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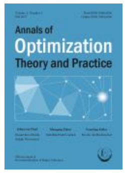

On planned time replacement of series-parallel system

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Abstract This paper investigate some characteristics of the age replacement model with minimal repair, by considering a series-parallel system with six units, such that the six units are having non-uniform failure rates and are subjected to two different types of failures, which are Type I and Type II failures. The six units of the system formed three subsystems, which are subsystems A, B and C. Subsystem A is having three parallel units, subsystem B is having a single unit and subsystem C is having two parallel units. We constructed age replacement model with minimal repair that will determine the optimal replacement time of the series-parallel system. Furthermore, we also considered some modifications of the age replacement model with minimal repair constructed. Finally, some numerical examples are given to illustrate the characteristics of the age replacement models with minimal repair constructed. From the results obtained, it was observed that the optimal replacement time of the system when the three units of A are in parallel is higher than when the three units of A are in series. It was also observed that, the optimal replacement time obtained from the standard age replacement model is higher than the optimal replacement time obtained from standard age replacement model with minimal repair

Keywords Optimal; Repair; Replacement; Rate; System; Time

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1. Introduction

Different stochastic models have been designed to study the behavior of a repairable system that are liable to fail. These models are associated with two types of repair i.e. the perfect repair and the minimal repair. Perfect repair restores a system to as good as new while in minimal repair the system will not be restored to as good as new. Though the motive of the repair is not to bring back the system to its normal self but to return it back to operation as soon as possible.

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Several researchers have presented tremendous works based on the idea of minimal repair. See for instance; Sandev and Aven (1999) studied the optimal replacement problem of a monotone system comprising n components, where the components are ‘‘minimally’’ repaired at failures. Pham (2003) presented age replacement model of a series and parallel systems.

Sharma *et al.* (2002) evaluated the expressions for expected cost for a system with replacement and minimal, and furthermore discussed the maintenance costs of various policies. Ouali and Yacout (2003) developed an optional replacement policy for the maintenance of two non-identical components connected in series configuration, where by each component is replaced correctively whenever it fails and preventively only if its age reaches or exceeds a preventive replacement age T when the other component fails. Nakagawa (2005) also discussed other modifications of age replacement model. Chien and Sheu (2006) proposed age replacement policy for an operating system which is subjected to shocks that arrive according to a non-homogeneous Poisson process, and as shocks occur the system has two types of failure: type I failure (minor) or type II failure (catastrophic). Chen (2007) constructed a cache document replacement policy which content can be tailored to the specific requirements of a caching system. Wang *et al.* (2008) presented a condition-based order-replacement policy for a single-unit system, aiming to optimize the condition-based maintenance and the spare order management jointly. Aven and Castro (2008) presented a minimal repair replacement model of a one unit system subjected to two types of failures. Yusuf and Ali (2012) considered two parallel units in which both units operate simultaneously, and the system is subjected to two types of failures. Type I failure is minor and occur with the failure of a single component and is checked by minimal repairs, while type II failure is catastrophic in which both components failed and the system is replaced. Xu et al. (2012) investigated on replacement scheduling for non-repairable safety-related systems (SRS) with multiple components and states, and their aim is to determine the cost-minimizing time for replacing SRS while meeting the required safety. Jain and Gupta (2013) investigated on the reliability analysis of a repairable system consisting of single repairman who can take multiple vacations, such that the system failure may occur due to two types of faults termed as major and minor. They assumed that the repairman can perform some other tasks when either the system is idle or waiting for recovery from the faults. Wang *et al.* (2014) introduced a two-level inspection policy model for a single component plant system based on a three-stage failure process, such that the failure process divide the system′s life into three stages: good, minor defective and severe defective stages. Malki *et al.* (2015) investigated on age replacement policies for two-component parallel system with stochastic dependence. The stochastic dependence considered, is model by a one-sided domino effect. Coria *et al.* (2015) proposed an analytical optimization method for preventive maintenance (PM) policy with minimal repair at failure, periodic maintenance, and replacement for systems with historical failure time data influenced by a current PM policy. Waziri *et al.* (2019) furthermore, extended the work of Aven and Castro (2008), by constructing some discounted age replacement models with minimal repair for a serial system, which is subjected to two types of failures.

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Mohammadi *et al.* (2018) presented reliability allocation problem of series-parallel systems by considering common cause failure. Chen *et al.* (2019) presented replacement policies with general models. Mannai and Gasmi (2018) dealt with the design of kk-out-of-nn system under first and last replacement in reliability theory. Okamra and Dohi (2017) studied moment-based approach for some age-based replacement problems. Shen *el al.* (2020) discussed the optimal switching policy for warm standby systems subjected to standby failure mode. Wang *et al.* (2019) presented an extended age maintenance models and its optimization for series and parallel systems. Zhao *et al.* (2020) discussed the preventive replacement policies with time of operations, mission durations, minimal repairs and maintenance triggering approaches. Zhao presented preventive replacement policies for parallel systems with deviation costs between replacement and failure. Zhu *et al.* (2018) dealt with redundancy allocation for serial-parallel system considering heterogeneity of components. Yusuf *et al.* (2019) analyzed the profit of a series-parallel system under partial and complete failures. Waziri *et al.* (2019) presented replacement analysis of a serial system based on discounted factor. Ling *et al.* (2019) discussed the optimal heterogeneous components grouping in series-parallel and parallel-series systems. Xie *el al.* (2020)presented reliability and and barrier assessment of series–parallel systems subject to cascading failures.

Mustafa (2017) presented reliability improvement of series-parallel system using modified Weibull distribution. Sibai (2014) dealt with the modelling and evaluation of series parallel photovoltaic modules. Chauhan and Malik (2016) dealt with reliability evaluation of series-parallel and parallel-series systems for arbitrary values of the parameters. Fallahnezhad and Najafian (2017) presented model of preventive maintenance for parallel, series, and single-item replacement systems based on statistical analysis. Xu *et al.* (2016) presented maintenance problem for series–parallel system under economic dependence. Peng *et al.* (2016) analyzed reliability of series-parallel phased-mission systems subject to fault-level coverage. Khatab *et al.* (2017) presented maintenance optimization for series-parallel systems alternating missions and scheduled breaks with stochastic durations.

This present study tends to develop age replacement models with minimal repair for parallel-series system subjected to two types of failures so as to address the problem of sudden failure of a multi-component systems, avoid rising maintenance cost of a multi-component system, and to provide some characteristics of the age replacement model with minimal repair. The organization of the paper is as follows: Section 2 presents the methodology of the study. Section 3 gives the proposed models. The numerical results are presented in section 4. Finally, the paper is concluded in section 5.

2. Methology

2.1 Notations

$r_i^*(t)$: Type I failure rate of unit A_i of subsystem A, for $i = 1, 2, 3$.
$r_b(t)$: Type II failure rate of subsystem B.
$r_i(t)$: Type II failure rate of unit C_i of subsystem C, for $i = 1, 2$.

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$R_i^*(t)$: reliability function of unit A_i of subsystem A, for $i = 1, 2, 3$.
C_b : cost of minimal repair of subsystem B due to Type II failure.
C_{1m} : cost of minimal repair of unit C_1 of subsystem C due to Type II failure.
C_{2m} : cost of minimal repair of unit C_2 of subsystem C due to Type II failure.
C_p : cost of planned replacement of the system at time T.
C_r : cost of un-planned replacement of the system due to Type I failure.
T^*: optimal replacement time.

2.2 Description of the system

Consider a system comprising of three subsystems A, B and C in series. Subsystem A consist of three active parallel units, which are A_1, A_2 and A_3. Subsystem B consist of a single active unit. While, subsystem C consist of two active units, which are $C_1$and C_2. See figure 1 below. The three units A_1, A_2 and A_3 are all subjected to Type I failure, which is an un-repairable failure. Subsystem B is only subjected to Type II failure, which is repairable failure. Also, the two units $C_1$and C_2, are only subjected to Type II failure. The system fails due to Type I failure, if all the three units of subsystem A fails due to Type I failure, at such failure, the system is replaced completely. While the system fails due to Type II failure, if subsystem B or all the two units of subsystem C fails due to Type II failure, at such failure the system is minimally repaired.

2.3 Assumptions

1. If subsystem B failed due to Type II failure, then the failed subsystem undergoes minimal repair, and allow the system operating from where it stopped.
2. If all the two units of subsystem C failed due to Type II failure, then the failed units will undergoes minimal repair, and allow the system operating from where it stopped.
3. The system is replaced at a planned replacement time $T (T > 0)$ after its installation or at a state, where all the three units of subsystem A fails due to Type I failure, whichever occurs first.
4. The cost of planned replacement of the system is less than the cost of un-planned replacement.
5. Both Type I failure rate and Type II failures rate arrives according to a non-homogeneous Poisson process
6. The cost of minimal repair, planned replacement and un-planned replacement are all positive numbers.

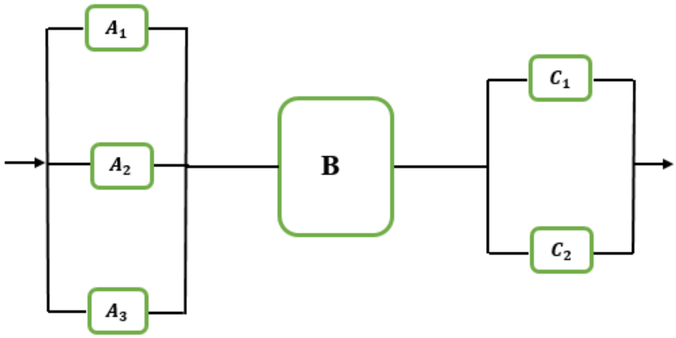

Figure 1.System (Series-parallel configuration)

3. The replacement models

Based on the assumptions, we have the probability that the system will be replaced at planned time T before Type I failure occurs as

$$R_A^*(T) = 1 - \left(1 - R_1^*(T)\right)\left(1 - R_2^*(T)\right)\left(1 - R_3^*(T)\right) \tag{1}$$

where

$$R_i^*(T) = e^{-\int_0^T r_i^*(t)dt} \qquad for \;\; i = 1, 2, 3. \tag{2}$$

The cost of unplanned replacement of the system in one replacement cycle is

$$C_r\left(1 - R_A^*(T)\right) \tag{3}$$

The cost of planned replacement of the system in one replacement cycle is

$$C_p R_A^*(T) \tag{4}$$

The cost of minimal repair of subsystem B in one replacement cycle is

$$\int_0^T C_b r_b(t) R_A^*(t)dt \tag{5}$$

The cost of minimal repair of unit c_1 of subsystem C in one replacement cycle is

$$\int_0^T C_{1m} r_1(t) R_A^*(t)dt \tag{6}$$

The cost of minimal repair of unit c_2 of subsystem C in one replacement cycle is

$$\int_0^T C_{2m} r_2(t) R_A^*(t)dt \tag{7}$$

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We have the total replacement cost rate of the system in one replacement cycle as

$$CA(T) = \frac{C_r\left(1 - R_A^*(T)\right) + C_p R_A^*(T) + \int_0^T C_b r_b(t) R_A^*(t)dt + \int_0^T C_{1m} r_1(t) R_A^*(t)dt + \int_0^T C_{2m} r_2(t) R_A^*(t)dt}{\int_0^T R_A^*(t)dt} \tag{8}$$

Noting that, $CA(T)$ is adopted as the objective function of an optimization problem, and the aim is to determine an optimal replacement time T^* that minimizes $CA(T)$.

Now, if we assumed that three units of subsystem A are arranged in series, then with this modification, this implies that the system is replaced completely, if at least one of the three units of subsystem A fails due to Type I failure. Then we have the probability that the system will be replaced a planned time T before at least one of the three units of subsystem A fails due to Type I failure as

$$R_B^*(T) = R_1^*(T)\, R_2^*(T) R_3^*(T) \tag{9}$$

So, for this modification, we have the total replacement cost rate of the system in one replacement cycle as

$$CB(T) = \frac{C_r(T)\left(1 - R_B^*(T)\right) + C_p(T) R_B^*(T) + \int_0^T C_b(t) r_b(t) R_B^*(t)dt + \int_0^T C_{1m}(t) r_1(t) R_B^*(t)dt + \int_0^T C_{2m}(t) r_2(t) R_B^*(t)dt}{\int_0^T R_B^*(t)dt} \tag{10}$$

Next, if we also assumed that the other two subsystems (subsystem B and subsystem C) of the system are not subjected to neither Type I nor Type II failure. So, for this modification, we have the total replacement cost rate of the system in one replacement cycle as

$$CC(T) = \frac{C_r\left(1 - R_A^*(T)\right) + C_p R_A^*(T)}{\int_0^T R_A^*(t)dt} \tag{11}$$

Noting that, equation (11) obtained is known as the standard age replacement model.

4. Numerical example

In this section, we will give two numerical examples so as to illustrate the characteristics of the age replacement models constructed. In try to do that, let the rate of Type I failure of the three units of subsystem A follows Weibull distribution:

$$r_i^*(t) = \lambda_i^* \propto_i^* t^{\propto_i^* - 1} \qquad t \geq 0, \qquad i = 1, 2, 3, \tag{12}$$

where $\propto_i^* > 1$.

Again, let the rate of Type II failure of subsystem B follows Weibull distribution:

$$r_b(t) = \lambda_b \propto_b t^{\propto_b - 1} \qquad t \geq 0 \tag{13}$$

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where $\propto_b > 1$.

Also, let the rate of Type II failure of the two units of subsystem C follows Weibull distribution:

$$r_i(t) = \lambda_i \propto_i t^{\propto_i - 1} \qquad t \geq 0\,, \qquad i = 1, 2, 3, \tag{14}$$

where $\propto_i > 1$.

Let set of parameters be used throughout this particular example:
$\propto_b = 4$, $\propto_2 = 3$, $\propto_3 = 3$, $\lambda_b = 0.03$, $\lambda_2 = 0.002$, $\lambda_3 = 0.03$, $\propto_1^* = 4$, $\propto_2^* = 3.5$, $\propto_3^* = 4$, $\lambda_1^* = 0.00033$, $\lambda_2^* = 0.00025$ and $\lambda_3^* = 0.00030$. Also, let the set of costs replacement/repair be used throughout this particular example : $C_r = 70$, $C_p = 50$, $C_b = 7$, $C_{1m} = 5$ and $C_{2m} = 5$. Consequently, by substituting the parameters in equations 12, we obtained the Type I failure rates of three units of subsystem A as

$$r_1^*(t) = 0.00132t^3 \tag{15}$$

And

$$r_2^*(t) = 0.000875t^{2.5} \tag{16}$$

And

$$r_3^*(t) = 0.0012t^3 \tag{17}$$

Also by substituting the parameters in equations 13, we obtained the Type II failure rate of subsystem B as

$$r_b(t) = 0.12t^3 \tag{18}$$

Also by substituting the parameters in equations 14, we obtained the Type II failure rates of the two units of subsystem C as

$$r_2(t) = 0.006t^2 \tag{19}$$

And

$$r_3(t) = 0.09t^2 \tag{20}$$

The tables and the graphs below, are the results obtained by substituting the cost of repair/replacement and equations (15) to (20) in the cost rates CA(T), CB(T) and CC(T).

Table 1.The values of CA(T), CB(T) and CC(T) versus planned replacement T.

T	1	2	3	4	5	6	7	8	9	10
CA(T)	250.42	126.84	87.84	71.20	64.71	64.61	69.42	77.68	86.67	93.00
CB(T)	250.61	128.28	92.40	81.19	81.79	87.40	90.59	92.42	92.98	94.06
CC(T)	250.00	125.00	83.33	62.51	50.10	42.19	37.60	36.12	37.33	40.00

Table 2.The optimal replacement time of the system from CA(T) as $C_p$decreases.

C_p	40	30	20	10
T^*	$T^* = 5$	$T^* = 5$	$T^* = 4$	$T^* = 3$

Table 3.The optimal replacement time of the system from CA(T) as C_r increases.

C_r	80	90	100	110
T^*	$T^* = 5$	$T^* = 5$	$T^* = 5$	$T^* = 5$

Table 4. The optimal replacement time of the system from CB(T) as C_p decreases.

C_p	40	30	20	10
T^*	$T^* = 4$	$T^* = 4$	$T^* = 3$	$T^* = 3$

Table 5.The optimal replacement time of the system from CB(T) as C_r increases.

C_r	80	90	100	110
T^*	$T^* = 4$	$T^* = 4$	$T^* = 4$	$T^* = 4$

Table 6.The optimal replacement time of the system from CC(T) as C_p decreases.

C_p	40	30	20	10
T^*	$T^* = 8$	$T^* = 7$	$T^* = 7$	$T^* = 6$

Table 7.The optimal replacement time of the system from CC(T) as C_r increases.

C_r	80	90	100	110
T^*	$T^* = 8$	$T^* = 8$	$T^* = 7$	$T^* = 7$

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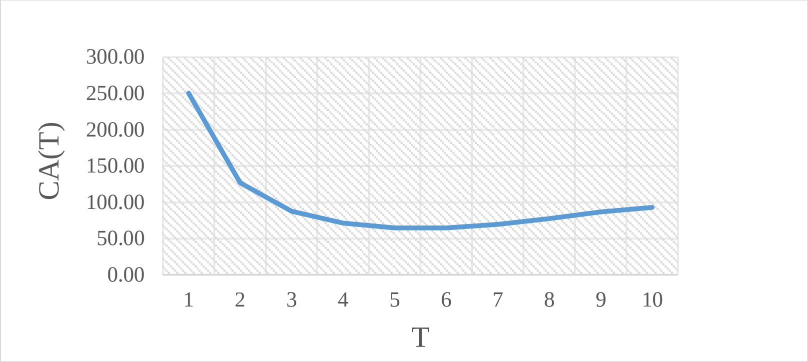

Figure 2.The plot of CA(T) versus T.

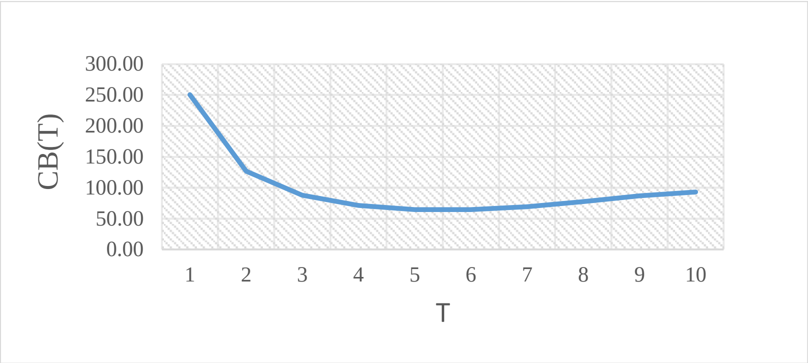

Figure 3.The plot of CB(T) versus T.

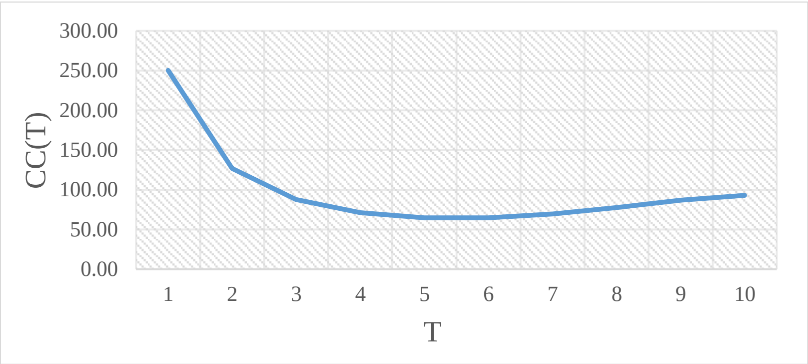

Figure 4.The plot of CC(T) versus T.

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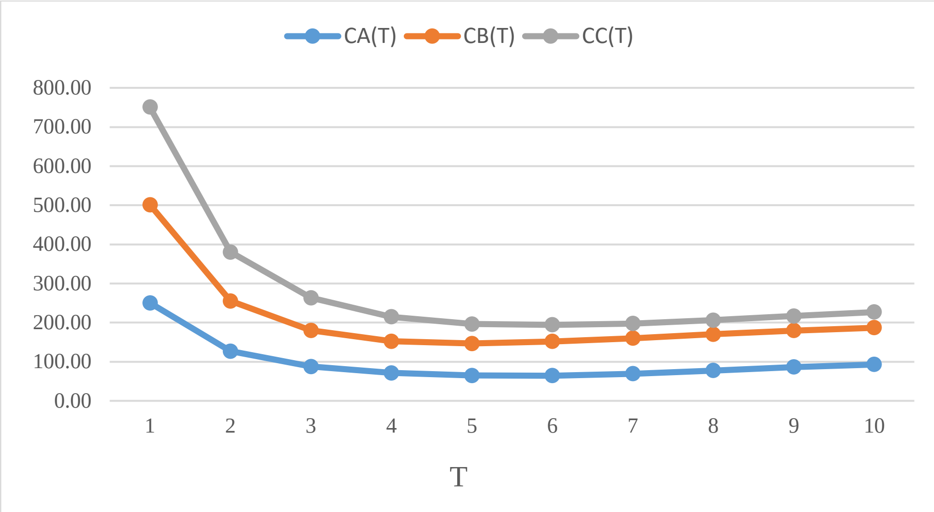

Figure 5.Comparing CA(T), CB(T) and CC(T).

Some observations from the results obtained are as follows

1. Observe that, we have the optimal replacement time for the system from the cost rate CA(T) as 6, that is, $T^* = 6$, with minimal cost rate $CA(T^* = 6) = 64.61$. See figure 2 below for the plots of $CA(T)$ versus T.
2. Observe from table 1, we have the optimal replacement time of the system from the cost rate CB(T) as 4, that is, $T^* = 4$, with minimal cost rate $CB(T^* = 4) = 81.19$. See figure 3 below for the plots of $CB(T)$ versus T.
3. Observe from table 1, we have the optimal replacement time of the system from the cost rate CC(T) as 8, that is, $T^* = 8$, with minimal cost rate $CC(T^* = 8) = 36.12$. See figure 3 below for the plots of $CC(T)$ versus T.
4. Observe from tables 2, 4, and 6, that the optimal replacement time of the system sometimes decreases slightly as the cost of planned replacement (C_p) decreases.
5. Observe from tables 3, 5 and 7, that the optimal replacement time of the system sometimes decreases slightly as the cost of un-planned replacement (C_r) increases.
6. Observe from table 1 and figure 5, we have $CA(T) \leq CB(T) \leq CC(T)$.
7. Observe from table 1, that, if the three units of subsystem A are in parallel, then the optimal replacement time $(T^* = 6)$, is higher than that of the optimal replacement time $(T^* = 4)$ when the three units of subsystem A are in series.
8. Observe from table 1, that the optimal replacement time obtained from CC(T) (which is $T^* = 8$) is higher than that is obtained from CA(T) (which is $T^* = 6$).

5. Conclusion and recommendations

The planned time replacement policy tells us when best to replace a component or system, so as to avoid failure during operation, and the policy minimizes the cost of

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maintenance. It is well known that the failure rate of components in a system may varies. In this paper, we constructed age replacement model with minimal repair of a series-parallel system, such that the system contained three subsystems, which are subsystem A, subsystem B and subsystem C. Furthermore, some modifications of the constructed age replacement model were also considered. Numerical example was given, so as to investigate the characteristics of the constructed age replacement model with minimal repair of the series-parallel system. It was also observed that, the optimal replacement time obtained from the standard age replacement model is higher than the optimal replacement time obtained from standard age replacement model with minimal repair. This paper will help reliability engineers and maintenance managers in choosing the best decisions.

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