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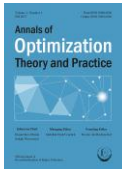

A credibility-constrained programming for closed-loop supply chain network design problem under uncertainty

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Abstract A closed-loop supply chain network (CLSCN) is consisted of both forward and reverse supply chains. In this paper, a CLSCN is including multiple plants, collection centers, demand markets, products and disposal centers. The plants manufacture the new products, then the new products are distributed to the demand market locations and the returned products are collected for sending to the collection centers. Collection centers have important role in recognizing the returned products conditions and the next action of supply chain as follows: inspection and/or separation of the collected products to check whether they are recoverable for sending to remanufacturing plants or unrecoverable ones to be sent to the disposal centers. A mixed-integer linear programming model is proposed to minimize the total cost. Since the uncertain parameters including cost, capacity, demand and the returned products influence the proposed CLSCN, a trapezoidal fuzzy model has been proposed to cope with the vagueness. The expected value is applied to the objective function and the chance constrained programming approach is used to model the uncertain constraint with fuzzy parameters. The numerical examples are coded and solved by GAMZ software. The computational results demonstrate the applicability of the proposed model and solution approach.

Keywords Closed-loop supply chain (CLSC); Mixed-integer linear programming (MILP); Possibilistic programming; Fuzzy mathematical programming; Credibility theory.

1. Introduction

Since the current market has been very dynamic and changeable, the organizations should be able to respond to the internal and external changes by considering the customers' needs for cost minimization and quality improvement. Obviously they need

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to achieve the competitive advantages in order to survive and develop themselves. Technological development, costumers' varied expectations and globalization cause the difficult business situations. In order to cope with the problem, the classical model of management led to a new modern management known as Supply Chain Management (SCM). Supply chain management has received a lot of attention by many authorities and researchers.

Supply chain is consisted of forward and reverse supply chain. The forward supply chain is known as all the activities from the production to the distribution of the products, while the reverse supply chain contains the collection and the recovery of the retuned products. The integration of forward and reverse supply chain provides the closed-loop supply chain. In recent decades, the advantage and the importance of the economic benefits have encouraged the most of companies to apply the closed-loop supply chain network design Srivastava (2007), Linton *et al.* (2007) and Pishvaee and Razmi (2012).

The organization showed the tendency to perform the closed-loop supply chain network (CLSCN) operations such as recovering, recycling, remanufacturing and disposing Devika *et al.* (2014). It was founded out that the integrated design of forward and reverse supply chain was an effective factor in reducing the costs, improving the quality of services and products; therefore many researchers focused on the CLSCN design problem. Kim *et al.* (2006) developed a multi-period, multi-product mix integer programming model for supply chain problem in which the returned products are dissembled to the remanufacturing centers.

The dynamic and complicated nature of supply chain is affected by a high degree of uncertainty and vagueness which influence the planning, decision making and strategic programming of supply chain Klibi *et al.* (2010) and Peidro *et al.* (2010). The cost, demand, capacity and the returned products are the examples of uncertain parameters. Since the quantity and quality of returned products are with higher degree of uncertainty comparing to the new products, uncertainty is significantly more important factor in reverse SC planning problems. The concern about the uncertain nature of supply chain network design has received the considerable attention of the researchers Fleischmann *et al.* (2001).

Tavana *et al.* (2017) proposed two hybrid metaheuristics genetic algorithm-variable neighborhood search (GA-VNS) and variable neighborhood search simulated annealing (VNS-SA) to solve the NP-hard problem in order to minimize the total cost composed of the opening and transportation costs in all three stages. In addition to the novelty of the proposed algorithms, they developed an innovative priority-based decoding method to design chromosomes and solutions related to the nature of the problem. A robust parameter and operator setting was implemented using the Taguchi experimental design method with several random test problems. The performance of the algorithms was evaluated and compared for different problem sizes. The experimental results indicated that the GA-VNS was robust and superior to the other competing methods.

Mahmoodirad *et al.* (2016) presented an effective optimization method based on meta-heuristics algorithms for the design of a multi-stage, multi-product solid supply chain network design problem. First, a mixed integer linear programming model was proposed. Second, because the problem was an NP-hard, three meta-heuristics algorithms, namely Dierential Evolution (DE), Particle Swarm Optimization (PSO), and Gravitational Search Algorithm (GSA), were developed for the first time for this kind of problem. Neither DE, nor PSO, nor GSA had been considered for the multi-stage solid supply chain network design problems. Furthermore, the Taguchi experimental design method was used to adjust the parameters and operators of the proposed algorithms. Finally, to evaluate the impact of increasing the problem size on the performance of the proposed algorithms, different problem sizes were applied and the associated results were compared with each other.

Amin *et al.* (2017) investigated the recovery options and CLSC network of tires in Toronto, Canada by introducing the closed-loop supply chain (CLSC) concept. They considered the effects of uncertainty in CLSC network using decision tree and cash flow in the analysis of the multi-period model and also utilized the real locations with real distances between the facilities using maps. They designed a tire remanufacturing CLSC network and optimized based on tire recovery options. The objective of the optimization model was to maximize the total profit for a model including multiple products, suppliers, plants, retailers, demand markets, and drop-off depots. The application of the model was discussed based on a realistic network in Toronto, Canada using map. In addition, a new decision tree-based methodology was provided to calculate the net present value of the problem in multiple periods under different sources of uncertainty such as demand and returns. Furthermore, the discount cash flow was considered in the methodology as a novel innovative approach.

Talaei *et al.* (2015) presented a bi-objective facility location-allocation model for a closed loop supply chain network design using a fuzzy programming approach to investigate the effects of uncertainties of the variable costs and the demand rate on the network. Özceylan *et al.* (2017) examined CLSCs in automotive industry, since many manufacturers are supposed to collect and recycle their end-of-life products. They employed deterministic linear programming to address their multi-echelon, multi-products, multi-period model. Al-Salem *et al.* (2016) employed an optimization model to minimize the total cost related to forward and reverse logistic networks. The non-linear objective function was linearized by performing the piecewise method. According to their findings, significant cost saving can be achieved because of integration the forward and reverse flows which leads to a CLSC. Kaya and Urek (2016) maximized the profit in a CLSC. They also developed a heuristic for the proposed multi-echelon model.

Ardalan *et al.* (2016) dealt with a supply chain network design with multi-mode demand. First, the problem is mathematically formulated as a mixed integer linear program. Since the supply chain network design was NP Hard, a new iterative lagrangian relaxation based heuristic is developed to solve the problem. Sanei *et al.* (2016) developed a typical supply chain network problem which is based on a two-stage single-product system under uncertain conditions such that both cost and constraint parameters are interval numbers.

Soto *et al.* (2017) applied a mixed integer non-linear programming to formulate a multi-period inventory lot sizing network.

In this paper, a mix-integer liner programming model has been used for the network formulation and in order to overcome the uncertainty parameters, a trapezoidal fuzzy model has been proposed. The aim is cost minimization and the credibility-based fuzzy chance constrained programming model Pishvaee *et al.* (2012) as an efficient fuzzy mathematical programming approach is applied in this paper. The expected value is applied to the objective function and the chance constrained programming approach is used to model the uncertain constraint with fuzzy parameters.

The rest of the paper is organized as follows: Sect.2 explains the fuzzy numbers and fuzzy equations, the problem description is presented in Sect.3, Sects.4 illustrates the mathematical model, Solution approach is described in Sct.5. Sect.6 provides the Numerical example and finally the Conclusion is presented in Sect.7.

2. Preliminary

The Possibilistic programming (PP) as one of the main branches of fuzzy mathematical programming is used in this paper to deal with the lack of knowledge about the exact values of the model parameters. also, the credibility-based fuzzy programming approach as a credible PP Liu and Liu (2002) and Liu (2004) is employed in order to handle the imprecise parameters in the proposed model. The mentioned approach relies on the strong mathematical concepts, i.e., the expected value of a Possibilistic number and the credibility measure, and enables the decision maker to control the confidence level of constraints'satisfactions besides supporting various kinds of Possibilistic numbers such as triangular and trapezoidal forms. It should be noted that despite the possibility and necessity measures that have no self-duality property, the credibility measure is a self-dual measure Li and Liu (2006). In other words, if the credibility value of a fuzzy event achieves 1, decision maker believes the fuzzy event will surely happen; however, when the corresponding possibility measure achieves 1, the fuzzy event may fail to happen and when its necessity measure is equal to 0 the fuzzy event may hold. Among the available credibility-based fuzzy programming models, the new PP method proposed by Pishvaee *et al.* (2012) is used to convert the concerned model into its crisp counterpart. This PP approach is actually a combination of two credibility measure based approaches, i.e., the fuzzy chance constrained programming Liu and Iwamura (1998) and the expected value Liu and Liu (2002) models as it applies the EV to convert the Possibilistic OFs into their crisp counterparts and the chance constrained programming approach to transform the Possibilistic chance constraints including imprecise parameters into their crisp counterparts Pishvaee *et al.* (2014).

Assume $\tilde{z}$ is a fuzzy variable with membership function $\mu(x)$, and r is a real number. The credibility measure is defined as bellows Liu and Liu (2002):

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$$Cr\{\tilde{z} \le r\} = \frac{1}{2}\left(\sup_{x \le 1} \mu(x) + 1 - \sup_{x > 1} \mu(x)\right) \tag{1}$$

Since $Pos\{\tilde{z} \le r\} = \underset{x \le r}{Sup}\ \mu(x)$ and $Nec\{\tilde{z} \le r\} = 1 - \underset{x > r}{Sup}\ \mu(x)$, the credibility measure can also define as bellows:

$$Cr\{\tilde{z} \le r\} = \frac{1}{2}\left(Pos\{\tilde{z} \le r\} + Nec\{\tilde{z} \le r\}\right) \tag{2}$$

As a result, the credibility is the average of the possibility (Pos) and the necessity (Nec) measures. In addition, the expected value of $\tilde{z}$ can be determined according to the credibility measures as follows:

$$E[\tilde{z}] = \int_0^{\infty} Cr\{\tilde{z} \ge r\}\, dr - \int_{-\infty}^{0} Cr\{\tilde{z} \le r\}\, dr \tag{3}$$

Now, consider $\tilde{z}$ as a trapezoidal fuzzy number including four prominent points as $\tilde{z} = (z_1 . z_2 . z_3 . z_4)$. According to the Eq.(3), the expected value of $\tilde{z}$ is $(z_1 + z_2 + z_3 + z_4)/4$ and the related credibility measures are as follows:

$$Cr\{\tilde{z} \le r\} = \begin{cases} 0, & r \in (-\infty, z_1], \\ \dfrac{r - z_1}{2(z_2 - z_1)}, & r \in (z_1, z_2], \\ \dfrac{1}{2}, & r \in (z_2, z_3], \\ \dfrac{r - 2z_3 + z_4}{2(z_4 - z_3)}, & r \in (z_3, z_4], \\ 1, & r \in (z_4, +\infty], \end{cases} \tag{4}$$

$$Cr\{\tilde{z} \ge r\} = \begin{cases} 1, & r \in (-\infty, z_1], \\ \dfrac{2z_2 - z_1 - r}{2(z_2 - z_1)}, & r \in (z_1, z_2], \\ \dfrac{1}{2}, & r \in (z_2, z_3], \\ \dfrac{z_4 - r}{2(z_4 - z_3)}, & r \in (z_3, z_4], \\ 0, & r \in (z_4, +\infty], \end{cases} \tag{5}$$

According to (4) and (5), it can be proved Zhu and Zhang (2009) that if $\tilde{z}$ is a trapezoidal fuzzy number and $\alpha > 0/5$ then:

$$Cr\{\tilde{z} \leq r\} \geq \alpha \Leftrightarrow r \geq (2-2\alpha) z_{(3)} + (2\alpha-1) z_{(4)} \quad , \tag{6}$$

$$Cr\{\tilde{z} \geq r\} \geq \alpha \Leftrightarrow r \leq (2\alpha-1) z_{(1)} + (2-2\alpha) z_{(2)} \quad , \tag{7}$$

The real number r and trapezoidal fuzzy number $\tilde{z}$ are indifferent or approximately equal if $z_2 \leq r \leq z_3$, that is:

$$\tilde{z} \approx r \Leftrightarrow z_2 \leq r \leq z_3 . \tag{8}$$

3. Problem description

The closed-loop supply chain network model (Fig.1) used in the paper is including manufacturing and remanufacturing plants, collection centers, demand markets and disposal centers. The plants manufacture the new products, then the new products are distributed to the demand market and finally the returned products are collected in order to be sent to the collection centers. Collection centers have important role in recognizing the returned products condition and the next action of supply chain as follows: inspection and/or separation of the collected products to check whether they are recoverable for sending to remanufacturing plants or unrecoverable ones to be sent to the disposal centers.

The assumptions of this paper are as bellows:

- A single period is designed for the model.
- All of the returned products are collected by collection center.
- Capacity, cost, demand and the returned product are fuzzy numbers.
- The number of plant locations and collection centers are determined in advance.

4. Mathematical formulation

A mix-integer liner programming model is used for the network formulation. The parameters and decision variables are defined as bellows Amin and Zhang (2013):

Sets:

I	set of potential manufacturing and remanufacturing plants location, index by $i = 1,...,I$
J	set of products, index by $j = 1,...,J$
K	set of demand markets locations ,index by, $k = 1,...,K$
L	sets of potential collection centers locations, index by , $l = 1,...,L$

Parameters:

$\tilde{A}_j$	fuzzy production cost of project j
$\tilde{B}_j$	fuzzy transportation cost of product j per km between plant and demand markets

$\tilde{C}_j$	fuzzy transportation cost of product *j* per km between demand and collection centers
$\tilde{D}_j$	fuzzy transportation cost of product *j* per km between collection centers and plants
$\tilde{O}_j$	fuzzy transportation cost of product *j* per km between collection centers and disposal centers
$\tilde{E}_i$	fuzzy fixed cost for opening plant *i*
$\tilde{F}_l$	fuzzy fixed cost for opening collection center *l*
$\tilde{H}_j$	fuzzy disposal cost of product *j*
$\tilde{P}_{ij}$	fuzzy capacity of plant *i* product *j*
$\tilde{Q}_{lj}$	fuzzy capacity of collection center *l* for product *j*
t_{ik}	the distance between plant *i* and market *k*
t_{kl}	the distance between location *k* and *l*
t_{li}	the distance between location *l* and *i*
t_l	the distance between location center *l* and disposal center
$\tilde{d}_{kj}$	fuzzy demand of customer *k* for product *j*
$\tilde{r}_{kj}$	fuzzy return of customer *k* for product *j*
α_j	minimum disposal fraction of product *j*

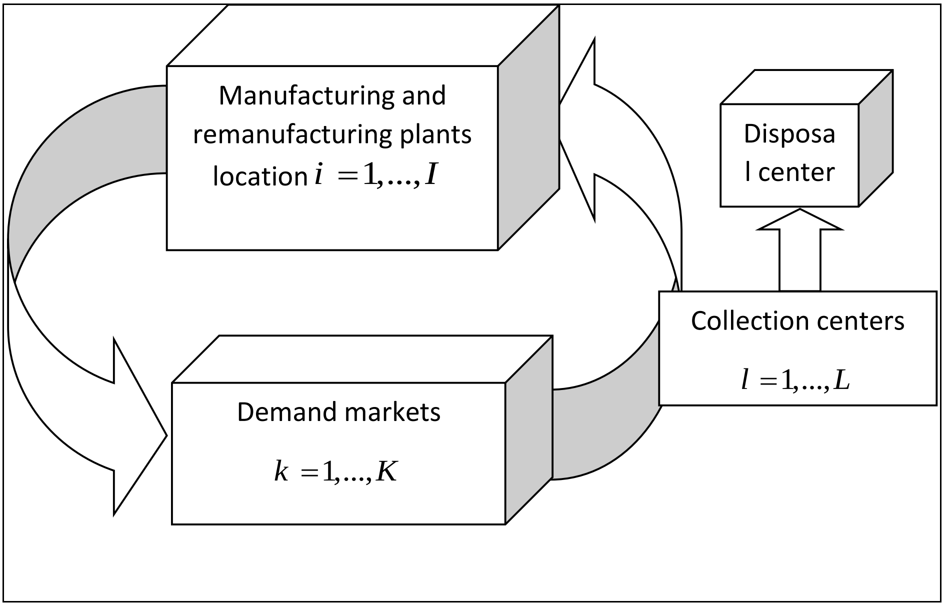

Figure 1. The closed-loop supply chain network Amin and Zhang (2013).

Variables:

W_{ikj}	quality of product *j* produced by plant *i* for demand market *k*
Y_{klj}	quality of returned product *j* from demand market *k* to collection center *l*
S_{lij}	quality of returned product *j* from collection center *l* to plant *i*
T_{lj}	quality of returned product *j* from collection center *l* to disposal center
Z_i	1, if a plant is located and set up at potential site *I*; 0, otherwise
W_l	1, if a collection center is located and set up at potential site *l*; 0, otherwise

The mathematical model is as follows:

$$Min \quad \tilde{Z} = \sum_{i=1}^{I} \tilde{E}_i Z_i + \sum_{l=1}^{L} \tilde{F}_1 W_l + \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{j=1}^{J} (\tilde{A}_j + \tilde{B}_j t_{ik}) X_{ikj} + \sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{j=1}^{J} \tilde{C}_j t_{kl} Y_{klj}$$
$$+ \sum_{l=1}^{L}\sum_{i=1}^{I}\sum_{j=1}^{J} \tilde{D}_j t_{li} S_{lij} + \sum_{l=1}^{L}\sum_{j=1}^{J} (\tilde{H}_j + \tilde{O}_j t_{lj}) T_{lj} \tag{9}$$

subject to

$$\sum_{i=1}^{I} X_{ikj} \geq \tilde{d}_{kj} \qquad \forall k, j \tag{10}$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J} S_{lij} + \sum_{k=1}^{K}\sum_{j=1}^{J} X_{ikj} \leq Z_i \sum_{j=1}^{J} \tilde{P}_{ij} \qquad \forall i \tag{11}$$

$$\sum_{l=1}^{L} Y_{kij} \leq \sum_{i=1}^{I} X_{ikj} \qquad \forall k, j \tag{12}$$

$$\alpha_j \sum_{k=1}^{K} Y_{klj} \leq T_{lj} \qquad \forall l, j \tag{13}$$

$$\sum_{k=1}^{K}\sum_{j=1}^{J} K_{klj} \leq W_l \sum_{j=1}^{J} \tilde{Q}_{lj} \qquad \forall l \tag{14}$$

$$\sum_{k=1}^{K} Y_{klj} = \sum_{i=1}^{I} S_{lij} + T_{lj} \qquad \forall l, j \tag{15}$$

$$\sum_{l=1}^{L} Y_{klj} = \tilde{r}_{kj} \qquad \forall k, j \tag{16}$$

$$Z_i, W_l \in \{0,1\} \qquad \forall i, l \tag{17}$$

$$X_{ikj}, Y_{klj}, S_{lij}, T_{lj} \geq 0 \qquad \forall i, k, l, j \tag{18}$$

The objective function (9) minimizes the total cost including the fixed cost of opening plants and collection centers; the production and transportation costs of new products;

the product recovery and transportation cost of the returned products; the total recovery and transportation cost of retuned products from collection centers to plants; disposal and transportation cost.

The constraint (10) ensures that the total number of each manufactured product for each demand market is equal or greater the demand. The constraint (11) is a capacity constraint of plants. The constraint (12) represents that the forward flow is greater than reverse flow. The constraint (13) enforces a minimum disposal fraction for each product. The constraint (14) is capacity constraint of the collection centers. The constraint (15) shows that the quality of retuned products from demand market is equal to the quantity of returned products to the plants and the quantity of products in disposal center for each collection center and each product. The constraint (16) shows the retuned products. The constraint (17) ensures the binary nature of decision variables. The constraint (18) preserves the non-negativity restriction on the decision variable.

5. Solution approach

Since the costs, capacities, demands and the returned products are uncertain parameters, the credibility-based fuzzy chance constrained programming model Pishvaee *et al.* (2012) as an efficient fuzzy mathematical programming approach is applied in this paper. The model relies on mathematical concepts including the expected value of a fuzzy number and the credibility measure which can support different kind of fuzzy numbers such as triangle and trapezoidal forms to provide some chance constraints in at least some given certainty level for the decision maker. The combination of the expected value and the chance constrained programming model, have been used in this paper. As the expected value is applied to the objective function and the chance constrained programming approach is used to model the uncertain constraint with fuzzy parameters.

$$\begin{aligned} Min \quad E\left[\tilde{Z}\right] = &\sum_{i=1}^{I} E\left[\tilde{E}_i\right] Z_i + \sum_{l=1}^{L} E\left[\tilde{F}_l\right] W_l \\ &+\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{j=1}^{J}\left(E\left[\tilde{A}_j\right] + E\left[\tilde{B}_j\right] t_{ik}\right) X_{ikj} + \sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{j=1}^{J} E\left[\tilde{C}_j\right] t_{kl} Y_{klj} \\ &+\sum_{l=1}^{L}\sum_{i=1}^{I}\sum_{j=1}^{J} E\left[\tilde{D}_j\right] t_{li} S_{lij} + \sum_{l=1}^{L}\sum_{j=1}^{J}\left(E\left[\tilde{H}_j\right] + E\left[\tilde{O}_j\right] t_{lj}\right) T_{lj} \end{aligned} \tag{19}$$

subject to

$$Cr\left\{\sum_{i=1}^{I} X_{ikj} \geq \tilde{d}_{kj}\right\} \geq \beta_{kj} \qquad \forall k, j \tag{20}$$

$$Cr\left\{\sum_{i=1}^{I}\sum_{j=1}^{J} S_{lij} + \sum_{k=1}^{K}\sum_{j=1}^{J} X_{ikj} \leq Z_i \sum_{j=1}^{J} \tilde{P}_{ij}\right\} \geq \lambda_{ij} \qquad \forall i \tag{21}$$

$$\sum_{l=1}^{L} Y_{kij} \leq \sum_{i=1}^{I} X_{ikj} \qquad \forall k, j \qquad (22)$$

$$\alpha_j \sum_{k=1}^{K} Y_{klj} \leq T_{lj} \qquad \forall l, j \qquad (23)$$

$$Cr\left\{\sum_{k=1}^{K}\sum_{j=1}^{J} K_{klj} \leq W_l \sum_{j=1}^{J} \tilde{Q}_{lj}\right\} \geq \theta_{lj} \qquad \forall l \qquad (24)$$

$$\sum_{k=1}^{K} Y_{klj} = \sum_{i=1}^{I} S_{lij} + T_{lj} \qquad \forall l, j \qquad (25)$$

$$r_{kj}^2 \leq \sum_{l=1}^{L} Y_{klj} \leq r_{kj}^3 \qquad \forall k, j \qquad (26)$$

$$Z_i, W_l \in \{0,1\} \qquad \forall i, l \qquad (27)$$

$$X_{ikj}, Y_{klj}, S_{lij}, T_{lj} \geq 0 \qquad \forall i, k, l, j \qquad (28)$$

Based on Eqs. (6) and (7) and according to the expected value of trapezoidal fuzzy numbers, the above credibility-based chance constraint programming model can be modified to the following crisp equivalent MILP model:

$$\begin{aligned} Min\ \ E(Z) = &\sum_{i=1}^{I}\left(\frac{E_1+E_2+E_3+E_4}{4}\right)Z_i + \sum_{l=1}^{L}\left(\frac{F_1+F_2+F_3+F_4}{4}\right)W_l \\ &+\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{j=1}^{J}\left(\left(\frac{A_1+A_2+A_3+A_4}{4}\right)+\left(\frac{B_1+B_2+B_3+B_4}{4}\right)t_{ik}\right)X_{ikj} \\ &+\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{j=1}^{J}\left(\frac{C_1+C_2+C_3+C_4}{4}\right)t_{kl}Y_{klj} \\ &+\sum_{l=1}^{L}\sum_{i=1}^{I}\sum_{j=1}^{J}\left(\frac{D_1+D_2+D_3+D_4}{4}\right)t_{li}S_{lij} + \sum_{l=1}^{L}\sum_{j=1}^{J}\left(\left(\frac{H_1+H_2+H_3+H_4}{4}\right)\right. \\ &\left.+\left(\frac{O_1+O_2+O_3+O_4}{4}\right)t_{lj}\right)T_{lj} \end{aligned} \qquad (29)$$

subject to

$$\sum_{i=1}^{I} X_{ikj} \geq \left(2-2\beta_{kj}\right)d_{kj_3} + \left(2\beta_{kj}-1\right)d_{kj_4} \qquad \forall k, j \qquad (30)$$

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$$\sum_{i=1}^{I}\sum_{j=1}^{J} S_{lij} + \sum_{k=1}^{K}\sum_{j=1}^{J} X_{ikj} \le Z_i \sum_{j=1}^{J}\left[\left(2\lambda_{ij}-1\right)P_{ij_1} + \left(2-2\lambda_{ij}\right)P_{ij_2}\right] \qquad \forall i \qquad (31)$$

$$\sum_{l=1}^{L} Y_{klj} \le \sum_{i=1}^{I} X_{ikj} \qquad \forall k,j \qquad (32)$$

$$\alpha_j \sum_{k=1}^{K} Y_{klj} \le T_{lj} \qquad \forall l,j \qquad (33)$$

$$\sum_{k=1}^{K}\sum_{j=1}^{J} y_{klj} \le w_l \sum_{j=1}^{J}\left[\left(2\theta_{lj}-1\right)q_{lj_1} + \left(2-2\theta_{lj}\right)q_{lj_2}\right] \qquad \forall l \qquad (34)$$

$$\sum_{k=1}^{K} Y_{klj} = \sum_{i=1}^{I} S_{lij} + T_{lj} \qquad \forall l,j \qquad (35)$$

$$r_{kj}^2 \le \sum_{l=1}^{L} Y_{klj} \le r_{kj}^3 \qquad \forall k,j \qquad (36)$$

$$Z_i, W_l \in \{0,1\} \qquad \forall i,l \qquad (37)$$

$$X_{ikj}, Y_{klj}, S_{lij}, T_{lj} \ge 0 \qquad \forall i,k,l,j \qquad (38)$$

It should be noted that in the above-mentioned formulation, we have assumed that the chance constraints should be satisfied with confidence level greater than 0.5 $\left(i.e., \beta_{kj}, \lambda_{ij}, \theta_{lj} > 0.5\right)$

6. Numerical example

This section presents an example to illustrate how the proposed model works in a closed-loop supply chain network design problem under an uncertain environment. The data of this problem are presented in Table 1-8. Also, we consider $\alpha_{j1} = 0/2$, $\alpha_{j2} = 0/3$ and $\beta.\lambda.\theta = 0/8$.

Table 1. Trapezoidal fuzzy values for fixed cost and capacity of plants.

	Plant 1	Plant 2	Plant 3
$\tilde{E}_i$	(200,300,400,500)	(150,300,400,600)	(200,200,300,350)
$\tilde{P}_{ij}$	(200,500,700,800) (300,400,500,600)	(200,250,350,450) (400,500,700,900)	(300,400,500,700) (320,420,500,700)

Table 2. Trapezoidal Fuzzy values for fixed cost and capacity of collection centers.

	Collection Center 1	Collection Center 2
$\tilde{F}_1$	(100,270,350,400)	(150,300,400,700)
$\tilde{Q}_{lj}$	(200,500,700,800) (300,400,500,600)	(200,250,350,450) (400,500,700,900)

Table 3. Trapezoidal Fuzzy values for cost.

	Product 1	Product 2
$\tilde{A}_j$	(100,200,300,400)	(200,400,500,600)
$\tilde{B}_j$	(30, 40, 50,70)	(20,50,55,60)
$\tilde{C}_j$	(100,200,250, 350)	(150,300,400, 500)
$\tilde{D}_j$	(10,50,70,80)	(15,30,40,60)
$\tilde{O}_j$	(20,40,60,70)	(25,30,40,60)
$\tilde{H}_j$	(20,40,60,80)	(50,60,70,80)

Table 4. Trapezoidal Fuzzy values of demand and returned products.

	$\tilde{d}_{kj}$	$\tilde{r}_{kj}$
Customer 1	(10,12,20,50)	(15,16,17,18)
	(70,90,100,110)	(14,16,18,19)
Customer 2	(3,5,6 ,8)	(13,14,16,18)

Table 4. Continued

	$\tilde{d}_{kj}$	$\tilde{r}_{kj}$
Customer 2		
	(4, 8,10,12)	(16,18, 19,20)
Customer 3	(10,12,15,18)	(17,19,20,21)
	(7,9,10,11)	(10,12,13,16)
Customer 4	(60,70,80, 90)	(80,10,12,13
	(50,60, 70, 90)	(90,10,11,13)
Customer 5	(50,60,70, 80)	(10,12,14,16)
	(6,7,8,10)	(15,18,30,35)

Table 5. Distance data t_{ik} .

	Customer 1	Customer 2	Customer 3	Customer 4	Customer 5
Plant 1	100	120	150	180	20
Plant 2	70	90	120	150	40
Plant 3	100	15	170	190	50

Table 6. Distance data t_{kl} .

	Collection Center 1	Collection Center 2
Customer 1	100	120
Customer 1	70	90
Customer 1	100	120
Customer 1	80	100
Customer 1	95	125

Table 7. Distance data t_{li} .

	Plant 1	Plant 1	Plant 1
Collection Center 1	85	100	110
Collection Center 2	90	110	125

Table 8. Distance data t_l .

	Disposal Center
Collection Center 1	20
Collection Center 2	50

The numerical examples of the model are coded and solved by GAMZ 24.9.1 software as follows: $s = 0$, $t_1 = 71, t_2 = 74$, z_1 *and* $z_2 = 1$, $w_l = 1$, the results of x and y are presented in Tables 9 and the other variables are equal to zero.

According to the computational results, the optimal objective function is 20,994,040.

Table 9. Summary of results.

Variable		Product 1	Product 2
X	1.5	76	18
	2.1	38	106
	2.2	14	18
	2.3	19	12
	2.4	86	82
Y	1.1	16	16
	2.1	14	18
	3.1	19	12
	4.1	10	10
	5.1	12	18

7. Conclusion

In this paper, a closed-loop supply chain network including manufacturing and remanufacturing plants, collection centers, demand markets and disposal centers has been proposed under uncertain conditions. Since the cost, capacities, demands and the retuned products were uncertain parameters, the credibility-based fuzzy chance constrained programming model was employed to cope with the vagueness. The expected value was applied to model the objective function and the chance constrained programming model was used to model the chance constraints with fuzzy parameters.

The numeral examples were also provided to show the applicability of the proposed model as well as the usefulness of its solution method.

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