Refueling station location problem under uncertain environment

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Abstract Development of the infrastructure of alternative fuel stations is one of the best ways to extend the use of alternative fuel vehicles. Hence, constructing refueling stations with minimum cost is an important issue. On the other hand, considering the exact value of cost is not match with real cases. In this regard, the cost of building station is considered as a trapezoidal fuzzy value and a mathematical fuzzy programming model is presented in this paper. In order to solve the fuzzy model, first the model is converted to an interval programming model, then the equivalent bi-objective crisp model of the interval programming problem is written. Finally, two interactive fuzzy solution approaches are used to solve the respective bi-objective crisp model. The results show that the performance of the solution approaches is the same.

Keywords Refueling station; Facility location; Fuzzy programming

1. Introduction

In response to concerns about limitation of fossil fuels and environmental problems, automotive companies have started to produce alternative-fuel vehicles (AFVs) using alternative forms of energy such as hydrogen, compressed natural gas (CNG), ethanol, liquefied petroleum gas (LPG, or propane), biodiesel, and electric vehicles (EVs). Researchers have shown that AFVs encounter with many obstacles. One of them is lack of refueling infrastructures due to a high cost of constructing a station Melendez and Milbrandt (2006) and Melendez et al. (2007). Therefore, building an easily accessible network of service stations with a minimum cost of establishing for refueling would increase more widespread application. Another obstacle that AFVs encounter with, is the limited driving range. Generally, refueling station location problem has gained many researcher's attention. Kuby and Lim (2005) extended the Flow-Refueling Location Model (FRLM) to find the optimal location of refueling stations for alternative-
fuel vehicles. The FRLM develops the Flow Capturing Location Model (FCLM), Hosseini et al. (2017) and Berman et al. (1992). The flow capturing model, which is structurally similar to the maximum-cover model problem Church and ReVelle (1974), locates a given number of facilities to maximize the traffic flow volume that passes by the facilities. However, the limited driving range of vehicles is not considered in FCLM. Short trips may be refuelable by a single facility, but longer trips require a combination of facilities spaced adequately along the shortest path. The main difficulty that actually limited the applicability of FRLM approach was the enormous number of facility combinations along the long paths with several nodes. It is impractical to generate and solve the problem even for medium-sized networks. Capar and Kuby (2011) proposed and test a Mixed Binary Integer Programming (MBIP) model for solving the FRLM. There are more other researches on FRLM, which have studied subjects such as dispersion of candidate sites on arcs Kuby and Lim (2007), capacitated facilities Upchurch et al. (2009) and Hosseini et al. (2017), and comparison of p-median and FRLM Upchurch and Kuby (2010). However, these models require the O–D flow data, which is not easy to obtain in practical applications Lin et al. (2008). Wang and Lin (2009) introduced the concept of set cover for proposing a refueling-station-location model using a mixed integer programming method, based on vehicle-routing logics. Wang and Wang (2010) also proposed a hybrid model with dual objectives, using a mixed integer programming method, to economically site refueling stations to simultaneously serve intercity and intra-city travel. MirHassani and Ebrazi (2013) reformulated the flow refueling location model and presented a mixed integer linear programming model with the objective of minimizing the cost of building refueling stations. Ferdowski et al. (2018) proposed a problem with a fuzzy bi-objective formulation to select on where to refuel a vehicle in order to have a minimum total cost of refueling as well as a minimum number of necessary refueling where the fuel consumption is of fuzzy values. They also presented a new hybrid fuzzy interactive approach for solving the model. Locations of charging stations for electric vehicles are also considered by some researches Chen et al. (2018). He et al. (2018) proposed a bi-level programming model to optimize charging station location with the consideration of electric vehicle’s driving range.

In the most of the real-world situations, the input parameter values of a problem are tained by uncertainty. Generally, researchers believe that uncertainty is an inherent part of measurements Ferdowski et al. (2018). Cost of building refueling stations is a parameter that is needed in the presented model in MirHassani and Ebrazi (2013). However, considering cost of building as a certain value may not match with real cases. It is due to the fact that the cost of building may vary with different economic conditions. This issue is not considered in MirHassani and Ebrazi (2013). To get closer to the real-world situations, we use the concept of fuzzy theory by considering the cost of building as a trapezoidal fuzzy number. Regarding the mentioned explanations, a fuzzy refueling station location model is presented in this paper. To solve the proposed fuzzy formulation an interval programming approach is applied. A numerical example is solved to illustrate the proposed problem.
In the remaining sections, the basic definitions are described in Section 2. The problem
description and its corresponding model are presented in Section 3. Section 4 is devoted
to solution approach. In Section 5 a numerical example is presented. Finally, conclusions
are given in Section 6.

2. Preliminaries

In this section some basic concepts and definitions are reviewed which will be applied
later.

Let $X$ be a universal set. $\tilde{A}$ is called a fuzzy set in $X$ if $\tilde{A}$ is a set of ordered pairs
$\tilde{A}=\{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$. Where $\mu_{\tilde{A}}(x)$ is the membership function of $x$ in $\tilde{A}$. Moreover, 
$\alpha$-cut of a fuzzy set $\tilde{A}$ is a subset of $\tilde{A}$ and define as $A_\alpha = \{x \mid \mu_{\tilde{A}}(x) \geq \alpha\}$, for all $\alpha \in [0,1]$.

Let $a = (a_1, a_2, a_3, a_4)$ fuzzy number, where be a trapezoidal $a_1 \leq a_2 \leq a_3 \leq a_4$ and its $\alpha$-cut is $A_\alpha = [a_1 + (a_2 - a_1)\alpha, a_4 - (a_4 - a_3)\alpha] = [A_\alpha^L, A_\alpha^U]$.

The fuzzy number can be approximated to an interval number. Uncertainties related to
the parameters can be effectively handled by using a suitable approximated interval
Sen and Pal (2015). Grezegorzewski introduced the metric "D" for approximating a fuzzy
number by the nearest interval as follow Grzegorzewski (2002):

$$N_D(\tilde{A}) = \left[ \int_0^1 A^L_\alpha d\alpha, \int_0^1 A^U_\alpha d\alpha \right]$$ (1)

Where $N_D(\tilde{A})$ represents the approximated interval of fuzzy number $\tilde{A}$ based on the
metric 'D'.

An interval is defined by an ordered pair of brackets as $B = [b^L, b^U] = \{b \mid b^L \leq b \leq
b^U, b \in R\}$. Where $b^L$ and $b^U$ are the lower and upper limits of $B$, respectively. The
interval is also denoted by its center and width as $B = \langle b_m, b_w \rangle = \{b \mid b_m - b_w \leq b \leq
b_m + b_w, b \in R\}$. Where $b_m = \frac{(b^U + b^L)}{2}$ and $b_w = \frac{(b^U - b^L)}{2}$ are the center and half width

3. Problem Statement

Vehicle range (the distance that vehicle can traverse with a full tank of fuel) is a critical
factor for determining locations and the number of refueling stations for completing a
trip. To find all valid combinations of stations on a specified path, a simple example is
considered in Figure 1. This figure shows the shortest path of 150 km between A and
D, with two intermediate nodes. It is assumed that a vehicle with the driving range of
R=100 km wants to have a round trip starting from A without running out of fuel and the
vehicle has a half tank of fuel at the origin, A, Melendez et al. (2007). If a station exists
in A and the vehicle begins at A and filled up its tank, it could reach at most C, and it
couldn't reach D, which means that there must also be a station at node C. If there is not any station at the origin, A, the vehicle starts its trip with half a tank and could reach at most node B, so one station should be at node B. Since the vehicle could not reach D with full tank from B, thus another station should be in node C. By these explanations \{A, C\} and \{B, C\} are valid combinations for refueling. For the return path, if there is no fuel station at destination D, the vehicle must reach the last station with at least a half-full tank. This fuel is enough for reaching C on the return path. There is no need to test both A-D and D-A direction because if one is not refuelable, neither will the other. Therefore, all valid combinations are, \{A, B, C, D\}, \{A, B, C\}, \{B, C, D\}, \{B, C\} and \{A, C\}.

![Figure 1. An example of simple path with R=100.](image-url)

To find valid combinations of refueling stations the concept of the expanded network is explained MirHassani and Ebrazi (2013). Firstly, some basic necessary notations are presented. The set of nodes on a specified path \(q\) is denoted by \(N^q\). Also, \(N^q\) is the set of nodes existing in its corresponding expanded network. Similarly, \(A^q\) is the set of arcs belonging to the expanded network. For any two nodes \(i\) and \(j\) on path \(q\), \(d_q(i, j)\) denotes the length of the sub path between. Finally, \(ord_q(i)\) defines an ordering index in the path sequence.

To construct the expanded network according to the specified path \(q\), two virtual nodes \(s\) and \(t\) are needed. Node \(s\) is referred to a source node and \(t\) is referred to a sink node. These two nodes are added to the path. Then \(s\) and \(t\) are connected to the origin node and destination node, respectively. After that, the source node \(s\) is connected to any other node \(i\) on path \(q\) if it is possible to begin at the origin node and get to node \(i\) with half of a tank or less. For example, in Figure 1, \(s\) is connected to node B because \(d_q(A, B) = 40\) is less than \(R/2\); so expanded network contains arc \((s, B)\). Further, any nodes \(i\) of path \(q\) is connected to node \(t\) whenever the vehicle can reach the destination node of path with a half-full tank or less. As a result \(C\) should be connected to node \(t\) because \(d_q(C, D) = 50\) is equal to \(R/2\), therefore the expanded network includes arc \((C, t)\). Finally, if the ordering index of each node \(i\) of path \(q\) is less than the ordering index of any other node \(j\) and the vehicle is able to start from \(i\) with a full tank and reach \(j\), node \(i\) is connected to node \(j\). Accordingly, the expanded network contains arcs \((A, B), (A, C), (B, C)\) and \((C, D)\).

In the expanded network, each arc can be traversed without running out of fuel and each directed path from the source \(s\) to the sink \(t\) corresponds to a feasible combination of fuel station that can refuel the path. The resulted expanded network according to Figure 1, is shown in Figure 2.
By doing the above process, all valid combinations for refueling are obtained. There are different criteria for selecting the best valid combination among all combinations. For refueling the pathCost of building stations is one of the most important factors for developing refueling infrastructures. Hence, a mixed integer linear programming model with the objective of minimizing the cost of building refueling stations was presented in MirHassani and Ebrazi (2013). In order to encounter this model, some parameters such as the cost of building refueling stations should be available. However, considering the cost of building as a certain value may not match with real cases. It is due to the fact that the cost of building may vary with different economic conditions. This issue is not considered in MirHassani and Ebrazi (2013). To get closer to the real-world situations, we use the concept of fuzzy theory by considering the cost of building as a trapezoidal fuzzy number. The new fuzzy refueling station location model with the following parameter and decision variables is formulated as below.

$$\text{Model 1}$$

$$\begin{align*}
\min Z &= \sum_{i \in N^q} \tilde{C}_i y_i \\
\text{subject to} & \\
\sum_{\{j|(i,j) \in A^q\}} x_{ij}^q - \sum_{\{j|(j,i) \in A^q\}} x_{ji}^q &= \begin{cases} 
1 & i = s \\
-1 & i = t \\
0 & i \neq s, t
\end{cases} \quad \forall i \in N^q \\
\sum_{\{j|(i,j) \in A^q\}} x_{ij}^q &\leq y_i \quad \forall i \in N^q \quad (4) \\
x_{ij}^q &\geq 0 \quad \forall (i,j) \in A^q \\
y_i &\in \{0,1\} \quad \forall i \in N^q \quad (6)
\end{align*}$$

The objective function (2) minimizes the total fuzzy cost of building refueling stations. Constraints (3) are the mass balance constraints, which state that the outflow minus inflow must equal the virtual supply and demand of the node. Constraints (4) let the flow...
pass through a node only when a fuel station is located at that node. Constraints (5) show that the flow on each arc is greater than or equal to zero and (6) define the facility location variables as binary variables.

4. Solution Method

In order to solve the proposed fuzzy model 1 the fuzzy objective function is to be defuzzified by using suitable defuzzification operator. In this regard, the metric "D" which is introduced by equation (1) is applied for approximating fuzzy numbers by the nearest intervals. Hence, the proposed model 1 can be written as,

**Model 2**

\[
\min Z = N_D \left( \sum_{i \in N^q} \tilde{C}_i y_i \right) = \sum_{i \in N^q} N_D (\tilde{C}_i) y_i
\]

subject to

\[
\sum_{\{j \mid (i,j) \in A'^q\}} x_{ij}^q - \sum_{\{j \mid (j,i) \in A'^q\}} x_{ij}^q = \begin{cases} 1 & i = s \\ -1 & i = t \\ 0 & i \neq s, t \end{cases} \quad \forall i \in N'^q \tag{3}
\]

\[
\sum_{\{j \mid (i,j) \in A'^q\}} x_{ij}^q \leq y_i \quad \forall i \in N^q \tag{4}
\]

\[
x_{ij}^q \geq 0 \quad \forall (i,j) \in A'^q \tag{5}
\]

\[
y_i \in \{0,1\} \quad \forall i \in N^q \tag{6}
\]

Where \([c_i^L, c_i^U]\) represents the nearest interval approximation of the fuzzy number \(\tilde{C}_i\). Moreover, \(c_i^L = \int_0^1 C_i^L d\alpha\) and \(c_i^U = \int_0^1 C_i^U d\alpha\).

Another challenge is solving the obtained interval mathematical model 2. To solve this model, it can be converted to a bi-objective model with crisp coefficient objective functions Das et al. (1999). The corresponding equivalent bi-objective refueling station location problem of model 2 can be written as follows:

**Model 3**

\[
\min Z_U(x) = \sum_{i \in N^q} c_{mi} y_i + \sum_{i \in N^q} c_{wi} y_i \tag{8}
\]

\[
\min Z_m(x) = \sum_{i \in N^q} c_{mi} y_i \tag{9}
\]

subject to

\[
\sum_{\{j \mid (i,j) \in A'^q\}} x_{ij}^q - \sum_{\{j \mid (j,i) \in A'^q\}} x_{ij}^q = \begin{cases} 1 & i = s \\ -1 & i = t \\ 0 & i \neq s, t \end{cases} \quad \forall i \in N'^q \tag{3}
\]
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\[ \sum_{(j,(j,i) \in A'^q)} x_{ij}^q \leq y_i \quad \forall i \in N^q \]  

(4)

\[ x_{ij}^q \geq 0 \quad \forall (i, j) \in A'^q \]  

(5)

\[ y_i \in \{0, 1\} \quad \forall i \in N^q \]  

(6)

Where, \( c_{mi} = \frac{c_i^u + c_i^l}{2} \) and \( c_{wi} = \frac{c_i^u - c_i^l}{2} \).

Now, with the help of fuzzy programming technique, the solution of the bi-objective model 3 can be obtained.

Two hybrid versions of fuzzy programming approaches which were introduced by Ferdowsi et al. (2018) (FMN approach) and Selim and Ozkarahan (2008) (SO approach) are explained shortly as follow,

**FMN approach:**

\[
\text{max } \lambda (x) = \sum_{k=1}^{K} \theta_k \mu_k (x) \\
\text{subject to } \lambda_0 \leq \mu_k (x) \\
\sum_{(j,(j,i) \in A'^q)} x_{ij}^q - \sum_{(j,(j,i) \in A'^q)} x_{ji}^q = \begin{cases} 
1 & i = s \\
-1 & i = t \\
0 & i \neq s, t 
\end{cases} \quad \forall i \in N'^q 
\]  

(3)

\[ \sum_{(j,(j,i) \in A'^q)} x_{ij}^q \leq y_i \quad \forall i \in N^q \]  

(4)

\[ x_{ij}^q \geq 0 \quad \forall (i, j) \in A'^q \]  

(5)

\[ y_i \in \{0, 1\} \quad \forall i \in N^q \]  

(6)

\[ \lambda_0 \in [0, 1] \quad k = 1, \ldots, K \]

**SO approach:**

\[
\text{max } \lambda (x) = \gamma \lambda_0 + (1 - \gamma) \sum_{k=1}^{K} \theta_k \lambda_k \\
\text{subject to } \lambda_0 + \lambda_k \leq \mu_k (x) \quad k = 1, \ldots, K \\
\sum_{(j,(j,i) \in A'^q)} x_{ij}^q - \sum_{(j,(j,i) \in A'^q)} x_{ji}^q = \begin{cases} 
1 & i = s \\
-1 & i = t \\
0 & i \neq s, t 
\end{cases} \quad \forall i \in N'^q 
\]  

(3)

\[ \sum_{(j,(j,i) \in A'^q)} x_{ij}^q \leq y_i \quad \forall i \in N^q \]  

(4)

\[ x_{ij}^q \geq 0 \quad \forall (i, j) \in A'^q \]  

(5)

\[ y_i \in \{0, 1\} \quad \forall i \in N^q \]  

(6)

\[ \gamma, \lambda_0, \lambda_k \in [0, 1] \quad k = 1, \ldots, K \]
In this model, \( \mu_k(x) \) indicates the level of satisfaction and \( \lambda_0 \) represents the minimum level of satisfaction. Also, \( \gamma \) is related to the coefficient of compensation which controls the compromise degree among the objectives as well as the minimum satisfaction level of objectives. Moreover, \( \theta_k \) is the weight representing the importance of \( k \)th objective function. The weight values are determined by the decision maker so that, \( \theta_k \in [0,1] \) and \( \sum_{k=1}^{K} \theta_k = 1 \).

5. Numerical example

To illustrate the efficiency of the proposed problem a numerical example is explained in the following.

Consider path \( q \) in figure 1. Supposed that Nodes \( A, B, C \) and \( D \) are candidate nodes with the different fuzzy cost of building refueling stations as below.

\[
\begin{align*}
\tilde{C}_A &= (1,2,3,3.5) \\
\tilde{C}_B &= (1,2,2.5,3) \\
\tilde{C}_C &= (3,3.5,4.5) \\
\tilde{C}_D &= (2,3,4,6)
\end{align*}
\]

The objective of the problem is to minimize the fuzzy cost of building refueling stations. In this fuzzy linear programming, fuzzy coefficients are involved in the objective function. For solving the fuzzy model, nearest interval approximation of fuzzy number is used. Therefore, by applying Eq. 1 the above fuzzy costs are converted to interval costs: \( N_D(\tilde{C}_A) = [1.5,3.25] \), \( N_D(\tilde{C}_B) = [1.5,2.25] \), \( N_D(\tilde{C}_C) = [3.25,4.5] \), \( N_D(\tilde{C}_D) = [2.5,5] \).

Now, we have an interval programming model. The corresponding equivalent bi-objective refueling station location model with crisp coefficient objective functions is written as follows:

\[
\begin{align*}
\min Z_U &= 3.25 y_A + 2.25 y_B + 4.5 y_C + 5 y_D \\
\min Z_m &= 2.37 y_A + 1.87 y_B + 3.87 y_C + 3.75 y_D \\
\text{subject to} & \quad x_{sA}^{q} + x_{sB}^{q} = 1 \\
& \quad x_{AB}^{q} + x_{AC}^{q} - x_{sA}^{q} = 0 \\
& \quad x_{BC}^{q} - x_{sB}^{q} - x_{AB}^{q} = 0 \\
& \quad x_{CD}^{q} = x_{ct}^{q} - x_{AC}^{q} - x_{BC}^{q} = 0 \\
& \quad x_{Dt}^{q} - x_{CD}^{q} = 0
\end{align*}
\]
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\[ x_{ct}^q + x_{dt}^q = 1 \]
\[ x_{sa}^q \leq y_A \]
\[ x_{sb}^q + x_{ab}^q \leq y_B \]
\[ x_{ac}^q + x_{bc}^q \leq y_C \]
\[ x_{cd}^q \leq y_D \]
\[ x_{ij}^q \geq 0 \quad \forall (i,j) \in A^q \]
\[ y_i \in \{0,1\} \quad \forall i \in N^q \]

To solve the above bi-objective model we use two hybrid versions of fuzzy programming approaches which were introduced by Ferdowsi et al. (2018) (FMN approach) and Selim and Ozkarahan (2008) (SO approach). The obtained results are presented in Table 2. The values of \( \theta_1 \) and \( \theta_2 \) representing the importance of the first and the second objective functions, respectively. The weight values are determined by a decision maker such that \( \theta_1 + \theta_2 = 1 \). Moreover, the value of the parameter \( \gamma \) should be tuned by the decision maker for SO method. \( \mu(Z_u) \) and \( \mu(Z_m) \) are the satisfaction degrees of the first and the second objective functions, respectively. As can be seen in Table 2, both of the solution methods obtain best values for the first and the second objective functions for different values of weights.

**Table 2. The results of different solution methods.**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Approaches</th>
<th>( \mu(Z_u) )</th>
<th>( \mu(Z_m) )</th>
<th>( Z_u )</th>
<th>( Z_m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\theta_1, \theta_2, \gamma) = (0.5, 0.5, 0.4))</td>
<td>FMN</td>
<td>1</td>
<td>1</td>
<td>11.75</td>
<td>9.49</td>
</tr>
<tr>
<td></td>
<td>SO</td>
<td>1</td>
<td>1</td>
<td>11.75</td>
<td>9.49</td>
</tr>
<tr>
<td>((\theta_1, \theta_2, \gamma) = (0.1, 0.9, 0.4))</td>
<td>FMN</td>
<td>1</td>
<td>1</td>
<td>11.75</td>
<td>9.49</td>
</tr>
<tr>
<td></td>
<td>SO</td>
<td>1</td>
<td>1</td>
<td>11.75</td>
<td>9.49</td>
</tr>
<tr>
<td>((\theta_1, \theta_2, \gamma) = (0.9, 0.1, 0.4))</td>
<td>FMN</td>
<td>1</td>
<td>1</td>
<td>11.75</td>
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</tr>
<tr>
<td></td>
<td>SO</td>
<td>1</td>
<td>1</td>
<td>11.75</td>
<td>9.49</td>
</tr>
</tbody>
</table>

6. Conclusion

Lack of refueling infrastructures due to the high cost of constructing a refueling station is a major problem. The uncertainty of the cost of constructing stations is an important issue in real-world situations. In this regard, the cost of building station is considered as a trapezoidal fuzzy value in this paper. In order to solve the fuzzy model, first we convert it to an interval programming model, then the corresponding equivalent bi-objective crisp model of the interval programming problem is written. Finally, two interactive fuzzy solution approaches SO and FMN are used to solve the respective bi-objective crisp model. The results show that the performance of the solution approaches is the same. The satisfaction degrees of the first and the second objective functions which were obtained by both of the solution methods are equal to one for different values of weights.
References


