Mathematical modeling of Imam Ali ibn Abi Ṭālib's war strategies with the Khawārij of Nahrawān using game theory

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Abstract Today, with the advancement of sciences in various fields, mathematics is being optimally used in all sciences, mathematics is therefore widely used in other sciences, and Islamic and military sciences are no exception, and mathematics can be used to design various offensive and defensive operations in battlefields and solve battlefield equations in order to gain absolute or relative superiority over the enemy as in the game theory of war, the application of mathematics can be clearly seen. This practical research, conducted through a library method, has modeled the challenges between the two armies of Imam ʿAlī ibn Abī Ṭālib (AS) and a group called the Māriqīn or Khawārij in the battle of Nahrawān using Game Theory and analyzed each army's strategies against each other, and finally explore the war utility to find the strategic equilibrium point between them. Therefore, we calculate the utility function for the set of strategies of each of the two armies including the army of Amir al-Mu'minin, ʿAlī ibn Abī Ṭālib (AS) and the Khawārij of Nahrawān and analyze it.

Keywords Strategies; Mathematical modeling; Game theory; Battle of Nahrawān

1. Introduction

In systems analysis, mathematical modeling is essential. Models are capable of understanding the behavior of the system at any given moment as well as modeling real systems. Models are always needed to design new processes and analyze existing processes in engineering science and mechanics. Therefore, one of the most up-to-date discussions in recent years is the modeling of war games Lezgi *et al.* (1398). An important principle in game theory is the rational behavior of most of the players. Rational behavior means every player is looking to maximize his profits and know how to maximize his profits. Therefore, the prediction of his behavior is based on the cost-

benefit graph. Game theory seeks to find the mathematical formulas necessary to simulate a strategic situation Akramizadeh *et al.* (1398). Although the effort in this research will be to extract and analyze prominent indices from the history of the battle of Nahrawān and to analyze and compare them with the algorithms and concepts of War Game. However, we note that since game theory is the result of human experience and computation, it may be combined with conceptual and computational errors and in some cases it cannot be merged totally with the hidden criteria in Islamic teachings based on divine traditions that are unchangeable and fixed. There may also be some differences in what the literature in this field describes in terms of profits and losses, with what emerges from the concept of profit and loss (the concept of the hereafter reward of profit and loss) based on the teachings of Islam. What will be analyzed in this research is within the scope of Alawite sīrat. And we don't seek to see the quoted material as the viewpoint of the whole religion, but regarding the living conditions of Imam ʿAlī ibn Abī Ṭālib (AS), this text will contain the most helpful guidelines. Imam ʿAlī ibn Abī Ṭālib's record of wars may open newer and clearer horizons to the question, what strategies have he used, whether these strategies can be analyzed in terms of game theory as a human and contractual artifact? And can it be considered a supporter for strategic decision-making in wars?

The purpose of this study is to propose a model, based on which the way can be paved for future research in the field of game theory and its applications based on the teachings of Islam.

2.   Theoretical foundations of research

A. Conceptology of strategy

There was no consensus among sociologists and scholars of international relations on the division of military strategies. They divide these strategies into traditional and modern strategies. Some divide the strategies into terrestrial, aerospace, and marine. Some have also divided strategies into five characteristics: direct and indirect threatening, indirect advance or operation through surprise, traditional destruction, and partisan strategy Azghandi, (1394). According to Colonel Arthur Lykke, determining military goals, adjusting military theories to achieve goals, and applying military resources to implement theories are called military strategies Roshandel, (1373).

B. Theory of games in the occurrence of war

Game theory is made up of two sciences of logic and mathematics and relies on abstract arguments. The rationality of the players is central to the game, in that players are involved in the game with meticulous calculation and trying to get the most points out of the game. In this theory, each player seeks to prevent the other from winning. The difference between a game and a real conflict situation is that the game is based on predetermined conditions, but in the conflict situation, there is not necessarily a predetermined situation Ebadizadeh, (1397).

The player is the person who is affected by the decisions and there is an interdependence between the decisions and their actions, the player who interacts with a particular player is called the "opponent" of that player.

C. The causes of the occurrence of Nahrawān war

The Khawārij were present in Imam Ali ibn Abi Ṭālib's army during the battles of Jamal and Ṣiffīn. After the battle of Ṣiffīn, Muʿāwiya ibn Abī Sufyān's army, with the deceit of his senior adviser raised the Qur'an over the spear and shouted a reference to the Qur'an, despite the objection of Imam Ali (AS), most of the Kufa Corps sought arbitration. Tolerant of physical and mental damage and fatigue and on the other hand the tribal texture of Kufa Corps, the superficiality of the ignorant people in Imam Ali's army made the deception of Amr ibn al-As successful and the same ignorant people in Imam Ali's Kufa Corps forced him to accept the arbitration. Imam ʿAlī ibn Abī Ṭālib (AS) first introduced his army's commander, Mālik Ashtar, as a judge who faced the objection of the same ignorant men, and then introduced one of his army commanders, Abdullah ibn Abbas, who was again rejected by the same ignorant people. So he was inevitably satisfied with the insistence of these ignorant low-minded people on the authority of Abu Musa al-Ash'ari, a naive person.

After the bitter story of arbitration ended with the astuteness of Abu Musa Ash'ari in favor of Muʿāwiya, the same ignorant people who had forced Imam Ali to accept the arbitration demanded him to repent of this disbelief Balādhurī, (1996). The Imam also said about arbitration: I did not agree with this arbitration from the beginning, and then I accepted it compulsively, stipulating that if Abu Musa al-Ash'ari and Amr ibn As rule according to the Holy Qur'an, I would abide by them, because in fact, we have accepted the judgment of the Qur'an, not the judgment of two people (Ibid., v. 2, P. 349). After the end of the Siffin war and the return of Imam Ali to Kufa, the opposition broke away from Imam Ali's army and formed a group called Khawārij (Nasr ibn Muzāḥim, 1382; Balādhurī, 1996; Ṭabarī, 1967; Mas'udi, nd).

The Khawārij assembled at a place in Kufa in the year 1337 AH and elected Abdullah ibn Wahab Rāsibī to organize their political and military affairs (Dīnawarī, 1330). At such times, the result of arbitration was determined, and Imam Ali ibn Abi Ṭālib (AS) opposed it and summoned his companions to the camp to fight Muʿāwiya. He also sent a message to the Khawārij, inviting them to join the war, which received a negative response from the Khawārij. The Khawārij chose Nahrawān as their destination (Ibid: 203-204). They committed many crimes along the Nahrawān route and killed many innocent people, including Abdullah ibn Khabbāb ibn Art, whose father was a companion of the Prophet (PBUH), Khawārij terribly murdered him along with his wife and her fetus (Ibn Qutaybah Dīnawarī, 1380). News of the crime

came to Imam Ali and he led the troops from the camp of fighting to Muʿāwiya toward Nahrawān.

Imam's reaction to the murder of Abdullah ibn Khabbāb

After that Imam Ali ibn Abi Ṭālib (AS) heard the news of Abdullah's martyrdom, he became very sad. Upon hearing the news, he, who had been tolerating the Khawārij, sent Harath ibn Marah Abdi to Khawārij to consider the issue, but the Khawārij also martyred him. The report of the martyrdom of Harath ibn Marah also reached the Imam and he went to Nahrawān asked them to return to peace, but the Khawārij were ready to fight him. Imam Ali (AS) asked them about the martyrdom of Abdullah ibn Khabbāb, and all the Khawārij confessed to killing him and reminded him: "As we killed Abdullah, we will also kill you. Imam (PBUH) asked them to hand over the murderer of Ibn Khabbāb for Qiṣāṣ so that he would forgive the rest of the Khawārij. But they responded: We were all partners in killing him, then the Khawārij shouted among themselves that no one should speak to Ali's companions and prepare themselves to fight them (Ibn Qutaybah, 1380).

Imam Ali's Enlightenment Before the War Began

Imam Ali ibn Abi Ṭālib (AS) used all his efforts in both the Jamal and Siffin wars, as well as the war with the Khawārij, to guide the main body of the opposition with enlightenment and awareness. But in the war with the Khawārij in Nahrawān he made further efforts because he believed that most of the body of the Khawārij Corps, not their commanders, were ignorant people who were on the wrong way, so he said about them: "do not fight with the Khawārij after me, for the one who is seeking the truth and goes astray is not like the one who was seeking the false and has achieved it. (Nahj al-Balāghah: Sermon 61).

Imam Ali ibn Abi Ṭālib (AS) began with the guidance of the Khawārij since the separation of Khawārij from his army. And in this way he sent his friends who were wise men to the Khawārij, and then he entered himself. The main question of the Khawārij was why Ali (AS) accepted "arbitration"? Ibn 'Abbas replied:' Ali (AS) opposed the judgment of Abu Musa Ash'ari, whose ignorance and naivety initiated sedition, the Imam wanted me or Malik Ashtar to be the judge, but you did not agree and insisted on Abu Musa Ash'ari. He then told the Khawārij: "don't disagree, Imam Ali forsaken the war with Muʿāwiya in the short term, but since Muʿāwiya deceived and violated the contract, he would again go to war with them. When the Khawārij did not find a rational answer to Ibn 'Abbas's arguments, by defaming the Ibn' Abbas, asked Imam himself speak to them.

The Imam Ali (AS) departed for Nahrawān with a hundred of his companions to speak to the Khawārij. The main points of his remarks were: First, I have officially stated from the beginning that putting the Qur'an on the spears is

the deceit of Muʿāwiya and Amr ibn ʿĀṣ and they intend to deceive you. I told you, the Levant troops are tired of the war, so don't accept arbitration and help win the war, but you insisted that they invite us to the Quran. You must accept the invitation, otherwise we will not fight and we will deliver you to Muʿāwiya.

After imposing the arbitration, I chose Abdullah ibn Abbas, who was a shrewd man and he was not a secularist and Amr ibn ʿĀṣ cannot deceive him in any way, but again you did not accept and insisted on Abu Musa Ash'ari for arbitration. I had no choice except accepting Abu Musa. Even though you imposed "Arbitration" and "Judge" on me, did I not order both of them to obey the laws of God in the Holy Quran and the Sunnah of Prophet Muhammad (PBUH) from beginning to end? The leaders of the Khawārij confirmed Imam Ali (AS) and confessed: "all your words are true and we have sinned and we must repent." These logical arguments of the Imam led nearly 11,000 out of the 14,000 Khawārij to leave the camp and join the Imam, and about 3,300, including 1,800 cavalry and 1,500 infantry, were ready for war against Imam Ali (AS) (Balādhurī, 1996).

Tolerating the opposition group of Khawārij

Imam Ali (AS) along with these enlightenments showed great patience and tolerance against Khawārij provocative programs. After returning from the Siffin war, they did not attend the congregational prayer, shouting slogans at the Kufa mosque against the Imam. But Imam Ali (AS) treated them with his greatness and did not prevent them from coming to the mosque of Kufa and paid them money from the Bayt al-mal until they made assassination and armed uprising against the Alawite regime, in fact tolerating the opposition was in line with the principles accepted by Imam Ali (AS) in his domestic policies, so long as he allowed the oath-breaker opposition, who quitted his army, to leave Medina.

Heads of Khawārij

Prominent figures in the Khawārij Corps were: Shurayh b. Awfi al-'Abasi, Hurqus b. Zuhayr al-Tamimi, 'Abd Allah b. Shajara al-Sulami, Farwa b. Nawfal al-Ashja'i, 'Abd Allah b. Wahb al-Rasibi and Hamza b. Sanan al-Asadi. A brief look at the names of the Khawārij heads, it is obvious that they were not Iraqi celebrities and were often from primitive tribes such as Banu Tamim and Bakr b. Wa'il, etc. (Balādhurī, 1996).

The war and its consequences

When the majority of Khawārij, by the rational arguments and valuable advice of Imam Ali ibn Abi Ṭālib (AS), became aware and went back from their wrong way, the war began with the rest of Khawārij who insisted on fighting with Imam Ali (AS). The battle of Nahrawān began at noon and

lasted no more than two hours, in this war, the number of Imam Ali's troop was 68,000 and they were ready to fight the Muʿāwiya bin Abi Sufyan's army, but the devious provocations of Khawārij diverted the war path (Ahmad ibn Abi Ya'qub, 1358). With the unique command of Imam Ali (AS) and the bravery of his companions, the majority of the Khawārij were killed quickly and some were wounded and very few escaped, 400 wounded were handed over to their families, fewer than ten escaped, including Abdul Rahman ibn Moljam Moradi, who was Imam Ali's killer, on the opposite side of the war, fewer than 10 people were martyred in the army of Imam Ali (AS) (Balādhurī, 1996).

Table 1.

Battle of Nahrawān	
Time	1338 AH
Place	Iraq - Nahrawān
Result	The victory of the Muslims
The cause of the war	Khawārij protesting Imam Ali (AS) on arbitration with respect to 'No judge except Allah'
Fighters	Khawārij - companions of Imam Ali (AS)
Commanders	'Abd Allah b. Wahb al-Rasibi - Imam Ali (AS)
Casualties	Khawārij: Following the advice and guidance of Imam Ali (AS) out of about 14,000 Khawārij, 3300 remained and the rest abandoned the war, with 400 wounded and 10 escaping and the remaining were killed. Kufa Corps: Less than 10 people were killed in Imam Ali's army.

3. Research methodology

This library-based research has an applied-research purpose, as it attempts to put the results of this research into practical use and to help organizations solve the problems.

Strategies of Khawārij and Imam Ali (AS)

We call the strategies set of Khawārij as S1 and the strategies set of Imam Ali (AS) as S2 so we will have: $S_1 = A_1$

A_1) is seeking repentance unjustly from Imam Ali,

A_2) is creating intimidation among the general population, especially creating intimidation among the supporters of Imam Ali (AS) by killing innocent people,

A_3) trying to deceive public opinion with the Quranic appearance,

A_4) trying to isolate and destroy the image of Imam Ali (AS) by speaking against them and insulting them among people in the mosque of Kufa,

A_5) Using sacred and apparent interpretations of the Quran in favor of the Khawārij,

A_6) Creating a division in the Kufa army,

A_7) direct war. $S_2 = I_1$

I_1) Rejecting the condition of repentance for not being guilty in the story of arbitration,

I_2) trying to satisfy the public opinion,

I_3) trying to enlighten the minds of uninformed Khawārij and advising them not to enter the war,

I_4) tolerating the opposition and trying to give them a chance to think more and return from their wrong path,

I_5) appropriate combat setup to the Corps,

I_6) creating a great motivation in the army with fiery speeches,

I_7) using talented, brave and warrior commanders,

I_8) a direct and brave presence on the battlefields to stir up the soldiers' offensive sentiments,

I_9) acting right during the war.

I_{10}) observing ethics In all scenes of war and the delivery of war casualties to their families for treatment,

Consequences:

The amount of win or loss or what a player gets at the end of a game is called "outcome or consequence". The consequence may be money, income or privilege, and so on. At this stage of the research, the possible consequences of each of the options (strategies) must be estimated. The key question in a game is which of the following strategies each player will use to achieve the highest possible outcome; the answer is the "balance point" of the game. But what is the point of "balance"?

The balance:

When each player uses a strategy that is the best answer to the strategy chosen by the other players, then they combine the chosen strategies, called "game balance". The balance point of a game is a combination of player strategies that happens in practice. On balance, players do not necessarily get the most results. This can be due to conflicts of interests or lack of knowledge of players toward each other. In fact, one of the main goals of game theory is to find the same point of balance for each game.

The utility function for game theory

The player i who is involved in the decision-making must fight against his rival to get the most achievement. The achievement value of the player i is based on the utility function. In this study, utility function model is calculated based on cost and benefit analysis.

According to rational decision-making theory, the use of the cost-benefit analysis model is nowadays common in macro-economic-political decision making. In this way, they analyze and evaluate all the benefits and costs of a decision-making, and then make a decision (Ebadizadeh, 1397).

There are many definitions of cost-benefit analysis. Berman argues that this type of analysis is a weighting method for decision-making and acts as a scales. That is, all the positive elements of a project (benefits) are placed in one side and all the negative elements and harms (costs) are placed on the other side, and the heavier side wins. Cost-

benefit analysis is a decision-making logic that acts based on calculating the likely outcomes of different decision options (Ibid: 20). The cost-benefit analysis will answer five basic questions:

1) What are the direct benefits that come from this expense?
2) What indirect benefits will come from this expense?
3) Whether the obtained benefits are greater than the incurred expenses?
4) What are the losses if no expense is incurred?
5) Is there a less expensive option if this action is necessary?

This analytical approach will help determine the best decision for maximizing benefits against minimum costs.

Modeling the utility function

Berman presented the following criterion for utility function in 2011:
For the utility function to be quantitatively predictive of war, this function considers three aspects: the gained or lost human cost (H), the economic cost (E) the socio-political influence value (I) (Berman *et al.* 2011).
 Missing costs include:

Human resources:

- Losing the allegiance of the loser forces.
- The elimination of effective and efficient human resources in social and defense activities.

Economic aspect:

- Paying pensions to the families of those killed during the war.
- Problems arising from the costs of war imposed on Bayt al-Mal.

Socio-political influences:

- The loss of the allegiance of the forces of the opposite army, which were the majority of the people of Kufa for the Khawārij Corps and some of the Khawārij supporters for the Kufa Corps.
- Loss of geographical-political boundaries of the loser.

 Incurred costs include:

Human resources:

- Obtaining the allegiance of the loser corps forces
- Gaining effective human resources in social and defense activities

Economic aspect:

- Obtaining war trophies.
- Adding the Human Resources of the other side to the effective and efficient force in the economic activities of the victorious side in the war.

Socio-political influences:

- Obtaining the allegiance of the forces of the opposite Corps, which were the majority of the people of Kufa for the Khawārij Corps and some of the Khawārij supporters for the Kufa Corps.
- Expanding the geographical-political boundaries of the winner side.

According to what Burman and his colleagues (2011) have done, the benefits of war can be calculated by:

$$u_i = \alpha_H(H_G - H_L) + \alpha_E(E_G - E_L) + (1 - \alpha_H - \alpha_E)(I_G - I_L) \tag{1}$$

In relation (1), H_L and H_G respectively represent the gained and lost costs of human resources resulting from the war. E_L and E_G are the value of the economic value gained and lost by the war, I_L and I_G are the value of the socio-political influences gained and lost by the war. α_E and α_G, respectively, represent the weight of economic and human factors in the decision-making of the player i, with values ranging from zero to one: $0 \leq \alpha_H, \alpha_E \leq 1$

Two things that should be considered in the model are that, first, the calculations toward time must be dynamic with respect to changes in the unit of time, and second, there must be a decision maker's opinion. Accordingly, and for dynamics toward time, we consider a coefficient for different times, indicating the importance of each time in the range 0 to t. The coefficient δ between 0 and 1 is assumed that if t is considered for this coefficient, the coefficient value for the utility function will be different and time dependent. In addition, the coefficient β is considered as the risk-to-utility coefficient (Sherman, 2015). Sherman concluded in his paper in 2015 that to make an important decision, the opinions of different people, each aiming to achieve maximum profit, are influential. Therefore, player i must consider a weight for each of the opinions in the calculations. With this description, the utility value is calculated from the following relation:

$$U_i(t) = (\sum_{t=1}^{T} \delta^{t-1} [\frac{\sum_{j=1}^{N} \gamma_j \alpha_{Hj}}{N}(H_{Gt} - H_{Lt}) + \frac{\sum_{j=1}^{N} \gamma_j \alpha_{Ej}}{N}(E_{Gt} - E_{Lt})$$
$$+ \frac{\sum_{j=1}^{N} \gamma_j(1 - \alpha_{Hj} - \alpha_{Ej})}{N}(I_{Gt} - I_{Lt})])^{1-\beta}$$

Another point to keep in mind is that each of the six economic values mentioned can have M subset and have different dimensions. As a result, relation (2) is rewritten as follows:

$$U_i(t) = (\sum_{t=1}^{T} \delta^{t-1}[\frac{\sum_{j=1}^{N}\gamma_j\alpha_{Hj}}{N}\sum_{k=1}^{M}(H_{Gkt} - H_{Lkt}) \tag{2}$$

$$+ \frac{\sum_{j=1}^{N}\gamma_j\alpha_{Ej}}{N}\sum_{k=1}^{M}(E_{Gkt} - E_{Lkt})$$

$$+ \frac{\sum_{j=1}^{N}\gamma_j(1 - \alpha_{Hj} - \alpha_{Ej})}{N}\sum_{k=1}^{M}(I_{Gkt} - I_{Lkt})])^{1-\beta}$$

Although the player i is more commonly known as one side of the conflict, he can be any actor or group. Different actors differ widely in how we evaluate weights α_H, α_E and $\alpha_H - \alpha_E - 1$ for human life value, economic value and influence value, respectively.

Utility table obtained for hypothetical war:

The table below shows the utility function in the states of victory of either Kufa or Sham (V), their failure (F), peace and acceptance of the terms and conditions of the parties' treaty (P), and finally staying in the former position and continuing political disputes (C).

Table 2.

Utility obtained for Kufa Corps (Player 2)	Utility obtained for Sham Corps (Player 1)	
$\{\sum_{t=1}^{T}(\delta^{t-1}{}_2[\alpha_{H2}(-H_{Lt}) + \alpha_{E2}(-E_{Lt}) + (1 - \alpha_{H2} - \alpha_{E2})(-I_{Lt})])\}^{1-\beta_2}$	$\{\sum_{t=1}^{T}(\delta^{t-1}{}_1[\alpha_{H1}(H_{Gt} - H_{Lt}) + \alpha_{E1}(E_{Gt} - E_{Lt}) + (1 - \alpha_{H1} - \alpha_{E1})(I_{Gt})])\}^{1-\beta_1}$	V_1 F_1
$\{\sum_{t=1}^{T}(\delta^{t-1}{}_2[\alpha_{H2}(H_{Gt} - H_{Lt}) + \alpha_{E3}(E_{Gt} - E_{Lt}) + (1 - \alpha_{H2} - \alpha_{E2})(I_{Gt} - I_{Lt})])\}^{1-\beta_2}$	$\{\sum_{t=1}^{T}(\delta^{t-1}{}_2[\alpha_{H1}(H_{Gt} - H_{Lt}) + \alpha_{E1}(E_{Gt} - E_{Lt}) + (1 - \alpha_{H1} - \alpha_{E1})(I_{Gt} - I_{Lt})])\}^{1-\beta_1}$	P
$\{\sum_{t=1}^{T}(\delta^{t-1}{}_2[\alpha_{H2}(H_{Gt} - H_{Lt}) + \alpha_{E2}(E_{Gt} - E_{Lt}) + (1 - \alpha_{H2} - \alpha_{E2})(I_{Gt} - I_{Lt})])\}^{1-\beta_2}$	$\{\sum_{t=1}^{T}(\delta^{t-1}{}_1[\alpha_{H1}(-H_{Lt}) + \alpha_{E2}(-E_{Lt}) + (1 - \alpha_{H1} - \alpha_{E1})(-I_{Lt})])\}^{1-\beta_1}$	V_2 F_2
0	0	C

$\alpha_{H1}, \alpha_{E1}, \delta^{t-1}{}_1$ and β_1, respectively, are the coefficient of human value, coefficient of economic value, the importance coefficient of different times, and coefficient of war risk for the Sham Corps (Player 1). Indicators 2 are used for the Kufa Corps (Player 2). Given that in the event of a war between the two armies and its failure, the loser will not gain any human, economic, and socio-political influence value, so the value of E_{Gt}, H_{GT} and I_{Gt} is set to zero. Situations 1 to 3 occur when one of the two corps prefers the strategy of starting a war to the strategy of not starting a war. Also, situation 4 represents a case in which the two armies consider the strategy of non-starting a war as rational.If the strategy of starting a war for one of the states 1 to 3, has a positive utility for one side, the player will adopt one of the strategies for starting or not starting the war, depending on the likelihood of that happening. These possibilities can determine the political position of the players, their political positions in the Islamic society of that time. We conclude by looking at these positions that if a player wants to avoid a fight, he or she must try to keep the utility value of starting the war for the opposite side in each of the 1 to 3 situations as negative, or keep the likelihood of occurring the negative utility in the event of a fight for the opposite side (the negative utility would be one to three states). It is also important to note that the end of a war can be terminated in a state of equilibrium that is a prelude to the start of the next war, or that one of the parties involved reaches a level of negative utility that justifies a new war.

4. Conclusion

The most important thing when modeling utility function is to estimate the benefits and losses of starting and continuing the war. What prevents the outbreak of war is called the War Costs, and also what drives the opposition and politicians to war is the benefit that war has for them. So we should try to estimate real costs and benefits. Therefore, it is necessary to estimate the costs and benefits of war in the scientific space and framework. War involves the human and economic costs and benefits, socio-political influences, or other benefits that will exist for the winner (loser) if they win (lose). Estimating each of these is essential for starting or continuing a particular war.

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