$.

N	C(N) with α=0.01	C(N) with α=0.05	C(N) with α=0.1	C(N) with α=0.15	C(N) with α=0.2
1	39873.107	7814.353	3807.13	2471.56	1803.95
2	20096.804	3857.069	1827.59	1151.77	814.59
3	13801.857	2592.478	1192.44	727.30	496.38
4	10985.513	2018.229	899.461	529.38	347.33
5	9644.837	1731.965	746.5227	422.79	265.75
6	9110.050	1598.002	664.9536	361.45	217.00
7	9092.142	1556.863	624.3298	324.94	185.86
8	9444.784	1577.671	608.6883	302.67	164.60
9	10084.963	1642.29	608.6077	288.59	148.98
10	10961.238	1738.983	618.0439	279.00	136.56
11	12039.312	1859.541	632.8908	271.61	125.94
12	13294.817	1997.849	650.2568	264.98	116.33
13	14709.440	2149.119	668.0691	258.25	107.31
14	16268.696	2309.456	684.8398	250.93	98.69
15	17960.608	2475.599	699.5171	242.81	90.39

Table 2. The values of C(N) versus N $(1, 2, 3 \dots)$ with different values of T, when $\propto = 0.01$.

N	C(N) with T=1	C(N) with T=2	C(N) with T=3	C(N) with T=4	C(N) with T=5
1	39873.107	20096.8	13801.857	10985.513	9644.837
2	20096.804	10985.51	9110.050	9444.784	10961.238
3	13801.857	9110.05	10084.963	13294.817	17960.608
4	10985.513	9444.784	13294.817	19774.876	28082.804
5	9644.837	10961.24	17960.608	28082.804	40275.090
6	9110.050	13294.82	23734.634	37713.010	53678.392
7	9092.142	16268.7	30384.074	48225.027	67464.851
8	9444.784	19774.88	37713.010	59192.881	80826.780
9	10084.963	23734.63	45537.744	70196.441	93004.464
10	10961.238	28082.8	53678.392	80826.780	103327.294
11	12039.312	32760.76	61956.662	90697.496	111257.053
12	13294.817	37713.01	70196.441	99458.636	116425.105
13	14709.440	42885.55	78225.827	106811.143	118656.500
14	16268.696	48225.03	85879.988	112520.084	117976.004
15	17960.608	53678.39	93004.464	116425.105	114594.122

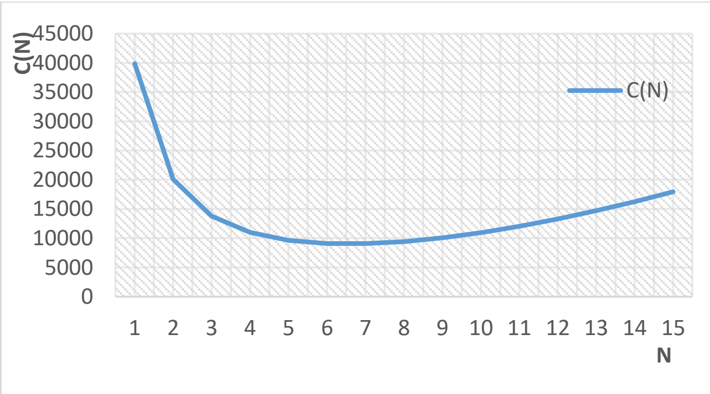

Figure 1. The plot of C(N) versus N, when T=1 and α=0.01

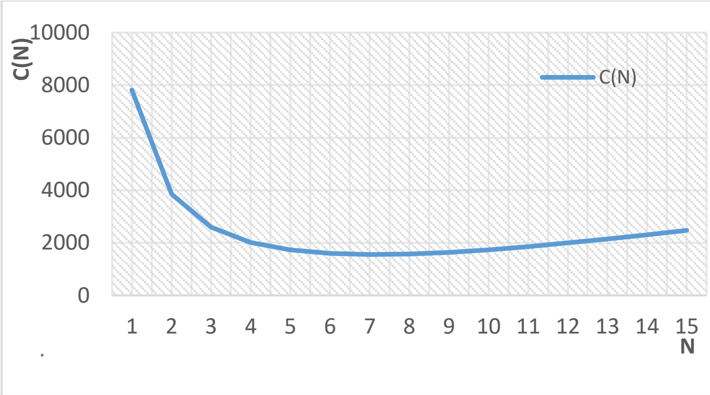

Figure 2. The plot of C(N) versus N, when T=1 and α=0.05

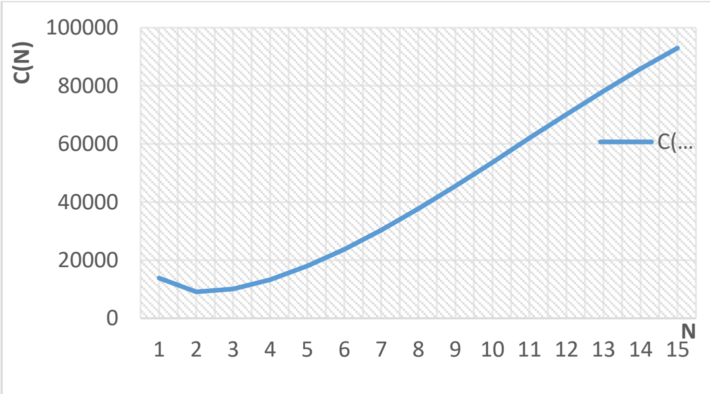

Figure 3. The plot of C(N) versus N, when T=1 and α=0.1.

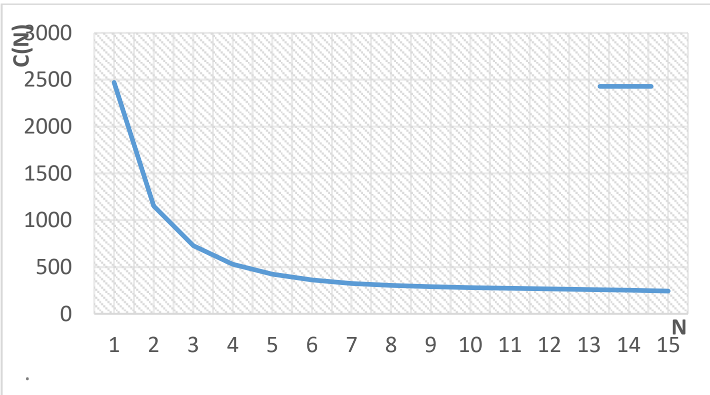

Figure 4. The plot of C(N) versus N, when T=1 and $\alpha = 0.15$

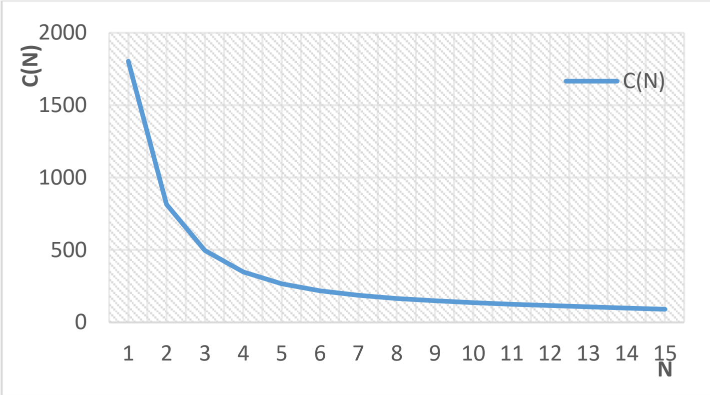

Figure 5. The plot of C(N) versus N, whenT=1 and α=0.2.

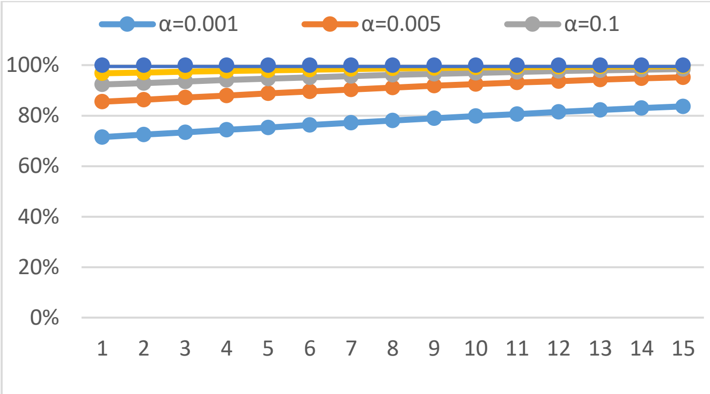

Figure 6. Comparing C(N) for α=0.01, α=0.05, α= 0.1, α=0.15, α=0.2.

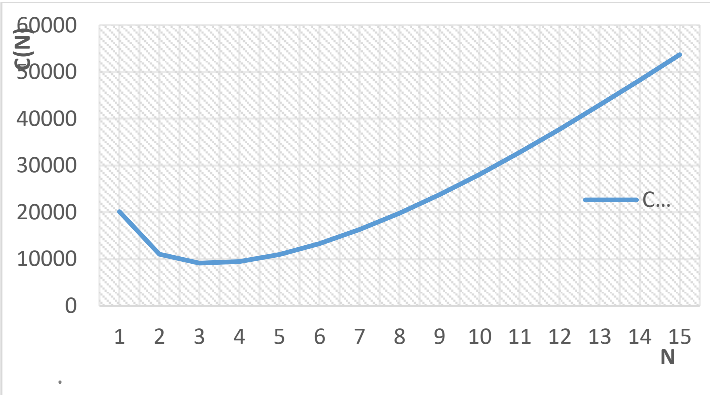

Figure 7. The plot of C(N) versus N, when T=2 and ∝=0.001

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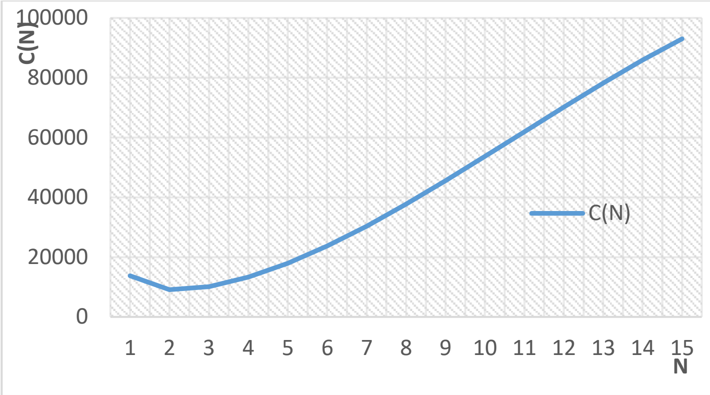

Figure 8 . The plot of C(N) versus N, when T= 3 and ∝=0.001.

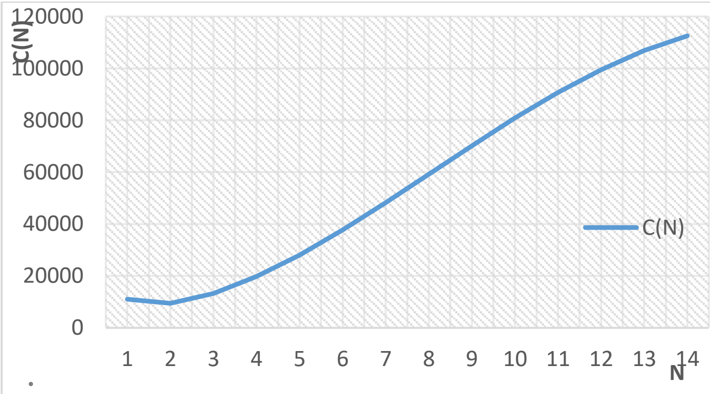

Figure 9. The plot of C(N) versus N, when T=4 and α=0.001

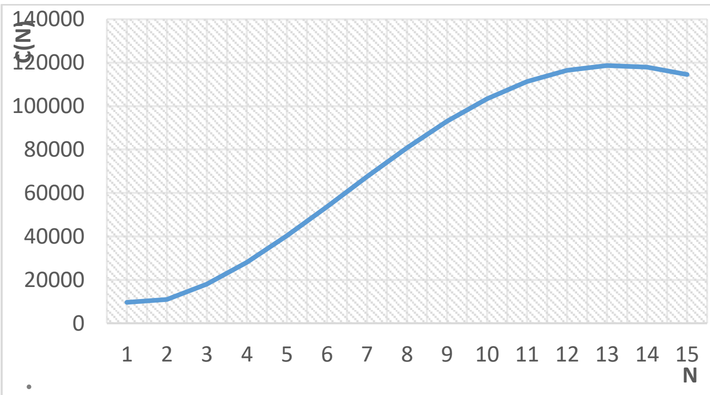

Figure 10. The plot of C(N) versus N, when T=5 and α=0.001

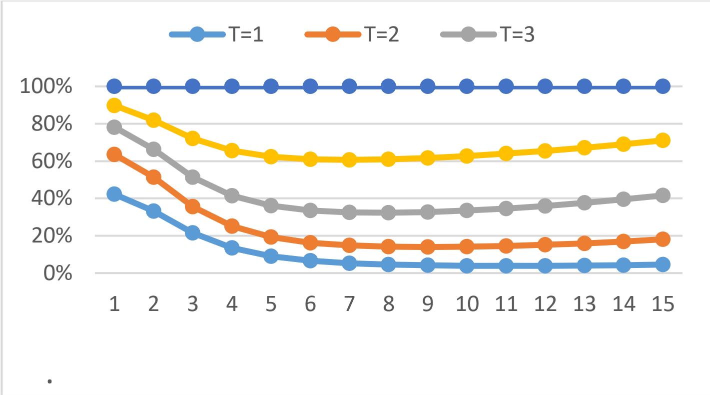

Figure 11. Comparing the values of C(N), for T=1, T=2, T=3, T=4, T=5

The following are some of the observations from the results obtained in the numerical example provided above:

- Observe from Table 1, the optimal discrete scheduled replacement time of the unit is $N^* = 7$, with cost rate $C(N^* = 7) = 9092.142$, when $T = 1$ and $\propto = 0.01$. See Fig. 1 below for the plot of $C(N)$ versus N, when $T = 1$ and $\propto = 0.01$.
- Observe from Table 1, the optimal discrete scheduled replacement time of the unit is $N^* = 7$, with cost rate $C(N^* = 7) = 1556.863$, when $T = 1$ and $\propto = 0.05$. See Fig. 2 below for the plot of $C(N)$ versus N, when $T = 1$ and $\propto = 0.05$.
- Observe from Table 1, the optimal discrete scheduled replacement time of the unit is $N^* = 9$, with cost rate $C(N^* = 9) = 1556.863$, when $T = 1$ and $\propto = 0.1$. See Fig. 3 below for the plot of $C(N)$ versus N, when $T = 1$ and $\propto = 0.5$.
- Observe from Table 1, the optimal discrete scheduled replacement time of the unit (N^*) is very large when $T = 1$ and $\propto = 0.15$. See Fig. 4 below for the plot of $C(N)$ versus N, when $T = 1$ and $\propto = 0.15$.
- Observe from Table 1, the optimal discrete scheduled replacement time of the unit (N^*) is very large when $T = 1$ and $\propto = 0.2$. See Fig. 5 below for the plot of $C(N)$ versus N, when $T = 1$ and $\propto = 0.2$.
- Observe from Fig.6, as the discounting rate $(\propto)$ increases, the cost rate $(C(N))$ also increases. That is, $(C(N), \propto = 0.01) < (C(N), \propto = 0.05) < (C(N), \propto = 0.1) < (C(N), \propto = 0.15) < (C(N), \propto = 0.2)$.
- Observe from Table 2, the optimal discrete scheduled replacement time of the unit is $N^* = 3$, with cost rate $C(N^* = 3) = 9110.05$, when $T = 2$ and $\propto = 0.01$. See Fig. 7 below for the plot of $C(N)$ versus N, when $T = 2$ and $\propto = 0.01$.
- Observe from Table 2, the optimal discrete replacement time of the unit is $N^* = 2$, with cost rate $C(N^* = 2) = 9110.050$, when $T = 3$ and $\propto = 0.01$. See Fig. 8 below for the plot of $C(N)$ versus N, when $T = 3$ and $\propto = 0.01$.
- Observe from Table 2, the optimal discrete scheduled replacement time of the unit is $N^* = 2$, with cost rate $C(N^* = 2) = 9444.784$, when $T = 4$ and $\propto = 0.01$. See Fig. 9 below for the plot of $C(N)$ versus N, when $T = 4$ and $\propto = 0.01$.

- Observe from Table 2, the optimal discrete scheduled replacement time of the unit is $N^* = 1$, with cost rate $C(N^* = 2) = 9644.837$, when $T = 5$ and $\propto = 0.01$. See Fig. 10 below for the plot of $C(N)$ versus N, when $T = 5$ and $\propto = 0.01$.
- Observe from Fig.11, as the value of T increases, the cost rate $(C(N))$ also increases. That is, $(C(N), T = 1) < (C(N), T = 2) < (C(N), T = 3) < (C(N), T = 4) < (C(N), T = 5)$.
- Observe all the curves of Fig. 1, Fig. 2, Fig. 3, Fig. 4, Fig. 5, Fig. 7, Fig. 8 and Fig. 9 are in full convex or partial convex shape, while Fig. 10 is in s-shape.

5. Discussion of the results obtained

In trying to evaluate the characteristics of the discrete age replacement model, one can clearly see from Table 1 obtained that, as the discounting rate $(\propto)$ increases, the optimal discrete scheduled replacement time of the unit (N^*) increases. Also, one can clearly see from Table 1, as the discounting rate $(\propto)$ increases, the average cost rate $C(N)$ also increases. Therefore, one can say that, the discounting rate $(\propto)$ has an effect in determining the optimal discrete scheduled replacement time of a unit. On the other hand, one can also clearly see from Table 2, as the value of T increases, the optimal discrete scheduled replacement time of the unit (N^*) decreases. While as the value of T increases, the average cost rate $C(N)$ also increases. Therefore, one can see that, as the choice of T is small, the optimal discrete scheduled replacement time of the unit (N^*) will be bigger, than when the choice of T is small.

6. Conclusion

This paper investigated the characteristics of discrete scheduled replacement model with minor repair. And also this paper evaluated the effect of discounting rate $(\propto)$ when it is involved in the construction of the discrete scheduled replacement model. It is assumed that, Category I failure is an un-repairable one, which occurs suddenly. While Category II and Category III failures are repairable failures, which occurs due to time and usage. Some numerical examples where provided to present the characteristics of the model constructed. Finally, the results obtained showed that, the discounting rate have an effect on the discrete scheduled replacement model in determining the optimal discrete scheduled replacement time of a unit or component. Also, the results showed the characteristics of the discrete scheduled replacement model. This paper is important to engineers, maintenance managers and plant management in replacing analog units at idle times, such as weekend, month-end or year-end.

For further studies, one can investigate the following suggested cases: (i) this paper can be extended to multi-component systems; (ii) this paper can also incorporate warranty period; (iii) this paper can also consider the repair time of a failure not negligible.

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