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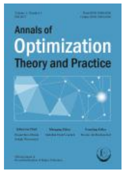

On the control of a dynamical system defined by a decreasing one-dimensional set-valued function

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Abstract Agricultural production can be described by discrete time as there is harvest in every year only once. The agricultural production is uncertain because of the weather and the ever changing technology. At the same time, the sector prefers stability which is reflected in the small changes in the prices. The uncertainty of the price may be modeled by a set-valued function in a single product market. The independent variable is the price expectation of the producer which is the future value of the price estimated by the producer. It can be assumed that the set-valued function is decreasing because in the case of higher price expectation, greater quantity appears on the market and thus the real market price becomes the lower. The stability of the market may require some control. In this paper the existence of an appropriate control to reach a target interval and to keep the trajectory in the interval is investigated from mathematical point of view. Necessary and sufficient conditions are given for the existence of the viable solution. The "striped structure" of the dynamical system is explored as well.

Keywords Set-valued function; Dynamical system; Control; Target interval; Viable solution.

1. Introduction

This paper is devoted to the analysis of certain discrete time dynamical systems defined by one-dimensional set-valued functions. The main motivation is the potential application in economics.

The produced quantity on arable land is uncertain because of the effect of the weather. The future price of a commodity is always uncertain as well, because it depends on the produced quantity and other factors. In most of the cases uncertainty is modeled by

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probability, see e.g. Flåm and Kaniovski (2002). Sometimes the system changes fast way and therefore it is not possible to learn its probabilistic properties if they exist at all. This is typical for agricultural markets where the parameters of the system do not remain the same from one harvest to the other one because of the ever improving technology. Therefore in modeling such a system one may use mathematical tools different from probability. One such tool is the set-valued function. Kovács *et al.* (2002) discusses a dynamical system defined by a setvalued function.

The mathematical model discussed in this paper assumes that the product in question has a seasonal production. Agriculture is again typical example of this. However, even industrial goods, e.g. jet ski and snowmobile, may have the same property. However, even the demand has a significant seasonal effect in the case of industrial products. For the sake of simplicity the market is supposed to be an agricultural one and its dynamics is described accordingly. It implies a smooth demand and no serious changes in the price during the whole time period. The change of the price takes place typically between time periods.

The basic dynamics of the market from one year to the next one is discussed in general in the frame called cobweb model. The harvested quantity is known for producers and customers in each year. This is the quantity which appears on the market until the next harvest. The equilibrium price of the market closely related to the harvested quantity. The higher the quantity is the lower the price is. Assume that the farmer can produce several different crops. The producer estimates the market price of the next year and according to the estimation he decides on the use of the arable land. That means at the same time a decision on the quantities produced in the next year. If the estimated price is higher, then larger area is used for the crop. The larger area implies that higher quantity will be produced and if higher quantity appears on the market then the equilibrium price becomes smaller. The final conclusion is that the market price is a decreasing function of the estimated price. The strategy of the famous financial tycoon George Soros is that he acts in an opposite way how the expectation of the majority suggest Freeland (2009). This type of effective (price) expectation goes back to Keynes (1963) and Kovács *et al.* (2001).

Generally, the methods to estimate price expectations are using only prices of the past. This fact leads to such phenomena. Typical cases are the adaptive price expectation of Nerlove (1958) and the extrapolative expectation of Szidarovszky and Molnár (1994). There are indications that the farmers do not estimate future prices numerically Imre and Tibor (1982), or make significant and skewed errors Kenyon (2001). Intensive research on price expectations is still going on Ahmad (2015), Dr. Agrarwissenschaften (2015), Kenyon (2001), Kovács *et al.* (2001) and Masuku *et al.* (2017). The price expectation determines the dynamic properties of the market Bacsi and Vizvári (1999).

In this paper a new approach is discussed, which more or less faces with all of the aforementioned difficulties: no stochastic distribution is known although the system is uncertain, the estimation is not a well-defined single numeric value. The new approach

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is to use dynamical system defined by a decreasing set-valued function as the model of a single commodity market.

2. Basic Assumptions

It is supposed that the dynamical system can be controlled. The basic properties of the system concerning control are summarized in seven assumptions.

The first four assumptions are describing the system in general, the next two ones concern to the control, and the last one excludes some degenerate cases.

(A1) The state of the system is a real number.

(A2) The time is discrete and it takes its value t from the set $\mathbb{Z}_+ = \{0.1.2. \ldots\}$.

(A3) Assume that the state of the system is x_t at time t. Then the state at time $t+1$ is an element of the set $G(x_t)$ if the control is switched off. $G: \mathbb{R} \to \mathbb{R}$ is a set-valued function and for all $x \in \mathbb{R} : G(x) = [a(x).b(x)]$, where $a(\cdot)$, and $b(\cdot)$ are real functions.

(A4) The functions $a(\cdot)$, and $b(\cdot)$ satisfy the conditions

1. $\forall\, x \in \mathbb{R}$: $a(x) \leq b(x)$.

2. Both $a(\cdot)$, and $b(\cdot)$ are continuous on the whole real line. It follows that the set-valued function G is continuous.

3. Both $a(\cdot)$, and $b(\cdot)$ are strictly decreasing on the whole real line.

(A5) The control is an additive term, say u_{t+1} in time t. Thus the state of the system in the case when control is switched on is

$$x_{t+1} = x'_{t+1} + u_{t+1} \in G(x_t) + u_{t+1} \tag{1}$$

i.e. x'_{t+1} an element of $G(x_t)$, becomes known only in time period $t+1$.

(A6) The value of the control may vary from period to period, however, it must belong to the interval $[d^-.\ d^+]$, where $d^- < 0 < d^+$.

(A7) The functions

$g(x) = b(a(x) + d^-) + d^+$,

and

$h(x) = a(b(x) + d^-) + d^+$

have only finite many fixed points. (Notice that $g(x)$ is the largest point, which might be reached by the trajectory at all from x in exactly two iterations, while $h(x)$ is the minimal point, which can be reached using control for sure in exactly two steps.)

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Equation (1) does not define the control uniquely. It is important to distinguish two types of control. It is called a priori if the control, i.e. u_{t+1}, is determined first and after that x'_{t+1} becomes known. The opposite case, when u_{t+1} can be determined after that x'_{t+1} becomes known is called a posteriori. Notice that the controller has no way to affect x'_{t+1}. In this paper the main emphasize is given to the a posteriori case. The value x'_{t+1} is called the realization of the system at time period $t + 1$.

It is also assumed that the starting point is known and it is x_0. The aim of the control is to reach the target interval $[V. W]$ in finite many steps, and keep the system in it, i.e. our goal is to give conditions for $[V. W]$ to be a viable domain by control. We suppose that

$x_0 < V < W.$

The following notation is used. Let f be an arbitrary real function. Then

$f^+(x) = f(x) + d^+$, and $f^-(x) = f(x) + d^-$.

The theorems use the combinations of the two extreme realizations $a(x)$, $b(x)$ and the two extreme control d^-, d^+. For example $a^-(x)$ means the minimal autonomous growth $a(x)$ and the minimal control d^-.

Then the functions g, and h in Assumption (A7) can be written in the following short form:

$g(x) = b^+(a^-(x))$, and $h(x) = a^+(b^-(x))$.

It is easy to see, that both g and h are strictly increasing.

3. On the Existence of Trajectories

There are cases when the goal of the control cannot be achieved. Therefore, the first issue to be discussed is the existence of the desired trajectories.

Theorem 1. No controlled trajectory can enter into the $[V. +\infty)$ interval in finite many steps if and only if $b^+(x_0) < V$ and there exists a point

$y \in [min\{x_0. b^+(x_0)\}. V]$

such that $g(y) \leq y$.

Proof. The necessity of the inequality $b^+(x_0) < V$ is obvious. The possible maximal value of a trajectory in step 1 is $b^+(x_0)$. Thus if the inequality does not hold then any trajectory, which has this value at step 1 enters the target interval.

In general we determine the possible maximal and minimal values of the trajectories at step t.

(a) If $t = 0$ then the only possible value is $x_0 < V$.

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(b) If $t = 1$ then
$x_1 \in G(x_0) + [d^-. \ d^+] = [a(x_0) + d^-. \ b(x_0) + d^+] = [a^-(x_0). b^+(x_0)]$.

(c) Let $t \geq 2$. At time t the system has the state x_t. The minimal and maximal value where the trajectory can step from it are $a^-(x_t)$ and $b^+(x_t)$, respectively. Hence it follows from the strictly monotone decreasing property of the functions $a(\cdot)$, and $b(\cdot)$ that we obtain the possible absolute minimal (maximal) value for x_t if x_{t-1} is the possible absolute maximal (minimal) value at step $t-1$. It means that if

$$x_2 \in [a^-\big(b^+(x_0)\big). b^+(a^-(x_0))] = [a^-\big(b^+(x_0)\big). g(x_0)]$$

and

$$x_3 \in [a^-\big(g(x_0)\big). b^+(a^-(b^+(x_0)))] = [a^-\big(g(x_0)\big). g(b^+(x_0))]$$

and in general for even t, say $t = 2k$,

$$x_{2k} \in [a^-\left(g^{k-1}\big(b^+(x_0)\big)\right). g^k(x_0)].$$

and for odd t, say $t = 2k+1$,

$$x_{2k+1} = [a^-\big(g^k(x_0)\big). g^k(b^+(x_0))].$$

For even values of t the possible absolute maximal values of the trajectories are

$$x_0. g(x_0). \dots . g^k(x_0). \dots .$$

This sequence is defined by the recursive equation

$$v_0 = x_0. \quad v_k = g(v_{k-1}).$$

First assume that the function $g(\cdot)$ does not have a fixed point. If for all real number z the inequality $z > g(z)$ holds then the sequence is strictly decreasing and converges to $-\infty$. Thus the trajectory is unable to enter the target interval, and at the same time the necessary and sufficient condition given in the statement holds. If the opposite inequality, i.e. $z < g(z)$, holds for all real numbers, then the sequence is strictly increasing and converges to $+\infty$. Thus the trajectory enters the target interval and the necessary and sufficient condition does not hold. If $g(\cdot)$ has at least one fixed point then there are the following cases:

(i) x_0 itself is a fixed point and the sequence is constant.

(ii) All of the fixed points are strictly less than x_0. If $g(x_0) < x_0$ then the sequence decreasing and converges to the maximal fixed point $x_f^{\max} < x_0$ and never enters the target interval. At the same time x_0 is appropriate for the value y of the statement. If $g(x_0) > x_0$ then the sequence is increasing and converges to $+\infty$ and the appropriate value y does not exists.

(iii) All fixed points are strictly greater than x_0. If $g(x_0) < x_0$ then the sequence decreasing and converges to $-\infty$. Again $y = x_0$ is a good choice. If $g(x_0) > x_0$ then the sequence is increasing and converges to the minimal fixed point $x_f^{\min} > x_0$. If $x_f^{\min} > V$ then the sequence enters the target interval and appropriate y does not exist. If $x_f^{\min} \leq V$ then the sequence does not reach the target interval in finite many steps and $y = x_f^{\min}$.

(iv) x_0 is between two fixed points, say x_f^l and x_f^u, such that $x_f^l < x_0 < x_f^u$ and there is no other fixed point in the interval $[x_f^l . x_f^u]$. Then the trajectory converges to x_f^l if $g(x_0) < x_0$ and $y = x_0$ is a good choice. If $g(x_0) > x_0$ then the sequence converges to x_f^u. If $x_f^u \leq V$ then the trajectory does not enter the target interval in finite many cases and $y = x_f^u$ is a good choice. Otherwise, i.e. if $x_f^u > V$, the trajectory enters the target interval and no appropriate y exists.

Hence one can obtain the conclusion that the trajectory cannot enter the target interval in even numbered steps if and only if the statement is true.

For odd values of t the sequence is

$$b^+(x_0). g\big(b^+(x_0)\big). \dots. g^k\big(b^+(x_0)\big). \dots$$

If $b^+(x_0) \geq V$ then there are trajectories entering the target interval. If $b^+(x_0) < V$ then the proof is similar to the case of even t's.

The next theorem discusses the viability of the interval $[V. \infty]$. The proof provides us with the necessary control, too.

Theorem 2. In the a posteriori case it is possible to choose the control for all trajectories such that the trajectory enters the interval $[V. +\infty]$ in finite many steps and it stays there forever if and only if

$$\forall\, y \in [x_0 . V]: h(y) \;>\; y \qquad and \qquad V \;\leq\; a^+(V). \tag{2}$$

Proof. First assume that (2) holds. Then an appropriate control is

$$u_{t+1} \;=\; \begin{cases} \min\{d^+ . V - x'_{t+1}\} & if \quad x'_{t+1} \leq V \\ \\ \max\{d^- . V - x'_{t+1}\} & if \quad x'_{t+1} > V. \end{cases} \tag{3}$$

Notice that u_{t+1} is determined such that if

$$V - d^+ \;\leq\; x'_{t+1} \;\leq\; V - d^-$$

then x_{t+1} becomes V. Outside this interval u_{t+1} controls the trajectory with maximal possible speed toward V.

It follows from the strictly decreasing property of function $a(\cdot)$, and the condition (2), and the inequality $x_0 < V$ that

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$V \le a^+(V) < a^+(x_0)$.

Hence starting from x_0 the first point of the trajectory, i.e. x_1, is at least V. It can be greater than V only if $x_1' > V - d^-$. In that case, (3) requires the application of the possible minimal control. Thus x_1 must be an element of the interval

$[V.\max\{V.b^-(x_0)\}]$.

The minimal value of x_2 is obtained if x_1 has its maximal value, i.e. $\max\{V.b^-(x_0)\}$, and the realization x_2' is minimal when the maximal control can be applied. Hence

$$x_2 \ge a^+(\max\{V.b^-(x_0)\}) = \min\{a^+(V).h(x_0)\} \ge \min\{V.h(x_0)\}.$$

Similarly, the maximal value of x_2 is obtained from the minimal value of x_1 with the maximal realization. Then it follows from the control (3) that

$$x_2 \le \max\{V.b^-(V)\}.$$

In a similar way, one can conclude that $x_3 \ge \min\{V.h(V)\}$. In general

$$x_{2k} \ge \min\{V.h^k(x_0)\} \quad \text{and} \quad x_{2k+1} \ge \min\{V.h^k(V)\}.$$

If $h(y) > y$ holds for all $y \in [x_0.V]$ then any trajectory of the form

$$y.h(y).\dots.h^k(y).\dots$$

must converge either to $+\infty$ or a fixed point of $h(\cdot)$ greater than V. Hence the trajectory with control (3) enters the target interval $[V.+\infty]$ in finite many steps.

At time t the system is at state x_t. There is a unique control such that the system will have the greatest possible state at time $t+2$: at time $t+1$ the smallest possible value must be applied, i.e. d^-, and at time $t+2$ the greatest possible one, i.e. d^+. The worst case of the realizations is $b(x_t)$ at time $t+1$, and $a(x_{t+1})$. Thus the greatest state what can be guaranteed at time $t+2$ is $h(x_t)$. If there is a $y \in [x_0.V]$ such that $h(y) \le y$ then it follows from the previous theorem that the $[V.+\infty)$ interval cannot be reached from x_0 in finite many steps.

If the interval $[V.\infty]$ is not viable then the target interval $[V.W]$ cannot viable either. The next theorem gives the necessary and sufficient conditions of the viability of the target interval.

Theorem 3. In the a posteriori case it is possible to choose the control for all trajectories such that the trajectory enters the $[V.W]$ interval in finite many steps and it stays there forever if and only if there exist a value $V_0 \in [V.W]$ such that

$$\forall y \in [x_0.V_0]: h(y) > y. \quad and \quad V_0 \le a^+(V_0). \quad and \quad b^-(V_0) \le W. \tag{4}$$

Proof. It follows from the previous theorem that if (4) holds then all trajectories can be controlled such that they enter the $[V_0.+\infty)$ interval and stays in it forever. Assume that control (3) is applied with $V = V_0$ and the trajectory just entered the interval $[V_0. +\infty)$, i.e. $x_k < V_0$ if $k < t$. Then it follows from the decreasing property of function $b(\cdot)$ and the definition of control (3) that x_{t+1} is at most $b^-(V_0)$.

Assume in an indirect way that V_0 does not exists, however, for every trajectory there is a control such that the trajectory enters the interval $[V.W]$ and stays there forever. Then it follows from the previous theorem that for all $y \in [x_0.V]$ the inequalities $h(y) > y$, and $V \leq a^+(V)$ hold. Then the indirect assumption is only true if $b^-(V) > W$. As the trajectory does not leave the interval $[V, W]$ for every $x_t \in [V.W]$ and for every realization of $G(x_t)$ there must be a control such that $x_{t+1} \leq W$. As $b(\cdot)$ is decreasing it is necessary that $b^-(W) \leq W$. Hence there is a unique $W_{b^-} \in [V.W]$ such that $b^-(W_{b^-}) = W$ and $W_{b^-} > V$. Here we distinguish two cases:

Case 1: $\forall\, y \in [V.W_{b^-}]$: $h(y) > y$. Hence

$$W_{b^-} < h(W_{b^-}) = a^+(b^-(W_{b^-})) = a^+(W) \leq a^+(W_{b^-}).$$

Then it follows from the equation $b^-(W_{b^-}) = W$ that W_{b^-} satisfies the properties that are claimed from V_0 in the statement.

Case 2: $h(W_{b^-}) \leq W_{b^-}$. Then there is at least one fixed point of function $h(\cdot)$ in the interval $(V.W_{b^-}]$. Let $x_{h.f}$ be the minimal among these fixed points. The function $h(\cdot)$ gives the possible maximal value of the state in every second steps. Let t be iteration when the state of the trajectory is at least V. Let us consider the sequence

$$\{h^{-1}(x_t). x_t. x_{t+2} = h(x_t). x_{t+4} = h(x_{t+2}). \dots\}.$$

It is the trajectory of a dynamic system defined by the (single-valued) function $h(\cdot)$ and started from $h^{-1}(x_t)$. The starting point exist as function $h(\cdot)$ is strictly increasing. A trajectory cannot stay in a fixed point unless it starts from the fixed point itself. Hence the elements of the sequence are never equal to $x_{h.f}$ just only converge to it. Then the trajectory may have a state strictly greater than W in every second step if the realization are provided alternatively from functions $a(\cdot)$, and $b(\cdot)$.

It is not necessary that a trajectory can reach the whole line. In certain cases, the trajectories determine a striped structure of the line as the following theorem shows.

Theorem 4. Assume that $x_0 \in [h_l. h_u]$, where the values h_l and h_u are two consecutive fixed points of function h. Let $p_l = b^-(h_l)$ and $p_u = b^-(h_u)$. Assume that

$$h_l < h_u < p_u < p_l.$$

Then there is a control for all trajectories that the trajectory does not leave the interval $[h_l. p_l]$.

Remarks. It follows from the relation $h_l < h_u$ and the decreasing property of the functions that $p_u < p_l$. The equations $h_l = a^+(p_l)$ and $h_u = a^+(p_u)$ are immediate consequences of the assumption that h_l and h_u are fixed points of the function h.

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Proof. The realization x_1' is in the interval $[a(x_0).b(x_0)]$. It follows from the decreasing property of the functions that $b(x_1') \leq b(h_l)$ and $a(x_1') \geq a(h_u)$. Notice that

$$p_u = (b^-)^{-1}(h_u).$$

As function $h(\cdot)$ is strictly monotone increasing we have the inequality:

$$h(x_0) \leq h(h_u) = h_\mathrm{u}.$$

Hence

$$a^+(b^-(x_0)) \leq h(h_u) = a^+(b^-(h_u)) = h_u.$$

By using again the monotone decreasing property of function $a^+(\cdot)$ one obtains that

$$b^-(x_0) \geq b^-(h_u) = p_u.$$

Similarly,

$$\boldsymbol{h_l = h(h_l) = a^+\left(b^-(h_l)\right) \leq h(x_0) = a^+\left(b^-(x_0)\right)}.$$

It follows from here that

$$b^-(h_l) = p_l \geq b^-(x_0).$$

Thus $\boldsymbol{x_1}$ can be kept in the interval $[\boldsymbol{h_l.p_l}]$. Then it can be proven with the same method that $\boldsymbol{x_2}$ can be kept in the interval $[\boldsymbol{h_l.p_l}]$, etc.

Remarks. If the trajectory enters the interval $[h_u.p_u]$ then it can be kept even in this interval. Otherwise in every odd step it is in $[p_u.p_l]$ and in every even step in $[h_u.g_u]$.

The theorem does not exclude that the trajectory enters the interval $[h_u.p_u]$. If it does so and within the interval there is a similar structure, then the trajectory can kept in an even narrower region. On the other hand if the realization x'_t is always $a(x_{t-1})$ if t is even and $b(x_{t-1})$ if t is odd then there are two cases. If h_u is an attractive fixed point of function $h(\cdot)$, i.e. for all $z \in (h_l.h_u)$ the inequality $h(z) > z$ holds, then trajectory with the control used in the proof converges to the interval $[h_u.p_u]$ in the sense that

$$\lim_{k\to\infty} x_{2k+1} = p_u. \quad \text{and} \quad \lim_{k\to\infty} x_{2k} = h_u.$$

In the case if h_u is a repelling fixed point, i.e. for all $z \in (h_l.h_u)$ the inequality $h(z) < z$ holds, then

$$\lim_{k\to\infty} x_{2k+1} = p_l. \quad \text{and} \quad \lim_{k\to\infty} x_{2k} = h_l.$$

If the condition $h_u < p_u$ does not hold then there are several cases and their discussion is omitted.

Example. Let $a(x) = -x^3 - 1$, $b(x) = -x^3 + 1$, $d^- = -1$, and $d^+ = 0.5$. Then

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$h(x) = a(b(x) + d^-) + d^+ = x^9 - 0.5$.

The three real fixed points of $h(x)$ are -0.904, -0.502, and 1.05. Let $h_l = -0.904$ and $h_u = -0.502$, then $p_l = b^-(h_l) = 0.739$ and $p_u = b^-(h_u) = 0.127$.

If $x_0 \in [-0.904; -0.520]$ then

$x'_1 \in [a(-0.502); b(-0.904)] = [-0.873; 1.739]$.

Thus x_1 can be kept in $[h_l . p_l] = [-0.904; 0.739]$. x_2 can be kept in this interval with same method, too.

Moreover, if $x_k \in [h_u . p_u] = [-0.502; 0.127]$ then

$x'_{k+1} \in [a(0.127); b(-0.502)] = [-1.002; 1.127]$.

Thus x_{k+1} can be kept in $[h_u . p_u]$.

For all $z \in (h_l . h_u)$ the inequality $h(z) > z$ holds ($z^9 - 0.5 > z$), ie. h_u is an attractive fixed point of function $h(\cdot)$, then trajectory with the control used in the proof converges to the interval $[h_u . p_u]$.

4. Conclusions

Production on arable land is one of the main sectors of agriculture. It is a process which can be described by discrete time as there is harvest every year only once with a few exceptions. An important parameter of the market is the market price which is determined by the harvested amount. If high future price is expected, then produced quantity will be higher as more farmers having suitable technology choose the crop in question to produce in the next season. High quantity on the market implies low future price. Thus, the market can be described by a decreasing function. Another important property of agricultural production is that it is uncertain. As the number of observations is very limited, it is difficult to determine the probabilistic distribution of the future harvested amount which depends not only on the random effect of the weather but also the price expectation and the ever changing technology. Therefore, set-valued functions are used as the mathematical tool for modelling the uncertainty.

The paper deals with a one-dimensional dynamical system described by an inclusion. It has all necessary properties to model the above-mentioned agricultural market. The set-valued function is decreasing. The model also contains a control variable. The meaning of the control variable is a quantity which still appear on the market, like import, if the value of the variable is positive, or is withdrawn to a reserve if the value of the control is negative. The paper discusses the a posteori control when the harvested amount is known and the value of the control is determined accordingly.

Two types of results are important as follows: (a) what are the condition for the existence of a control such that the trajectory of the system reaches a target interval, (b) what are the condition for the existence of a control such that the trajectory can be kept in the

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target interval. The latter one called viability result. Theorem 1 belongs to category (a). Theorems 2, 3, and 4 belong to both categories (a), and (b).

In the future a priori control can be analyzed as well. The extension of the model to a multi-dimensional case is also possible.

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