Annals of Optimization Theory and Practice

Volume 3, Number 2, 49-64

November 2020

DOI: 10.22121/aotp.2020.250749.1043

Decision making under intuitionistic fuzzy metric distances

Kousik Bhattacharya . Sujit Kumar De ✉

Department of Mathematics, Midnapore College (Autonomous), 721101, West Bengal, India

✉ skdemamo2008.com@gmail.com

(Received: September 30, 2020 / Accepted: November 9, 2020)

Abstract This article deals with qualitative difference between two intuitionistic fuzzy sets with the help of standard pseudo metric and metric spaces. Some definitions over metric spaces, pseudo metric spaces, intuitionistic fuzzy sets, indeterminacy and the formula of measuring metrices have been incorporated. Numerical illustrations, graphical illustrations, area of applications and ranking for decision making are discussed to show the novelty of this article. Finally, conclusions and scope of future works are mentioned.

Keywords Metric distance; Pseudo metric distance; Intuitionistic fuzzy set; Ranking

1. Introduction

In traditional set theory (classical), the idea of member and non-member of an element in a set was sudden i.e., an element either belongs to a set or not belongs to the set. But there was no knowledge about the transition of an element from member to non-member of the set and vice-versa. Zadeh (1965) has solved these ambiguities through his new invention, the fuzzy set theory. Since then, numerous research articles have been studied over the fuzzy set itself to explain the real-world phenomenon. Bellman and Zadeh (1970) introduced a new concept of Decision making in a fuzzy environment. Piegat (2005) gives us a new definition of fuzzy set. The concepts of dense fuzzy set studied by De and Beg (2016) to discuss the frequent learning effect of the fuzzy parameters. Analysing the behaviour of human thinking process De (2018) developed a new inexact set which is known as Triangular dense fuzzy lock set and its new defuzzification method. After this invention many articles have been made by eminent researchers Maity *et al.* (2020), Maity *et al* (2020), De and Mahata (2019) . to control the individual or group decision making problems on pollution sensitive inventory modelling. Baez-Sancheza *et al.* (2012) discussed polygonal fuzzy sets and numbers extensively.

Published by Firouzabad Institute of Higher Education, Firouzabad, Fars, Iran

Trapezoidal approximations of fuzzy numbers and their existence as well as uniqueness and continuity are exclusively discussed by Ban and Coroianu (2014). Chutia *et al.* (2010) contributes to find membership function of a fuzzy number. De and Mahata (2017) designed a fuzzy backorder model where demand rate is considered as cloudy inexactness. Decision making in a bi-objective inventory problem was discussed by eminent researchers like De and Pal (2016). Mahanta *et al.* (2010) made a new approach of fuzzy arithmetic without using alpha cuts. Mao *et al.* (2018) extensively analysed about the relation between cloud aggregation operators and multi attribute group decision making in interval valued hesitant fuzzy linguistic environment. However, Atanassov (1986), Atanassov (1983) introduces a new approach of fuzzy set namely Intuitionistic fuzzy sets in terms of membership and non-membership function. Some notable works over the EOQ models on fuzzy environments as well as IFS may be pointed out over here. A step order fuzzy approach discussed by Das *et al.* (2015). Recently, Maity *et al.* (2019) studied an intuitionistic dense fuzzy model where the learning- forgetting or agreement-disagreement are considered. De and Sana (2018) discussed a stochastic demand model under aggregation with Bonferroni mean in intuitionistic fuzzy environment. Deli and Broumi (2015) worked in Neutrosophic soft matrices and NSM-decision making. Recently, Kaur *et al.* (2019) contributed to find relation between interval type intuitionistic trapezoidal fuzzy sets and decision making with incomplete weight information. Liang and Wang (2019) considered a linguistic intuitionistic cloudy fuzzy model with sentiment analysis in E-commerce. Xu (2007) studied about Intuitionistic fuzzy aggregation operators. From the above discussion, it is observed that none of the researchers have been studied over the metric distances of intuitionistic fuzzy numbers. In this study we develop the theory of distances between two non-linear intuitionistic fuzzy sets (numbers) with respect to (pseudo) metrics. We give some definitions of metric spaces and the formula of distance measure of two different sets via cumulative aggregated formula. To show the novelty of this article a numerical illustration has been analysed through the ranking of distances with the existing metrics.

2. Preliminaries

2.1 Here we shall introduce some definitions over metric and pseudo metric spaces.

Definition 2.1.1 Let A be a non-empty set. A function $d: A \times A \to \mathbb{R}$ is said to be a 'metric' or a distance function on A if it satisfies the following properties:

i. $d(x, y) \geq 0$ *for all* $x, y \in A$;

ii. $d(x, y) = 0$ *if and only if* $x = y$;

iii. $d(x, y) = d(y, x)$ *for all* $x, y \in A$;

iv. $d(x, z) = d(x, y) + d(y, z)$ *for all* $x, y, z \in A$

Any non-empty set A together with a metric d defined on it is said to be a 'metric space'.

Definition 2.1.2 Let A be a non-empty set. A function $d^p: A \times A \to \mathbb{R}$ is said to be a 'pseudo-metric' on A if it satisfies the following properties:

Published by Firouzabad Institute of Higher Education, Firouzabad, Fars, Iran

i. $d^p(x,y) \geq 0$ *for all* $x,y \in A$;
ii. $x,y \in A$ and $x = y \Rightarrow d^p(x,y) = 0$;
iii. $d^p(x,y) = d^p(y,x)$ *for all* $x,y \in A$;
iv. $d^p(x,z) = d^p(x,y) + d^p(y,z)$ *for all* $x,y,z \in A$

Any non-empty set A together with a pseudo-metric d^p defined on it is said to be a 'pseudo-metric space'.

Definition 2.1.3 Atanassov (1983) An intuitionistic fuzzy set A defined in the universe of discourse X is given by $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle | x \in X\}$, where $\mu_A, \nu_A: X \to [0,1]$ denote the degree of membership and non-membership of x in A respectively, satisfying the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. The indeterminacy degree $\psi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ expresses the lack of knowledge of whether x belongs to A or not, also $0 \leq \psi_A(x) \leq 1$ for $x \in X$. An intuitionistic fuzzy number $\omega = (\mu_\omega, \nu_\omega)$ is an ordered pair which satisfies the conditions: $0 \leq \mu_\omega \leq 1$, $0 \leq \nu_\omega \leq 1$ and $0 \leq \mu_\omega + \nu_\omega \leq 1$, where μ_ω and ν_ω are called membership degree and non-membership degree respectively.

2.2 Pseudo-metrics in Intuitionistic Fuzzy Set

Let us consider F be the set of all intuitionistic fuzzy numbers and each $\omega = (\mu_\omega, \nu_\omega) \in F$ is called a point of F. However, $\omega = (\mu_\omega, \nu_\omega, \psi_\omega)$ only has two degrees of freedom because $\mu_\omega + \nu_\omega + \psi_\omega \equiv 1$. So, we observe such a system by keeping one variable constant when the other variable is changing.

Definition 2.2.1 Xu (2007) Given two Intuitionistic fuzzy numbers (IFN) ρ and σ,

$$\rho \cap \sigma = (min(\mu_\rho, \mu_\sigma), max(\nu_\rho, \nu_\sigma)) \text{ and } \rho \cup \sigma = (max(\mu_\rho, \mu_\sigma), min(\nu_\rho, \nu_\sigma)).$$

Lemma 1 If $(\mu_\rho - \mu_\sigma)(\nu_\rho - \nu_\sigma) \geq 0, \rho \cap \sigma \in F$ and $\rho \cup \sigma \in F$.

Proof $\mu_{\rho\cap\sigma} + \nu_{\rho\cap\sigma} = \mu_\rho + \nu_\sigma \leq \mu_\sigma + \nu_\sigma \leq 1$,

$$\mu_{\rho\cup\sigma} + \nu_{\rho\cup\sigma} = \mu_\sigma + \nu_\rho \leq \mu_\sigma + \nu_\sigma \leq 1, if\ \mu_\rho \leq \mu_\sigma;$$

$$\mu_{\rho\cap\sigma} + \nu_{\rho\cap\sigma} = \mu_\sigma + \nu_\rho < \mu_\rho + \nu_\rho \leq 1,$$

$$\mu_{\rho\cup\sigma} + \nu_{\rho\cup\sigma} = \mu_\rho + \nu_\sigma < \mu_\rho + \nu_\rho \leq 1, if\ \mu_\rho > \mu_\sigma;$$

Definition 2.2.2 An ordered pair (F, d_ψ^p) is called the intuitionistic fuzzy indeterminacy pseudo metric space on F, where $d_\psi^p: F^2 \to \mathbb{R}$ is the indeterminacy pseudo metric, for any $\rho, \sigma \in F$, $d_\psi^p(\rho,\sigma) = \frac{|\psi_\rho^2 - \psi_\sigma^2|}{2}$.

It is easy to verify that d_ψ^p satisfies the properties of pseudo-metric. Similarly, we can define the intuitionistic fuzzy membership pseudo metric space on F, where $d_\mu^p: F^2 \to \mathbb{R}$ is the membership pseudo metric, for any $\rho, \sigma \in F$, $d_\mu^p(\rho,\sigma) = \frac{|\mu_\rho^2 - \mu_\sigma^2|}{2}$ and the

Published by Firouzabad Institute of Higher Education, Firouzabad, Fars, Iran

intuitionistic fuzzy non-membership pseudo metric space on F, where $d_\nu^p: F^2 \to \mathbb{R}$ is the non-membership pseudo metric, for any $\rho, \sigma \in F$, $d_\nu^p(\rho,\sigma) = \frac{|\nu_\rho^2 - \nu_\sigma^2|}{2}$.

It is also easy to verify that d_μ^p and d_ν^p satisfies the properties of pseudo-metric.

Definition 2.2.3 An ordered pair (F, d_ψ) is called the intuitionistic fuzzy indeterminacy metric space on F, where $d_\psi: F^2 \to \mathbb{R}$ is the indeterminacy metric, for any $\rho, \sigma \in F$, $d_\psi(\rho,\sigma) = |\psi_\rho - \psi_\sigma|$.

It is easy to verify that d_ψ satisfies the properties of metric. Similarly, we can define the intuitionistic fuzzy membership metric space on F, where $d_\mu: F^2 \to \mathbb{R}$ is the membership metric, for any $\rho, \sigma \in F$, $d_\mu(\rho,\sigma) = |\mu_\rho - \mu_\sigma|$ and the intuitionistic fuzzy non-membership metric space on F, where $d_\nu: F^2 \to \mathbb{R}$ is the non-membership metric, for any $\rho, \sigma \in F$, $d_\nu(\rho,\sigma) = |\nu_\rho - \nu_\sigma|$.

It is also easy to verify that d_μ and d_ν satisfies the properties of metric.

Lemma 2 If $\alpha, \beta \in F$, $d_\Sigma(\alpha,\beta) = max(d_\mu(\alpha,\beta), d_\nu(\alpha,\beta), d_\psi(\alpha,\beta))$ satisfies the four properties of pseudo-metric as well as metric.

Lemma 3 Let $\tilde{P}$ and $\tilde{Q}$ be two intuitionistic fuzzy sets defined over the interval $[L, R]$, then the summative metric distance (summative pseudo metric distance) between $\tilde{P}$ and $\tilde{Q}$ is denoted by $d(\tilde{P},\tilde{Q})$ (for pseudo metric $d_p(\tilde{P},\tilde{Q})$) and defined as

$$d(\tilde{P},\tilde{Q}) = \frac{1}{R-L}\int_L^R d_\Sigma\left(\left(\mu_{\tilde{P}}(x), \nu_{\tilde{P}}(x)\right), \left(\mu_{\tilde{Q}}(x), \nu_{\tilde{Q}}(x)\right)\right) dx \tag{1}$$

3. Representation of IFSs $\tilde{P}$ *and* $\tilde{Q}$ over the interval $[a_1, a_3]$ Maity and Mondal (2020).

(2)

$$\mu_{\tilde{P}}(x) = \begin{cases} \left(\frac{x-a_1}{a_2-a_1}\right)^m & if\ a_1 \le x \le a_2 \\ \left(\frac{a_3-x}{a_3-a_2}\right)^m & if\ a_2 \le x \le a_3 \\ 0 & otherwise \end{cases} \quad \text{and} \quad \mu_{\tilde{Q}}(x) = \begin{cases} \left(\frac{x-a_1}{a_2-a_1}\right)^n & if\ a_1 \le x \le a_2 \\ \left(\frac{a_3-x}{a_3-a_2}\right)^n & if\ a_2 \le x \le a_3 \\ 0 & otherwise \end{cases}$$

$$\nu_{\tilde{P}}(x) = \begin{cases} \left(\frac{a_2-x}{a_2-a_1}\right)^m & if\ a_1 \le x \le a_2 \\ \left(\frac{x-a_2}{a_3-a_2}\right)^m & if\ a_2 \le x \le a_3 \\ 0 & otherwise \end{cases} \qquad \nu_{\tilde{Q}}(x) = \begin{cases} \left(\frac{a_2-x}{a_2-a_1}\right)^n & if\ a_1 \le x \le a_2 \\ \left(\frac{x-a_2}{a_3-a_2}\right)^n & if\ a_2 \le x \le a_3 \\ 0 & otherwise \end{cases}$$

Now as per equation (1), the pseudo metric between two IFSs is given by

(3)

$$d^p(\tilde{P},\tilde{Q}) = \frac{1}{a_3 - a_1}\int_{a_1}^{a_3} d_\Sigma\left((\mu_{\tilde{P}}(x),\nu_{\tilde{P}}(x)),\left(\mu_{\tilde{Q}}(x),\nu_{\tilde{Q}}(x)\right)\right)dx$$

$$= \frac{1}{a_3-a_1}\int_{a_1}^{a_2} d_\Sigma\left((\mu_{\tilde{P}}(x),\nu_{\tilde{P}}(x)),\left(\mu_{\tilde{Q}}(x),\nu_{\tilde{Q}}(x)\right)\right)dx + \frac{1}{a_3-a_1}\int_{a_2}^{a_3} d_\Sigma\left((\mu_{\tilde{P}}(x),\nu_{\tilde{P}}(x)),\left(\mu_{\tilde{Q}}(x),\nu_{\tilde{Q}}(x)\right)\right)dx$$

$$= \frac{1}{a_3-a_1}\int_{a_1}^{a_2} d_\Sigma\left(\left(\left(\frac{x-a_1}{a_2-a_1}\right)^m,\left(\frac{a_2-x}{a_2-a_1}\right)^m\right),\left(\left(\frac{x-a_1}{a_2-a_1}\right)^n,\left(\frac{a_2-x}{a_2-a_1}\right)^n\right)\right)dx + \frac{1}{a_3-a_1}\int_{a_2}^{a_3} d_\Sigma\left(\left(\left(\frac{a_3-x}{a_3-a_2}\right)^m,\left(\frac{x-a_2}{a_3-a_2}\right)^m\right),\left(\left(\frac{a_3-x}{a_3-a_2}\right)^n,\left(\frac{x-a_2}{a_3-a_2}\right)^n\right)\right)dx$$

$$= \frac{1}{a_3-a_1}\int_{a_1}^{a_2} max\left\{\frac{1}{2}\left|\left(\frac{x-a_1}{a_2-a_1}\right)^{2m} - \left(\frac{x-a_1}{a_2-a_1}\right)^{2n}\right|,\frac{1}{2}\left|\left(\frac{a_2-x}{a_2-a_1}\right)^{2m} - \left(\frac{a_2-x}{a_2-a_1}\right)^{2n}\right|,\frac{1}{2}\left|\left(1 - \left(\frac{x-a_1}{a_2-a_1}\right)^m - \left(\frac{a_2-x}{a_2-a_1}\right)^m\right)^2 - \left(1 - \left(\frac{x-a_1}{a_2-a_1}\right)^n - \left(\frac{a_2-x}{a_2-a_1}\right)^n\right)^2\right|\right\}dx + \frac{1}{a_3-a_1}\int_{a_2}^{a_3} max\left\{\frac{1}{2}\left|\left(\frac{x-a_2}{a_3-a_2}\right)^{2m} - \left(\frac{x-a_2}{a_3-a_2}\right)^{2n}\right|,\frac{1}{2}\left|\left(\frac{a_3-x}{a_3-a_2}\right)^{2m} - \left(\frac{a_3-x}{a_3-a_2}\right)^{2n}\right|,\frac{1}{2}\left|\left(1 - \left(\frac{x-a_2}{a_3-a_2}\right)^m - \left(\frac{a_3-x}{a_3-a_2}\right)^m\right)^2 - \left(1 - \left(\frac{x-a_2}{a_3-a_2}\right)^n - \left(\frac{a_3-x}{a_3-a_2}\right)^n\right)^2\right|\right\}dx$$

Similarly, for the metric distance we have

(4)

$$d(\tilde{P},\tilde{Q}) = \frac{1}{a_3 - a_1}\int_{a_1}^{a_3} d_\Sigma\left((\mu_{\tilde{P}}(x),\nu_{\tilde{P}}(x)),\left(\mu_{\tilde{Q}}(x),\nu_{\tilde{Q}}(x)\right)\right)dx$$

$$= \frac{1}{a_3-a_1}\int_{a_1}^{a_2} d_\Sigma\left((\mu_{\tilde{P}}(x),\nu_{\tilde{P}}(x)),\left(\mu_{\tilde{Q}}(x),\nu_{\tilde{Q}}(x)\right)\right)dx + \frac{1}{a_3-a_1}\int_{a_2}^{a_3} d_\Sigma\left((\mu_{\tilde{P}}(x),\nu_{\tilde{P}}(x)),\left(\mu_{\tilde{Q}}(x),\nu_{\tilde{Q}}(x)\right)\right)dx$$

$$= \frac{1}{a_3-a_1}\int_{a_1}^{a_2} d_\Sigma\left(\left(\left(\frac{x-a_1}{a_2-a_1}\right)^m,\left(\frac{a_2-x}{a_2-a_1}\right)^m\right),\left(\left(\frac{x-a_1}{a_2-a_1}\right)^n,\left(\frac{a_2-x}{a_2-a_1}\right)^n\right)\right)dx + \frac{1}{a_3-a_1}\int_{a_2}^{a_3} d_\Sigma\left(\left(\left(\frac{a_3-x}{a_3-a_2}\right)^m,\left(\frac{x-a_2}{a_3-a_2}\right)^m\right),\left(\left(\frac{a_3-x}{a_3-a_2}\right)^n,\left(\frac{x-a_2}{a_3-a_2}\right)^n\right)\right)dx$$

$$= \frac{1}{a_3-a_1}\int_{a_1}^{a_2} max\left\{\left|\left(\frac{x-a_1}{a_2-a_1}\right)^m - \left(\frac{x-a_1}{a_2-a_1}\right)^n\right|, \left|\left(\frac{a_2-x}{a_2-a_1}\right)^m - \left(\frac{a_2-x}{a_2-a_1}\right)^n\right|, \left|\left(1-\left(\frac{x-a_1}{a_2-a_1}\right)^m - \left(\frac{a_2-x}{a_2-a_1}\right)^m\right) - \left(1-\left(\frac{x-a_1}{a_2-a_1}\right)^n - \left(\frac{a_2-x}{a_2-a_1}\right)^n\right)\right|\right\} dx + \frac{1}{a_3-a_1}\int_{a_2}^{a_3} max\left\{\left|\left(\frac{x-a_2}{a_3-a_2}\right)^m - \left(\frac{x-a_2}{a_3-a_2}\right)^n\right|, \left|\left(\frac{a_3-x}{a_3-a_2}\right)^m - \left(\frac{a_3-x}{a_3-a_2}\right)^n\right|, \left|\left(1-\left(\frac{x-a_2}{a_3-a_2}\right)^m - \left(\frac{a_3-x}{a_3-a_2}\right)^m\right) - \left(1-\left(\frac{x-a_2}{a_3-a_2}\right)^n - \left(\frac{a_3-x}{a_3-a_2}\right)^n\right)\right|\right\} dx$$

4. Formulation of EOQ Model

Let the inventory starts with the order quantity Q and the demand rate is D per month. The items are exhausted after time T and new re-order has been placed. Items are received instantly and the next cycle time begins. Thus, we have the average inventory cost of the classical EOQ model as $z = \frac{1}{2}hDT + \frac{c}{T}$ with $= DT$, where h is the unit holding cost and c is the set-up cost.

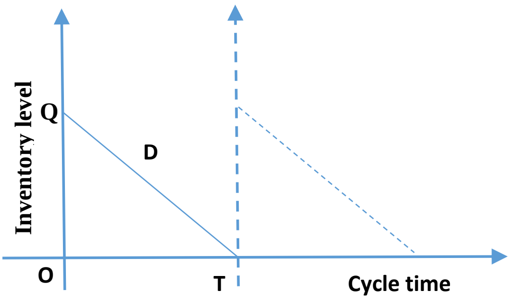

Figure 1.Basic EOQ model

4.1 Crisp mathematical model

The crisp mathematical model of the above discussed problem can be formulated as

$$\begin{cases} Minimize\ \boldsymbol{z} = \frac{1}{2}hDT + \frac{c}{T} \\ Q = DT \end{cases} \quad (5)$$

4.2 Fuzzy mathematical model

Let the costs associated with the inventory are fuzzy parameters. Then the fuzzy mathematical model can be formulated as

$$\begin{cases} Minimize\ \tilde{z} = \frac{1}{2}\tilde{h}DT + \frac{\tilde{c}}{T} \\ Q = DT \end{cases} \quad (6)$$

Here this will be discussed in the following three cases.

Published by Firouzabad Institute of Higher Education, Firouzabad, Fars, Iran

Case 1: The holding cost $\tilde{h}$ is considered as fuzzy parameter.

Let the holding cost $\tilde{h} = \langle h_1, h_2, h_3\rangle$ be considered as triangular intuitionistic fuzzy number whose membership and non-membership functions are given by

$$\mu_{\tilde{h}}(x) = \begin{cases} \left(\frac{x-h_1}{h_2-h_1}\right)^m & if\ h_1 \le x \le h_2 \\ \left(\frac{h_3-x}{h_3-h_2}\right)^m & if\ h_2 \le x \le h_3 \\ 0 & otherwise \end{cases} \text{ and } \nu_{\tilde{h}}(x) = \begin{cases} \left(\frac{h_2-x}{h_2-h_1}\right)^m & if\ h_1 \le x \le h_2 \\ \left(\frac{x-h_2}{h_3-h_2}\right)^m & if\ h_2 \le x \le h_3 \\ 0 & otherwise \end{cases}$$

Using this we get $\tilde{z} = \langle z_1, z_2, z_3\rangle = \langle \frac{1}{2}h_1DT + \frac{c}{T}, \frac{1}{2}h_2DT + \frac{c}{T}, \frac{1}{2}h_3DT + \frac{c}{T}\rangle$ as a triangular intuitionistic fuzzy number whose membership and non-membership functions are given by

$$\mu_{\tilde{z}}(z) = \begin{cases} \left(\frac{z-z_1}{z_2-z_1}\right)^m & if\ z_1 \le z \le z_2 \\ \left(\frac{z_3-z}{z_3-z_2}\right)^m & if\ z_2 \le z \le z_3 \\ 0 & otherwise \end{cases} \ \&\ \nu_{\tilde{z}}(z) = \begin{cases} \left(\frac{z_2-z}{z_2-z_1}\right)^n & if\ z_1 \le z \le z_2 \\ \left(\frac{z-z_2}{z_3-z_2}\right)^n & if\ z_2 \le z \le z_3 \\ 0 & otherwise \end{cases}$$

Case 2: The set-up cost $\tilde{c}$ is considered as fuzzy parameter.

Let the set-up cost $\tilde{c} = \langle c_1, c_2, c_3\rangle$ be considered as triangular intuitionistic fuzzy number whose membership and non-membership functions are given by

$$\mu_{\tilde{c}}(x) = \begin{cases} \left(\frac{x-c_1}{c_2-c_1}\right)^n & if\ c_1 \le x \le c_2 \\ \left(\frac{c_3-x}{c_3-c_2}\right)^n & if\ c_2 \le x \le c_3 \\ 0 & otherwise \end{cases} \text{ and } \nu_{\tilde{c}}(x) = \begin{cases} \left(\frac{c_2-x}{c_2-c_1}\right)^n & if\ c_1 \le x \le c_2 \\ \left(\frac{x-c_2}{c_3-c_2}\right)^n & if\ c_2 \le x \le c_3 \\ 0 & otherwise \end{cases}$$

Using this we get $\tilde{z} = \langle z_1, z_2, z_3\rangle = \langle \frac{1}{2}hDT + \frac{c_1}{T}, \frac{1}{2}hDT + \frac{c_2}{T}, \frac{1}{2}hDT + \frac{c_3}{T}\rangle$ as a triangular intuitionistic fuzzy number whose membership and non-membership functions are given by

$$\mu_{\tilde{z}}(z) = \begin{cases} \left(\frac{z-z_1}{z_2-z_1}\right)^m & if\ z_1 \le z \le z_2 \\ \left(\frac{z_3-z}{z_3-z_2}\right)^m & if\ z_2 \le z \le z_3 \\ 0 & otherwise \end{cases} \ \&\ \nu_{\tilde{z}}(z) = \begin{cases} \left(\frac{z_2-z}{z_2-z_1}\right)^n & if\ z_1 \le z \le z_2 \\ \left(\frac{z-z_2}{z_3-z_2}\right)^n & if\ z_2 \le z \le z_3 \\ 0 & otherwise \end{cases}$$

Case 3: Both holding cost $\tilde{h}$ and set-up cost $\tilde{c}$ are considered as fuzzy parameter.

Let the holding cost $\tilde{h} = \langle h_1, h_2, h_3\rangle$ and the set-up cost $\tilde{c} = \langle c_1, c_2, c_3\rangle$ are considered as triangular intuitionistic fuzzy number whose membership and non-membership functions are given by

$$\mu_{\tilde{h}}(x) = \begin{cases} \left(\frac{x-h_1}{h_2-h_1}\right)^m & if\ h_1 \le x \le h_2 \\ \left(\frac{h_3-x}{h_3-h_2}\right)^m & if\ h_2 \le x \le h_3 \\ 0 & otherwise \end{cases} \ \& \ \nu_{\tilde{h}}(x) = \begin{cases} \left(\frac{h_2-x}{h_2-h_1}\right)^m & if\ h_1 \le x \le h_2 \\ \left(\frac{x-h_2}{h_3-h_2}\right)^m & if\ h_2 \le x \le h_3 \\ 0 & otherwise \end{cases}$$ and

$$\mu_{\tilde{c}}(x) = \begin{cases} \left(\frac{x-c_1}{c_2-c_1}\right)^n & if\ c_1 \le x \le c_2 \\ \left(\frac{c_3-x}{c_3-c_2}\right)^n & if\ c_2 \le x \le c_3 \\ 0 & otherwise \end{cases} \ \& \ \nu_{\tilde{c}}(x) = \begin{cases} \left(\frac{c_2-x}{c_2-c_1}\right)^n & if\ c_1 \le x \le c_2 \\ \left(\frac{x-c_2}{c_3-c_2}\right)^n & if\ c_2 \le x \le c_3 \\ 0 & otherwise \end{cases}$$

Using this we get $\tilde{z} = \langle z_1, z_2, z_3 \rangle = \langle \frac{1}{2} h_1 DT + \frac{c_1}{T}, \frac{1}{2}\ h_2 DT + \frac{c_2}{T}, \frac{1}{2} h_3 DT + \frac{c_3}{T} \rangle$ as a triangular intuitionistic fuzzy number whose membership and non-membership functions are given by

$$\mu_{\tilde{z}}(z) = \begin{cases} \left(\frac{z-z_1}{z_2-z_1}\right)^m & if\ z_1 \le z \le z_2 \\ \left(\frac{z_3-z}{z_3-z_2}\right)^m & if\ z_2 \le z \le z_3 \\ 0 & otherwise \end{cases} \ \& \ \nu_{\tilde{z}}(z) = \begin{cases} \left(\frac{z_2-z}{z_2-z_1}\right)^n & if\ z_1 \le z \le z_2 \\ \left(\frac{z-z_2}{z_3-z_2}\right)^n & if\ z_2 \le z \le z_3 \\ 0 & otherwise \end{cases}$$

4.3 Defuzzification of the fuzzy mathematical model

Here we consider the third case of fuzzy mathematical model for defuzzification because in this case both costs are intuitionistic fuzzy numbers. Using the formula of α *and* β cut from De *et al*. (2016) we solve the model as

$$\begin{cases} Maximize\ \alpha - \beta \\ \alpha \ge \beta, \alpha + \beta \le 1, \alpha, \beta \ge 0 \\ \mu_{\tilde{z}}(z) \ge \alpha, \nu_{\tilde{z}}(z) \le \beta, \\ \mu_{\tilde{h}}(h) \ge \alpha, \nu_{\tilde{h}}(h) \le \beta, \mu_{\tilde{c}}(c) \ge \alpha, \nu_{\tilde{c}}(c) \le \beta \\ z = \frac{1}{2} hDT + \frac{c}{T} \end{cases} \quad (7)$$

Table-3: we consider the holding cost $\tilde{h}$ as intuitionistic fuzzy set

Criteria	Pseudo metric distance	Metric distance
A (m, n integer) m=2, n=4	0.146	0.267
B (m integer n fraction) m=2, n=1/2	0.150	0.667
C (m, n fraction) m=1/2, n=1/4	0.125	0.267

Table-4: we consider the set-up cost $\tilde{c}$ as intuitionistic fuzzy set

Criteria	Pseudo metric distance	Metric distance
A (m, n integer) m=2, n=4	0.146	0.267
B (m integer n fraction) m=2, n=1/2	0.150	0.667
C (m, n fraction) m=1/2, n=1/4	0.125	0.267

Published by Firouzabad Institute of Higher Education, Firouzabad, Fars, Iran

Table-5: we consider the holding cost $\tilde{h}$ and set up cost $\tilde{c}$ as intuitionistic fuzzy set

Criteria	Pseudo metric distance	Metric distance
A (m, n integer) m=2, n=4	0.146	0.267
B (m integer n fraction) m=2, n=1/2	0.150	0.667
C (m, n fraction) m=1/2, n=1/4	0.125	0.267

Table-6: Ranking Table

Type of cost	Pseudo metric distance	Metric distance
Fuzzy holding cost	*B> A> C*	*B> A=C*
Fuzzy set up cost	*B> A> C*	*B> A=C*
Fuzzy holding cost and Fuzzy set up cost	*B> A> C*	*B> A=C*

4.4 Numerical Illustration

Let us assume the holding cost $h = 2.5$, set-up cost $c = 1200$, constant demand rate $D = 100$. Also let us consider the fuzzy (holding and set up) costs as $\tilde{h} = \langle 1.75, 2.5, 3 \rangle$ and $\tilde{c} = \langle 1000, 1200, 1300 \rangle$. The following table shows that the result in different scenarios:

Published by Firouzabad Institute of Higher Education, Firouzabad, Fars, Iran

Table: 7 Result in different scenarios under pseudo metric distances

Scenario		Order quantity	Cycle time	Average inventory cost
Crisp		309.84	3.098	774.596
Intuitionistic fuzzy	Case-B (m=2, n=1/2)	311.69	3.117	770.000
	Case-A (m=2, n=4)	310.54	3.105	770.053
	Case-C (m=1/2,n=1/4)	311.62	3.116	770.090

Table 7 shows that the average inventory cost is minimum when we consider fuzzy cost parameters. Moreover, in fuzzy case we see that the ranking of the costs is exactly same manner with respect to pseudo metric distances. From Table 6 we see that the pseudo metric distance is maximum in case B and minimum in case C. In case of decision making, Table 7 shows that the average inventory cost is minimum in case B and maximum in case C. So we can conclude that when we consider minimization problem maximum pseudo metric distance implies minimum average inventory cost and minimum pseudo metric distance implies maximum average inventory cost. Here we do not consider metric distances because we see that the metric distances are always greater than pseudo metric distances. But the distance between two fuzzy sets should not be large that is why we consider here pseudo metric distances.

3.2 Graphical Illustration

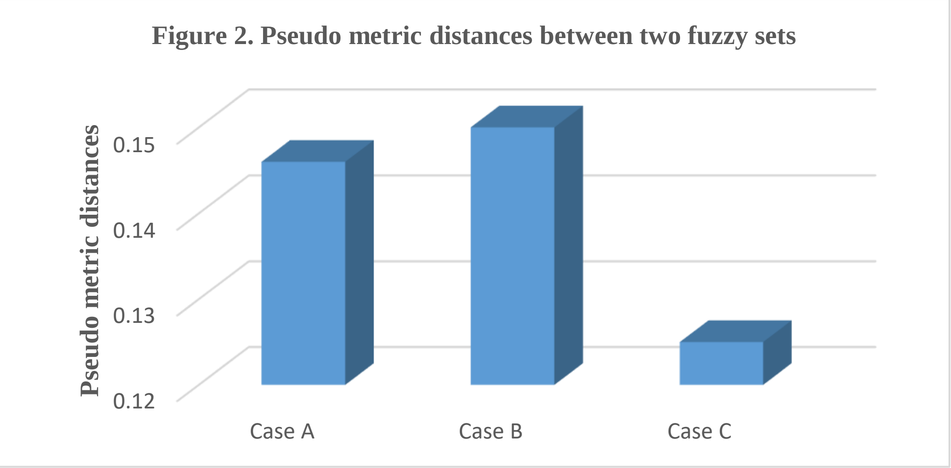

Figure2. shows that the pseudo metric distance is maximum in case B and that is minimum in case C.

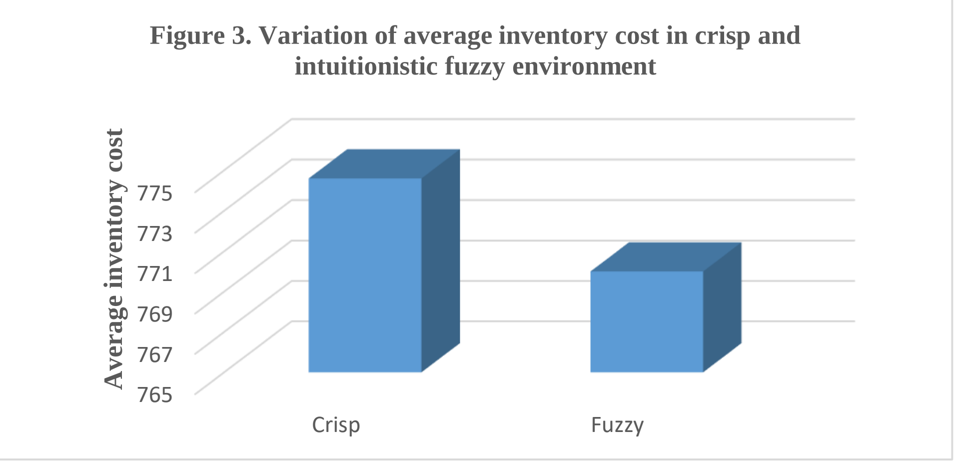

Figure 3. Shows that the average inventory cost is minimum in intuitionistic fuzzy environment rather than crisp environment.

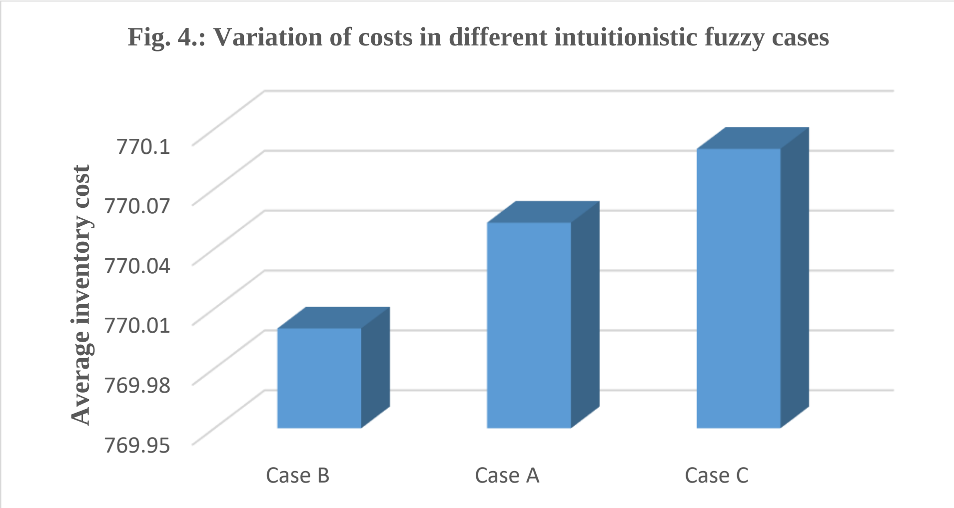

Figure 4. Shows that hierarchy of average inventory cost in different cases of intuitionistic fuzzy environments.

5. Area of application

The area of applications of this proposed approach are stated below:

- **i.** It is used to measure the qualitative differences among various subjects like any supply chain modelling or decision-making problem.
- **ii.** To know the degrees of membership, non-membership and indeterminacy, this method can be applied.
- **iii.** Any kind of ranking of different subjects over several disciplines is possible with its help.

5.1 Merits and Demerits

After the study of numerical and graphical illustrations, we see that there exists some merits and demerits of the proposed approach. They are stated as follows:

5.1.1 *Merits*

- i. This approach is very useful to find difference between two IFSs whose membership, non-membership and indeterminacy functions are non-linear.
- ii. This method told us that the pseudo metric distance is more user friendly for decision making.
- iii. This method helps us to study different nature of intuitionistic fuzzy sets drawn over physical problems with more detailing.

5.1.2 *Demerits*

i. This method is not applicable for the IFSs which are defined in a discrete space.
ii. This method is silent for higher dimensional intuitionistic fuzzy sets.
iii. This method might be complicated to handle when the membership function is complicated.
iv. The results may vary when the IFSs are assumed to be different.

6. Conclusion

In this study we have discussed about the qualitative differences of the subjects of real-world problem by means of metric distances between two non-linear intuitionistic fuzzy sets. Here we see that the distance under pseudo metric is more effective and extensive rather than the distance under conventional standard metric. The ranking of (pseudo) metric distances gives us a clear idea of quality measurement of two IFSs with non-linear membership and non-membership function. This idea will be very useful to solve a decision-making problem. Also, graphical illustrations show the fluctuation of differences in several criteria of exponents in membership and non-membership function.

Scope of future work

This study of IFS with the help of metric and pseudo metric is innovative. In future, various types of works can be done using this approach. This method can be applied for decision making problems such as supply chain modelling or inventory modelling.

Conflicts of interest

It is declared by the authors that there is no conflict of interest regarding the publication of this article.

References

1. Atanassov, K.T.: Intuitionistic fuzzy sets. Fuzzy Sets Syst. 20, 87–96 (1986)
2. Atanassov, K.T.: Intuitionistic fuzzy sets. VII ITKR’s Session, Sofia (1983)
3. Báez-Sánchez, A. D., Moretti, A. C., & Rojas-Medar, M. A. (2012). On polygonal fuzzy sets and numbers. Fuzzy Sets and Systems, 209, 54-65.
4. Ban, A. I., & Coroianu, L. (2014). Existence, uniqueness and continuity of trapezoidal approximations of fuzzy numbers under a general condition. Fuzzy sets and Systems, 257, 3-22.
5. Bellman, R. E., & Zadeh, L. A. (1970). Decision-making in a fuzzy environment. Management science, 17(4), B-141
6. Chutia, R., Mahanta, S., & Baruah, H. K. (2010). An alternative method of finding the membership of a fuzzy number. International journal of latest trends in computing, 1(2), 69-72.

Published by Firouzabad Institute of Higher Education, Firouzabad, Fars, Iran

7. Das, P., De, S. K., & Sana, S. S. (2015). An EOQ model for time dependent backlogging over idle time: a step order fuzzy approach. International Journal of Applied and Computational Mathematics, 1(2), 171-185.
8. De, S. K. (2018). Triangular dense fuzzy lock sets. Soft Computing, 22(21), 7243-7254.
9. De, S. K., & Beg, I. (2016). Triangular dense fuzzy neutrosophic sets. Neutrosophic Sets and Systems, 13, 24-37.
10. De, S. K., & Beg, I. (2016). Triangular dense fuzzy sets and new defuzzification methods. Journal of Intelligent & Fuzzy systems, 31(1), 469-477.
11. De, S. K., & Mahata, G. C. (2017). Decision of a fuzzy inventory with fuzzy backorder model under cloudy fuzzy demand rate. International Journal of Applied and Computational Mathematics, 3(3), 2593-2609.
12. DE, S. K., & Mahata, G. C. (2019). A comprehensive study of an economic order quantity model under fuzzy monsoon demand. Sādhanā, 44(4), 89.
13. De, S. K., & Pal, M. (2016). An intelligent decision for a bi-objective inventory problem. International Journal of Systems Science: Operations & Logistics, 3(1), 49-62.
14. De, S. K., & Sana, S. S. (2018). The (p, q, r, l) model for stochastic demand under intuitionistic fuzzy aggregation with Bonferroni mean. Journal of Intelligent Manufacturing, 29(8), 1753-1771.
15. De, S. K., Goswami, A., & Sana, S. S. (2014). An interpolating by pass to Pareto optimality in intuitionistic fuzzy technique for a EOQ model with time sensitive backlogging. Applied Mathematics and Computation, 230, 664-674.
16. Deli, I., & Broumi, S. (2015). Neutrosophic soft matrices and NSM-decision making. Journal of Intelligent & Fuzzy Systems, 28(5), 2233-2241.
17. Kaur, A., Kumar, A., & Appadoo, S. S. (2019). A note on “approaches to interval intuitionistic trapezoidal fuzzy multiple attribute decision making with incomplete weight information”. International Journal of Fuzzy Systems, 21(3), 1010-1011.
18. Liang, R., & Wang, J. Q. (2019). A linguistic intuitionistic cloud decision support model with sentiment analysis for product selection in E-commerce. International Journal of Fuzzy Systems, 21(3), 963-977.
19. Mahanta, S., Chutia, R., & Baruah, H. K. (2010). Fuzzy arithmetic without using the method of α–cuts. International journal of latest trends in computing, 1(2), 73-80.
20. Maity, S., Chakraborty, A., De, S. K., Mondal, S. P., & Alam, S. (2020). A comprehensive study of a backlogging EOQ model with nonlinear heptagonal dense fuzzy environment. RAIRO: Recherche Opérationnelle, 54.
21. Maity, S., De, S. K., & Mondal, S. P. (2020). A study of a backorder EOQ model for cloud-type intuitionistic dense fuzzy demand rate. International Journal of Fuzzy Systems, 22(1), 201-211.
22. Maity, S., De, S. K., & Prasad Mondal, S. (2019). A study of an EOQ model under Lock Fuzzy Environment. Mathematics, 7(1), 75.
23. Mao, X. B., Hu, S. S., Dong, J. Y., Wan, S. P., & Xu, G. L. (2018). Multi-attribute group decision making based on cloud aggregation operators under interval-valued

hesitant fuzzy linguistic environment. International Journal of Fuzzy Systems, 20(7), 2273-2300.

24. Piegat, A. (2005). A new definition of the fuzzy set. Int. J. Appl. Math. Comput. Sci, 15(1), 125-140.
25. Xu, Z. (2007). Intuitionistic fuzzy aggregation operators. IEEE Transactions on fuzzy systems, 15(6), 1179-1187
26. Zadeh, L.A.: Fuzzy sets. Inf. Control 8(3), 338–356 (1965)

Published by Firouzabad Institute of Higher Education, Firouzabad, Fars, Iran