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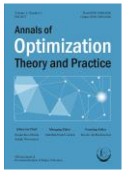

A novel solution approach for solving intuitionistic fuzzy transportation problem of type-2

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Abstract In the cases that decision maker faces with uncertainty and hesitation together for determining a parameter of an optimization problem, considering intuitionistic fuzzy parameters is useful. A transportation problem with triangular intuitionistic fuzzy unit transportation costs is focused in this study. The fuzzy costs are crisped using the accuracy function of the literature. Then the algorithms e.g. the north west corner method, the least cost method, and the Vogel's approximation method are applied to obtain an initial basic feasible solution for the crisp version of the problem. After that the modified distribution method is used to obtain the optimal solution of the problem. The performed computational experiments show the superiority of the proposed approach over those of the literature from the results' quality and the computational difficulty point of views.

Keywords Triangular intuitionistic fuzzy number; Accuracy function; Intuitionistic fuzzy transportation problem of type-2; Optimal solution

1. Introduction

The classic transportation problem consists of decision on how much product to send from each source to each destination in order to minimize the total transportation cost which is affected by the unit transportation cost between each source and each destination. In this classic problem the capacities of each source and each destination is given. For the cases that the sum of capacities of the sources equals the sum of capacities of the destinations, the problem is balanced. On the other hand, it can be converted to a balanced problem using dummy source or destination.

Fuzzy theory which was first introduced by Zadeh (1965) has been employed to formulate many real-life engineering and non-engineering problems. The fuzzy theory

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became more popular in the case of optimization problems by the excellent study of Bellman and Zadeh (1970). Some years later than this work, Atanassov (1986) introduced the theory of intuitionistic fuzzy set (IFS) that has been a very crucial concept of fuzzy theory. The most important advantage of IFS comparing to fuzzy set is that it isolates the membership and non-membership degrees of a number of the set. Therefore, this theory seems to be very applicable when considering vagueness. In the case of transportation problem some information about the membership degree and non-membership degree of the transportation cost can be obtained when this cost is of intuitionistic fuzzy numbers.

The literature of optimization problems is full of the works applying fuzzy and intuitionistic fuzzy set theory for formulating and solving real-life optimization problems like production planning and scheduling, transportation, manufacturing, etc. (see Xu, 1988; Cascetta *et al.*, 2006; Ganesan and Veeramani, 2006; Asunción *et al.*, 2007; Kaur and Kumar, 2012; De and Sana, 2013; Mahmoodirad *et al.*, 2014; Mahmoodi-Rad *et al.*, 2014; Niroomand *et al.*, 2016; Taassori *et al.*, 2016; Niroomand *et al.*, 2016; Mahmoodirad *et al.*, 2019). In transportation problems considered for real cases, in many situations parameters like transportation costs, supply values, demand values, etc. may be of uncertainty because of some reasons (see Dempe and Starostina, 2006). To cope with the uncertainty of the parameters of a transportation problem many studies have been done. Nagoorgani and Razak (2006) focused on a two stage cost minimizing transportation problem in fuzzy environment for supply and demand values. Dinager and Palanivel (2009) proposed an approach for solving fuzzy transportation problem with trapezoidal fuzzy parameters. Pandian and Natarajan (2010) proposed an algorithm to obtain fuzzy optimal solution of fuzzy transportation problem. Mohideen and Kumar (2010) performed a comparative study on fuzzy transportation problems. Basirzadeh (2011) proposed a method to solve transportation problem of fuzzy environment. Kaur and Kumar (2012) tackled fuzzy transportation problem with trapezoidal fuzzy parameters.

As for determining the unit transportation costs in a transportation problem the decision maker may hesitate, it would be more realistic to consider the cost as intuitionistic fuzzy number to consider both the uncertainty and the hesitation in the cost determination procedure. A transportation problem with intuitionistic fuzzy supply and demand values and crisp costs is called intuitionistic fuzzy transportation problem of type-1 (presented by Singh and Yadav, 2014). On the other hand, a transportation problem with intuitionistic fuzzy unit transportation costs and crisp supply and demand values is called intuitionistic fuzzy balanced transportation problem of type-2 (IFBTP-2) (presented by Singh and Yadav, 2016). In this study, we consider the IFTP-2 with triangular intuitionistic fuzzy numbers (TIFN). We propose a new solution approach which uses an accuracy function to crisp and rank the TIFNs. Then the classic solution algorithms of transportation problem (the same as what Singh and Yadav (2016) applied) are applied to solve the crisp version of the IFBTP-2. The computational results prove the effectiveness of the proposed approach comparing to that of Singh and Yadav (2016) from the results' quality and the computational difficulty point of views as the study of Singh and Yadav (2016) do the procedure of the algorithms on TIFNs.

The reminder of this paper is organized by the following sections. Section 2 presents some initial definitions of intuitionistic fuzzy numbers. Section 3 describes the mathematical formulation of the IFBTP-2. The proposed solution approach and its comparison with the existing approaches of the literature is presented by Section 4. Computational experiments are done in Section 5. Finally, conclusions are drawn by Section 6.

2. Basic definitions

Some basic definitions from fuzzy theory which will be applied later in this paper are explained in this section. For more detailed version of these definitions the resources e.g. Singh and Yadav (2016) can be referred.

Definition 1. If X be a universal set, a fuzzy set $\tilde{A}$ on the set X is defined as $\tilde{A}=\{(x,\mu_{\tilde{A}}(x)):x\in X\}$ where $\mu_{\tilde{A}}:X\to[0,1]$.

Definition 2. If X be a universe of discourse, then the following set of ordered triples defines an intuitionistic fuzzy set (IFS) $\tilde{A}^I$.

$$\tilde{A}^I=\{\langle x,\mu_{\tilde{A}^I}(x),\upsilon_{\tilde{A}^I}(x)\rangle:x\in X\} \tag{1}$$

where, $\mu_{\tilde{A}^I},\upsilon_{\tilde{A}^I}:X\to[0,1]$ and $0\le\mu_{\tilde{A}^I}(x)+\upsilon_{\tilde{A}^I}(x)\le 1$ ($x\in X$) are held. In this definition $\mu_{\tilde{A}^I}(x),\upsilon_{\tilde{A}^I}(x)$ ($x\in X$) are called degree of membership and degree of non-membership respectively. Also for $x\in X$ the following relation calculates degree of hesitation $(h(x))$.

$$h(x)=1-\mu_{\tilde{A}^I}(x)-\upsilon_{\tilde{A}^I}(x) \tag{2}$$

Definition 3. An intuitionistic fuzzy set (IFS) $\tilde{A}^I=\{\langle x,\mu_{\tilde{A}^I}(x),\upsilon_{\tilde{A}^I}(x)\rangle:x\in X\}$ must have the following two conditions,

1) There should be a real number r such that $\mu_{\tilde{A}^I}(r)=1$ and $\upsilon_{\tilde{A}^I}(r)=0$,
2) $\mu_{\tilde{A}^I}(x)$ and $\upsilon_{\tilde{A}^I}(x)$ are piecewise continuous mapping from the set of real numbers to the interval $[0,1]$ where $0\le\mu_{\tilde{A}^I}(x)+\upsilon_{\tilde{A}^I}(x)\le 1$.

The membership and non-membership functions of the triangular intuitionistic fuzzy number (TIFN) $\tilde{A}^I=(a_1,a_2,a_3;a_1',a_2,a_3')$ are defined as follow,

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$$\mu_{\tilde{A}^I}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1} & a_1<x\le a_2\\ \dfrac{a_3-x}{a_3-a_2} & a_2\le x<a_3\\ 0 & \text{otherwise}\end{cases} \tag{3}$$

$$\upsilon_{\tilde{A}^I}(x)=\begin{cases}\dfrac{a_2-x}{a_2-a_1'} & a_1<x\le a_2\\ \dfrac{x-a_2}{a_3'-a_2} & a_2\le x<a_3\\ 1 & \text{otherwise}\end{cases} \tag{4}$$

where $a_1'\le a_1<a_2<a_3\le a_3'$. These functions are schematically shown by Figure 1.

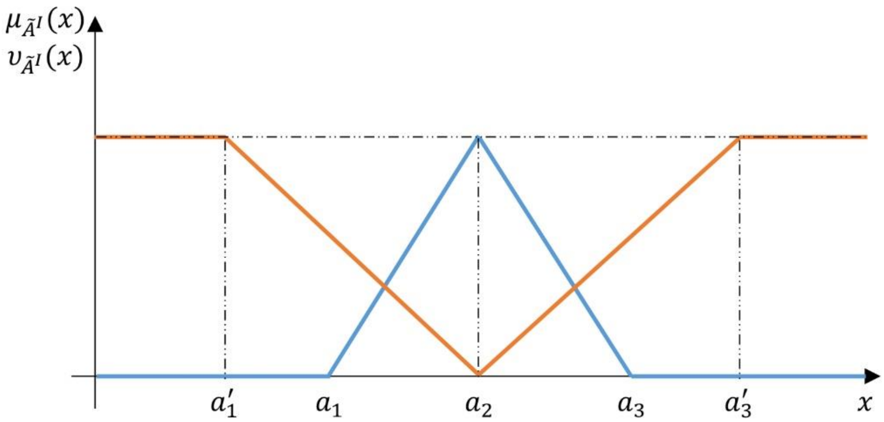

Figure 1. Membership and non-membership functions for a TIFN.

Definition 4. The following operations can be done on TIFNs $\tilde{A}^I=(a_1,a_2,a_3;a_1',a_2,a_3')$ and $\tilde{B}^I=(b_1,b_2,b_3;b_1',b_2,b_3')$,

$$\tilde{A}^I\oplus\tilde{B}^I=(a_1+b_1,a_2+b_2,a_3+b_3;a_1'+b_1',a_2+b_2,a_3'+b_3') \tag{5}$$

$$\tilde{A}^I\Theta\tilde{B}^I=(a_1+b_3,a_2+b_2,a_3+b_1;a_1'+b_3',a_2+b_2,a_3'+b_1') \tag{6}$$

$$\tilde{A}^I\otimes\tilde{B}^I=(l_1,l_2,l_3;l_1',l_2,l_3') \tag{7}$$

$$k\tilde{A}^I=(ka_1,ka_2,ka_3;ka_1',ka_2,ka_3') \qquad k\ge 0 \tag{8}$$

$$k\tilde{A}^I=(ka_3,ka_2,ka_1;ka_3',ka_2,ka_1') \qquad k<0 \tag{9}$$

where $l_1 = \min\{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$, $l_2 = a_2b_2$, $l_3 = \max\{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$, $l'_1 = \min\{a'_1b'_1, a'_1b'_3, a'_3b'_1, a'_3b'_3\}$, $l'_3 = \max\{a'_1b'_1, a'_1b'_3, a'_3b'_1, a'_3b'_3\}$.

Definition 5. Considering $\tilde{A}^I = (a_1, a_2, a_3; a'_1, a_2, a'_3)$, the score function of the membership and non-membership functions $\mu_{\tilde{A}^I}$ and $\upsilon_{\tilde{A}^I}$ (shown by $S(\mu_{\tilde{A}^I})$ and $S(\upsilon_{\tilde{A}^I})$ respectively), is calculated as $\frac{a_1 + 2a_2 + a_3}{4}$ and $\frac{a'_1 + 2a_2 + a'_3}{4}$, respectively. Now, the accuracy function of $\tilde{A}^I = (a_1, a_2, a_3; a'_1, a_2, a'_3)$ is calculated by the formula $f(\tilde{A}^I) = \frac{S(\mu_{\tilde{A}^I}) + S(\upsilon_{\tilde{A}^I})}{2}$.

Theorem 1. Let $k_i \in \mathbb{R}, i = 1, 2, ..., n$ **.** The accuracy function $f : IF(\mathbb{R}) \to \mathbb{R}$ is a linear function, namely: $f\left(k_1\tilde{A}^{I1} + k_2\tilde{A}^{I2} + ... + k_n\tilde{A}^{In}\right) = k_1 f\left(\tilde{A}^{I1}\right) + k_2 f\left(\tilde{A}^{I2}\right) + ... + k_n f\left(\tilde{A}^{In}\right)$.

Proof. Let $\tilde{A}^{Ii} = \left(a_1^i, a_2^i, a_3^i; a_1'^i, a_2^i, a_3'^i\right)$ for $i = 1, \ldots, n$ be n TIFNs. Then for $k_i > 0,\ i = 1, \ldots, n$, we have,

$$f\left(k_1\tilde{A}^{I1} + k_2\tilde{A}^{I2} + ... + k_n\tilde{A}^{In}\right)$$
$$= f\left(\left(k_1a_1^1, k_1a_2^1, k_1a_3^1; k_1a_1'^1, k_1a_2^1, k_1a_3'^1\right) + ... + \left(k_na_1^n, k_na_2^n, k_na_3^n; k_na_1'^n, k_na_2^n, k_na_3'^n\right)\right)$$
$$= f\left(\left(k_1a_1^1 + ... + k_na_1^n, k_1a_2^1 + ... + k_na_2^n, k_1a_3^1 + ... + k_na_3^n; k_1a_1'^1 + ... + k_na_1'^n, k_1a_2^1 + ... + k_na_2^n, k_1a_3'^1 + ... + k_na_3'^n\right)\right)$$

$$= \frac{\left(\left(k_1a_1^1 + ... + k_na_1^n\right) + 2\left(k_1a_2^1 + ... + k_na_2^n\right) + \left(k_1a_3^1 + ... + k_na_3^n\right) + \left(k_1a_1'^1 + ... + k_na_1'^n\right) + 2\left(k_1a_2^1 + ... + k_na_2^n\right) + \left(k_1a_3'^1 + ... + k_na_3'^n\right)\right)}{8}$$

$$= \frac{\left(k_1\left(a_1^1 + 2a_2^1 + a_3^1 + a_1'^1 + 2a_2^1 + a_3'^1\right) + ... + k_n\left(a_1^n + 2a_2^n + a_3^n + a_1'^n + 2a_2^n + a_3'^n\right)\right)}{8}$$
$$= \frac{k_1\left(a_1^1 + 2a_2^1 + a_3^1 + a_1'^1 + 2a_2^1 + a_3'^1\right)}{8} + ... + \frac{k_n\left(a_1^n + 2a_2^n + a_3^n + a_1'^n + 2a_2^n + a_3'^n\right)}{8}$$
$$= k_1 f\left(\tilde{A}^{I1}\right) + ... + k_n f\left(\tilde{A}^{In}\right)$$

Definition 6. The following comparisons can be done on TIFNs $\tilde{A}^I = (a_1, a_2, a_3; a'_1, a_2, a'_3)$ and $\tilde{B}^I = (b_1, b_2, b_3; b'_1, b_2, b'_3)$,

- $\tilde{A}^I \geq \tilde{B}^I$ if $f(\tilde{A}^I) \geq f(\tilde{B}^I)$,
- $\tilde{A}^I \leq \tilde{B}^I$ if $f(\tilde{A}^I) \leq f(\tilde{B}^I)$,

- $\tilde{A}^I = \tilde{B}^I$ if $f\left(\tilde{A}^I\right) = f\left(\tilde{B}^I\right)$,
- $\min\left\{\tilde{A}^I, \tilde{B}^I\right\} = \tilde{A}^I$ if $\tilde{A}^I \le \tilde{B}^I$ or $\tilde{B}^I \ge \tilde{A}^I$,
- $\max\left\{\tilde{A}^I, \tilde{B}^I\right\} = \tilde{A}^I$ if $\tilde{A}^I \ge \tilde{B}^I$ or $\tilde{B}^I \le \tilde{A}^I$.

3. Intuitionistic fuzzy transportation problem of type-2

In a primal transportation problem some products are to be sent from some sources to some destinations in a way that the supplying amount of the sources and the demand values of the destinations are respected. Each route from the sources to the destinations has a transportation cost for each unit of transported product. The objective of this problem is to minimize the total transportation cost. In most of real cases of this problems, it is almost impossible to determine an exact value for the unit transportation cost of each route. Therefore, use of interval or fuzzy values can be of interest to model the real cases of this problem. As for determining the unit transportation costs the decision maker may hesitate, it would be more realistic to consider the cost as IFN to consider both the uncertainty and the hesitation of the cost determination procedure. In this study, the costs of the above-mentioned transportation problem are TIFNs. To model such problem the following notations are defined.

- m is the number of sources being indexed by i .
- n is the number of destinations being indexed by j .
- a_i is the amount of product supplied by source i .
- b_j is the amount of product demanded by destination j .
- $\tilde{c}_{ij}^I = \left(c_{ij,1}, c_{ij,2}, c_{ij,3}; c'_{ij,1}, c'_{ij,2}, c'_{ij,3}\right)$ is the TIFN for the cost of sending one unit of product from source i to destination j .
- X_{ij} is a continuous variable showing the amount of product sent from source i to destination j .

Now the mathematical formulation of the transportation problem with unit costs of TIFN is given as (see Singh and Yadav, 2016),

$$\min \tilde{Z}^I = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}^I X_{ij} \tag{10}$$

subject to

$$\sum_{j=1}^{n} X_{ij} = a_i \qquad \forall i \tag{11}$$

$$\sum_{i=1}^{m} X_{ij} = b_j \qquad \forall j \tag{12}$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \qquad (13)$$

$$X_{ij} \geq 0 \qquad \forall i, j \qquad (14)$$

The above problem is also called intuitionistic fuzzy balanced transportation problem of type-2 which is summarized as IFBTP-2 (see Singh and Yadav, 2016).

If $\tilde{u}_i^I$ and $\tilde{v}_j^I$ are defined as the intuitionistic fuzzy dual variables of i-th row constraint and j-th column constraint of the IFBTP-2, the intuitionistic fuzzy dual of the IFBTP-2 is obtained by the following formulation.

$$\max \tilde{W}^I = \sum_{i=1}^{m} a_i \tilde{u}_i^I \oplus \sum_{j=1}^{n} b_j \tilde{v}_j^I \qquad (15)$$

subject to

$$\tilde{u}_i^I \oplus \tilde{v}_j^I \leq \tilde{c}_{ij}^I \qquad \forall i, j \qquad (16)$$

In the next section, a simple method which has many advantages comparing to the existing approaches of the literature, is proposed to solve the IFBTP-2.

4. Solution methodology

In this section, a simple method which has many advantages comparing to the existing approach of the literature Singh and Yadav (2016), is proposed to solve the IFBTP-2.

4.1. On the approach of Singh and Yadav (2016)

Singh and Yadav (2016), using the accuracy function and the relations of definitions 5 and 6, applied three approaches of intuitionistic fuzzy north west corner method (IFNWCM), intuitionistic fuzzy least cost method (IFLCM) and intuitionistic fuzzy Vogel's approximation method (IFVAM) to obtain an initial basic feasible solution (IBFS) for the IFBTP-2. After that the intuitionistic fuzzy modified distribution method (IFMODIM) was used to obtain the intuitionistic fuzzy optimal solution of the IFBTP-2 employing the IBFS. This method is constructed based on duality and complementary slackness theorem. In the proposed approach, all the arithmetic operations are done on the TIFNs. The approach of Singh and Yadav (2016) is summarized by the following steps.

Step 1. Find an IBFS of the IFBTP-2 by IFNWCM or IFLCM or IFVAM.

Step 2. Calculate $\tilde{z}_{ij}^I = \tilde{u}_i^I \oplus \tilde{v}_j^I$, $\tilde{d}_{ij}^I = \tilde{z}_{ij}^I \Theta \tilde{c}_{ij}^I$, and $f\left(\tilde{d}_{ij}^I\right)$ for each non-basic variable. Stop (when all $f\left(\tilde{d}_{ij}^I\right) \leq 0$) or select an entering column.

Step 3. Determine an existing column.

Step 4. Calculate the new basic feasible solution and go to Step 2.

4.2. Proposed solution approach

The main contribution of this study is to reduce the computational difficulty of the proposed approach of Singh and Yadav (2016). To do that, we propose the following procedure to solve the IFBTP-2.

Step 1. Use the accuracy function of Definition 5 to calculate the rank of each intuitionistic fuzzy cost element of objective function (10) in the IFBTP-2 ($f(\tilde{c}_{ij}^{I})$). Then use the obtained ranks instead of their corresponding intuitionistic fuzzy cost in the IFBTP-2. Therefore, the IFBTP-2 is converted to the following crisp balanced transportation problem (CBTP),

$$\min Z = \sum_{i=1}^{m}\sum_{j=1}^{n} f\left(\tilde{c}_{ij}^{I}\right) X_{ij} \tag{17}$$

subject to

$$\sum_{j=1}^{n} X_{ij} = a_i \quad \forall i \tag{18}$$

$$\sum_{i=1}^{m} X_{ij} = b_j \quad \forall j \tag{19}$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \tag{20}$$

$$X_{ij} \geq 0 \quad \forall i, j \tag{21}$$

If u_i and v_j are defined as the intuitionistic fuzzy dual variables of i-th row constraint and j-th column constraint of the CBTP, the dual of the CBTP is obtained by the following formulation.

$$\max W = \sum_{i=1}^{m} a_i u_i \oplus \sum_{j=1}^{n} b_j v_j \tag{22}$$

subject to

$$u_i + v_j \leq f\left(\tilde{c}_{ij}^{I}\right) \quad \forall i, j \tag{23}$$

Step 2. Find an IBFS of the CBTP by North West corner method (NWCM), least cost method (LCM) and Vogel's approximation method (VAM).

Step 3. Calculate $z_{ij} = u_i + v_j$, $d_{ij} = z_{ij} - f\left(\tilde{c}_{ij}^I\right)$ for each basic and non-basic variables, respectively. Stop (when all $d_{ij} \leq 0$) or select an entering column.

Step 4. Determine an existing column.

Step 5. Calculate the new basic feasible solution and go to Step 2.

Step 6. Using the obtained optimal solution from Steps 1-5 and the intuitionistic fuzzy costs of the problem ($\tilde{c}_{ij}^I$), the intuitionistic fuzzy optimal value of objective function ($\tilde{Z}^I$) is obtained.

According to Theorem 1, the amount of $f(\tilde{Z}^I)$ obtained by the proposed approach will not be worse than what is obtained by the approach of Singh and Yadav (2016).

4.3. Comparing the approaches

The following paragraphs can prove the efficiency and simplicity of the proposed approach of this study comparing to the method proposed by Singh and Yadav (2016).

In the approach of Singh and Yadav (2016), the Step 1 can be done in three ways. To use the IFLCM for this step it is needed to find the minimum intuitionistic fuzzy transportation cost in intuitionistic fuzzy transportation tableau (IFTT) of order $m \times n$ until the order of 1×1 from the IFTT is remained. For this issue, the intuitionistic fuzzy transportation costs of the IFTT are compared applying the accuracy function and relations of definitions 5 and 6. It is notable that in the proposed approach of this study the comparison of intuitionistic fuzzy costs is done once. To do the Step 1 of the approach of Singh and Yadav (2016) using the IFVAM, first an intuitionistic fuzzy penalty for each row (also each column) of the IFTT with order of $m \times n$ by subtracting the minimum entry from the second minimum entry in each step. Then, the maximum intuitionistic fuzzy penalty is found using the accuracy function and relations of definitions 5 and 6. Of course, these calculations must be continued until the order of 1×1 from the IFTT is remained. Therefore, too much number of intuitionistic fuzzy operators e.g. additions, subtractions and comparison of the accuracy function values of the TIFNs are required where in the proposed approach of this study the comparison of intuitionistic fuzzy costs is done once.

Similar difficulty happens when the Steps 2-4 of the approach of Singh and Yadav (2016) is performed. First, the system of $\tilde{u}_i^I \oplus \tilde{v}_j^I = \tilde{c}_{ij}^I$ for $m + n - 1$ intuitionistic fuzzy equations related to the basic variables is solved. Then for each non-basic variable the values $\tilde{z}_{ij}^I = \tilde{u}_i^I \oplus \tilde{v}_j^I$, $\tilde{d}_{ij}^I = \tilde{z}_{ij}^I \ominus \tilde{c}_{ij}^I$, and $f(\tilde{d}_{ij}^I)$ are calculated. Therefore, too much number of intuitionistic fuzzy operators e.g. additions, subtractions and comparison of the accuracy function values of the TIFNs are required where in the proposed approach of this study the comparison of intuitionistic fuzzy costs is done once.

5. Results and discussion

To study the performance of the proposed approach of this study, the same numerical examples considered by Singh and Yadav (2016) is solved in this section. These common examples make us able to compare the approaches in a correct way.

Example 1. This example consists of four sources and four destinations with given crisp supply and demand values and intuitionistic fuzzy costs of Table 1.

Table 1. The IFTT for the intuitionistic costs of the Example 1.

		Destinations				a_i
		1	2	3	4	
Sources	1	(2,4,5;1,4,6)	(2,5,7;1,5,8)	(4,6,8;3,6,9)	(4,7,8;3,7,9)	11
	2	(4,6,8;3,6,9)	(3,7,12;2,7,13)	(10,15,20;8,15,22)	(11,12,13;10,12,14)	11
	3	(3,4,6;1,4,8)	(8,10,13;5,10,16)	(2,3,5;1,3,6)	(6,10,14;5,10,15)	11
	4	(2,4,6;1,4,7)	(3,9,10;2,9,12)	(3,6,10;2,6,12)	(3,4,5;2,4,8)	12
b_j		16	10	8	11	45

Performing the Step 1 of the proposed approach of this study (Section 4.2) on this example, the crisp transportation tableau of Table 2 is obtained.

Table 2. The obtained crisp costs for transportation tableau of the Example 1.

		Destinations				a_i
		1	2	3	4	
Sources	1	3.75	4.75	6	6.5	11
	2	6	7.25	15	12	11
	3	4.25	10.25	3.25	10	11
	4	4	7.875	6.375	4.25	12
b_j		16	10	8	11	45

Now the classic north west corner method is applied to find a basic feasible solution for the Example 1. This solution is represented in the Table 3.

Table 3. The basic feasible solution obtained by the classic north west method for the Example 1. The right side number of each cell show its corresponding X_{ij} value.

		Destinations								a_i
		1		2		3		4		
Sources	1	3.75	11	4.75	-	6	-	6.5	-	11
	2	6	5	7.25	6	15	-	12	-	11
	3	4.25	-	10.25	4	3.25	7	10	-	11
	4	4	-	7.875	-	6.375	1	4.25	11	12
b_j		16		10		8		11		45

Now, the initial solution of Table 3 is to be optimized. Applying the classic modified distribution method this issue is done and the obtained optimal solution is shown by Table 4.

Table 4. The optimal solution obtained by the classic modified distribution method for the Example 1. The right side number of each cell show its corresponding X_{ij} value.

		Destinations								a_i
		1		2		3		4		
Sources	1	3.75	1	4.75	10	6	-	6.5	-	11
	2	6	11	7.25	-	15	-	12	-	11
	3	4.25	3	10.25	-	3.25	8	10	-	11
	4	4	1	7.875	-	6.375	-	4.25	11	12
b_j		16		10		8		11		45

In the case of this example, interestingly, the basic feasible solution (Table 3) and the optimal one (Table 4) are exactly the same as what obtained by the method of Singh and Yadav (2016) with many fuzzy ranking operations. The similarity of the objective function values are shown by Table 5.

Table 5. The optimal objective function values for Example 1.

Objective function value	Method	
	Proposed approach	Singh and Yadav (2016)
Fuzzy form ($\tilde{Z}^I$)	(126,204,282;78,204,352)*	(126,204,282;78,204,352)
Crisp form	206.75**	206.75***

* Obtained by multiplying the solution of Table 4 and the costs of Table 1.
** Obtained by multiplying the solution of Table 4 and the costs of Table 2.
***Obtained by calculating the accuracy function value of the intuitionistic fuzzy objective function value.

According to the similarity of the solutions obtained in Table 3 and Table 4 with those of Singh and Yadav (2016) method and also similarities of objective function values of Table 6, the similarity of the performances of the proposed method of this study and the method of Singh and Yadav (2016) is proved. Notably, the computational difficulty of the proposed approach of this study is much less than the computations of the approach of Singh and Yadav (2016).

Example 2. This example is the application given by Singh and Yadav (2016). It consists of three sources and four destinations with given crisp supply and demand values and intuitionistic fuzzy costs of Table 6.

Table 6. The IFTT for the intuitionistic costs of the Example 2.

		Destinations				a_i
		1	2	3	4	
Sources	1	(210,250,270; 200,250,280)	(600,700,750; 600,700,800)	(950,1000,1050; 900,1000,1100)	(3500,3700,3900; 3400,3700,4100)	4500
	2	(650,750,800; 600,750,850)	(350,400,450; 340,400,480)	(1000,1050,1100; 950,1050,1150)	(3600,3900,4600; 3500,3900,4600)	3500
	3	(2600,2800,3000; 2500,2800,3100)	(2100,2200,2300; 2100,2200,2350)	(2900,3100,3300; 2800,3100,3400)	(5400,5600,5800; 5300,5600,6000)	2000
b_j		3500	3000	2000	1500	10000

The solution obtained for this example by the proposed approach of this study has the positive variables of $X_{11} = 3500$, $X_{14} = 1000$, $X_{22} = 1500$, $X_{23} = 2000$, $X_{32} = 1500$, and $X_{34} = 500$ which is different than what has been obtained by Singh and Yadav (2016).

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On the other hand, another method of the literature (Antony *et al.*, 2014) has been used for this aim and the objective function value of all the methods are compared in Table 7.

Table 7. The optimal objective function values for Example 2.

		Objective function value	
		Fuzzy form $(\tilde{Z}^I)$	Accuracy function value $f(\tilde{Z}^I)$
Method	Antony *et al.* (2014)	(12585000, 13425000, 14395000; 12290000, 13425000, 14860000)	13478750
	Singh and Yadav (2016)	(12710000, 13425000, 14070000; 12400000, 13425000, 14605000)	13435625
	Proposed approach	(12610000, 13375000, 12310000; 13375000, 13425000, 14625000)*	13389375

*Obtained by multiplying the optimal solution and the costs of Table 6.

The obtained results shown by Table 7 prove the superiority of the proposed approach of this study comparing to the approaches of Singh and Yadav (2016) and Antony *et al.* (2014) in the terms of ranked intuitionistic objective function values. It is notable to remind that the proposed method of this study has much less computational difficulties comparing to the two other approaches.

6. Concluding remarks

A transportation problem in its balanced form with triangular intuitionistic fuzzy unit transportation costs was solved in this study. The fuzzy costs were crisped using the accuracy function of the literature. Then the algorithms e.g. the north west corner method, the least cost method, and the Vogel's approximation method was applied to obtain an initial basic feasible solution for the crisp version of the problem. After that the modified distribution method was used to obtain the optimal solution of the problem. Using the optimal solution and the initial triangular intuitionistic fuzzy unit transportation costs, the triangular intuitionistic fuzzy objective function value was calculated. The performed computational experiments showed the superiority of the proposed approach over those of the literature from the results' quality and the computational difficulty point of views.

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