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Linear programming problem with interval-valued generalized trapezoidal fuzzy numbers

Rohollah Taghaodi[1]✉ • Fatemeh Kardani[2]

[1] Department of Mathematics, Kashan Branch, Islamic Azad University, Kashan, Iran

[2] Department of Industrial Engineering, Masjed-Soleiman Branch, Islamic Azad University, Masjed-Soleiman, Iran

✉ taghaode@yahoo.com

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Abstract In this paper, we concentrate on linear programming problems in which the cost vector, the technological coefficients and the right-hand side values are interval-valued generalized trapezoidal fuzzy numbers. To the best of our knowledge, till now there is no method described in the literature to find the optimal solution of the linear programming problems with interval-valued generalized trapezoidal fuzzy numbers. We apply the signed distance for defuzzification of this problem. The crisp problem obtained after the defuzzification is solved by the linear programming methods. Finally, we give an illustrative example and its numerical solutions.

Keywords Linear programming problem; Generalized trapezoidal fuzzy number; Interval-valued generalized trapezoidal fuzzy number

1. Introduction

The fuzzy set theory was, for the first time, introduced by Zadeh (1965) and has found extensive applications in various fields. Bellman and Zadeh (1970) were the first to consider the application of the fuzzy set theory in solving optimization problems. The fuzzy set theory is a powerful tool to handle imprecise data and fuzzy expressions that are more natural for humans than rigid mathematical rules and equations. It is obvious that much knowledge in real world situations is fuzzy rather than precise. This theory is being applied extremely in many fields these days. One of these is linear programming problems. The linear programming problem is one of the most frequently applied operation research techniques. Although it has been investigated and expanded for more than six decades by many researchers and from the various point of views, it is still useful to develop new approaches in order to better fit the real world problems within the framework of linear programming Allahviranloo *et al*. (2008) and Baranifar (2018).

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In conventional approach, parameters of linear programming problems must be well defined and precise. However, in real world environment, this is not a realistic assumption. In such cases, using imprecise data such as interval numbers, triangular fuzzy numbers, trapezoidal fuzzy numbers, generalized trapezoidal fuzzy numbers (GTFNs) and interval-valued generalized trapezoidal fuzzy numbers (IVGTFNs) for modeling the problem is quite appropriate. In this paper, we focus on GTFNs and IVGTFNs.

The concept of interval-valued fuzzy sets is initially proposed by Gorzalczany (1987) and Turksen (1996). Then based on this achievement, Wang and Li (1988) defined the expansion operation of interval-valued fuzzy numbers, and proposed the concept and properties of similarity coefficient based on the interval-valued fuzzy numbers. Hong and Lee (2002) and Wang and Li (2001), proposed the distance of interval-valued fuzzy numbers. Chen and Chen (2006) presented the concept of IVGTFN based on the concepts of generalized trapezoidal fuzzy number proposed by Chen (1985) and interval-valued fuzzy number proposed by Wang and Li (1998), and proposed a new method for ranking the IVGTFNs. Farhadinia (2014) consider the Sensitivity analysis in interval-valued trapezoidal fuzzy number linear programming problems. Ebrahimnejad (2016) proposed the new method for solving transportation problems with interval-valued trapezoidal fuzzy numbers.

Obviously, the IVGTFN is more general form of fuzzy numbers, almost all fuzzy numbers can be viewed as its special case, for example, trapezoidal fuzzy number, generalized trapezoidal fuzzy number, interval-valued triangular fuzzy, etc.

For the linear programming problem that the cost vector, the technological coefficients and the right-hand side is IVGTFNs, this paper proposes a new method to obtain the optimal solution. It applies the signed distance for defuzzification of this problem. So, the crisp problem obtained after the defuzzification may be solved by the linear programming methods.

The rest of this paper is organized as follows: In Section 2, we review the basic definitions and concepts the generalized trapezoidal fuzzy numbers and IVGTFNs. Section 3 gives the definition of Linear programming with IVGTFNs problem. We give a new method for solving linear programming problem with IVGTFNs in Section 4. A numerical example is given in section 5. The conclusions are discussed in Section 6.

2. Preliminaries

2.1. Generalized trapezoidal fuzzy numbers

In this section, we briefly review some of the concept of generalized trapezoidal fuzzy numbers. They are crucial for the remainder of this paper.

2.1.1. The concept of generalized trapezoidal fuzzy numbers

Definition 1: A fuzzy set $\widetilde{A}$, defined on $\mathbb{R}$, is said to be generalized fuzzy number if the following conditions hold:

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(a) Its membership function is piecewise continuous function.

(b) There exist two intervals [a, b] and [c, d] such that $\widetilde{A}$ is strictly increasing on [a, b] and strictly decreasing on [c, d].

(c) $\widetilde{A}(x) = w$, For all $x \in [b, c]$, where $0 < w \leq 1$.

Definition 2: A fuzzy number $\widetilde{A}$ = (a, b, c, d; w) is said to be a generalized trapezoidal fuzzy number (as shown in Figure 1) if its membership function is given by

$$\tilde{A}(x) = \begin{cases} w\dfrac{x-a}{b-a}, & a \leq x < b \\ w\,, & b \leq x \leq c \\ w\dfrac{x-a}{b-a}\;, & c < x \leq d \\ 0\,, & otherwise \end{cases}$$

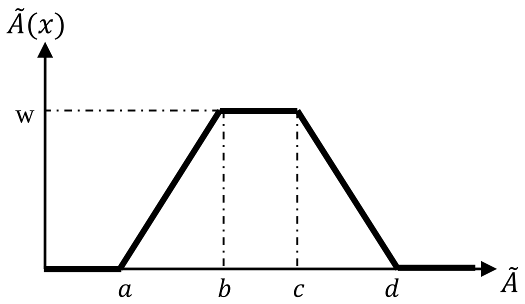

Figure 1. Generalized trapezoidal fuzzy numbers.

The elements of the generalized trapezoidal fuzzy numbers are real numbers. If $-1 \leq a \leq b \leq c \leq d \leq 1$, then $\widetilde{A}$ is called the normalized trapezoidal fuzzy number. Especially, if w = 1, then the generalized trapezoidal fuzzy number $\widetilde{A}$ is called a trapezoidal fuzzy number and denoted as $\widetilde{A} = (a, b, c, d)$, If $a < b = c < d$, then $\widetilde{A}$ is reduced to a triangular fuzzy number. If $a = b = c = d$. then $\widetilde{A}$ is reduced to a real number.

2.1.2. Arithmetic operations

In this subsection, we review arithmetic operations on generalized trapezoidal fuzzy numbers Chen and Chen (2006) and Chen (2012).
Let $\widetilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\widetilde{B} = (a_2, b_2, c_2, d_2; w_2)$ be two generalized trapezoidal fuzzy numbers. Define,

- $\widetilde{A} \oplus \widetilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; w)$, where w= minimum $\{w_1, w_2\}$,
- $\widetilde{A} \ominus \widetilde{B} = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2; w)$, where w= minimum $\{w_1, w_2\}$,
- $\theta\widetilde{A} = \begin{cases} (\theta a_1, \theta b_1, \theta c_1, \theta d_1; w_1) & \theta > 0, \\ (\theta d_1, \theta c_1, \theta b_1, .\theta a_1; w_2) & \theta < 0, \end{cases}$

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2.2. The interval-valued generalized trapezoidal fuzzy numbers

In this section, we review some of the concept, notations and arithmetic operations generalized interval-valued trapezoidal fuzzy numbers.

2.2.1. The concept of the interval-valued generalized trapezoidal fuzzy numbers

Definition 3: Let $\widetilde{A}^L$ and $\widetilde{A}^U$ be two generalized trapezoidal fuzzy numbers. A IVGTFN (as shown in Figure 2) represented by the following:

$$\tilde{\tilde{A}} = \left[\widetilde{A}^L, \widetilde{A}^U\right] = [\left(a_1^l, a_2^l, a_3^l, a_4^l; w^l\right), (a_1^u, a_2^u, a_3^u, a_4^u; w^u)]$$

Where, $0 \le a_1^l \le a_2^l \le a_3^l \le a_4^l \le 1$, $0 \le a_1^u \le a_2^u \le a_3^u \le a_4^u \le 1$, $0 \le w^l \le w^u \le 1$ and $\widetilde{A}^L \subset \widetilde{A}^U$.

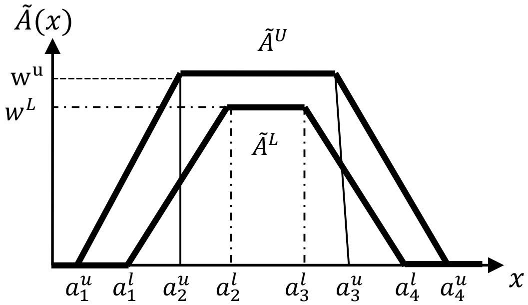

Figure 2. Interval-valued generalized trapezoidal fuzzy numbers.

From Figure 2, we can see that an interval-valued generalized trapezoidal fuzzy number $\tilde{\tilde{A}}$ consist of the lower generalized trapezoidal fuzzy number $\widetilde{A}^L$ and the upper generalized trapezoidal fuzzy number $\widetilde{A}^U$.

2.2.2. Arithmetic operations

The following we review arithmetic operations between two IVGTFNs.

Let $\tilde{\tilde{A}} = \left[\widetilde{A}^L, \widetilde{A}^U\right] = \left[\left(a_1^l, a_2^l, a_3^l, a_4^l; w^l\right), (a_1^u, a_2^u, a_3^u, a_4^u; w^u)\right]$, where
$0 \le a_1^l \le a_2^l \le a_3^l \le a_4^l \le 1$, $0 \le a_1^u \le a_2^u \le a_3^u \le a_4^u \le 1, 0 \le w^l \le w^u \le 1$, and,
$\tilde{\tilde{B}} = \left[\widetilde{B}^L, \widetilde{B}^U\right] = [\left(b_1^l, b_2^l, b_3^l, b_4^l; v^l\right), (b_1^u, b_2^u, b_3^u, b_4^u; v^u)]$ where
$0 \le b_1^l \le b_2^l \le b_3^l \le b_4^l \le 1$, $0 \le b_1^u \le b_2^u \le b_3^u \le b_4^u \le 1, 0 \le v^l \le v^u \le 1$, be two IVGTFNs, then, the operations are shown as follows [20, 21].

$$\tilde{\tilde{A}} \oplus \tilde{\tilde{B}} = [(a_1^l + b_1^l, a_2^l + b_2^l, a_3^l + b_3^l, a_4^l + b_4^l; \min(w^l, v^l)), (a_1^u + b_1^u, a_2^u + b_2^u, a_3^u + b_3^u, a_4^u + b_4^u; min(w^u, v^u))]$$

$$\tilde{\tilde{A}} \ominus \tilde{\tilde{B}} = [\left(a_1^l - b_4^l, a_2^l - b_3^l, a_3^l - b_2^l, a_4^l - b_1^l; \min\left(w^l, v^l\right)\right), (a_1^u - b_4^u, a_2^u - b_3^u, a_3^u - b_2^u, a_4^u - b_1^u; \min(w^u, v^u))]$$

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$\lambda\tilde{\tilde{A}} = \left[\left(\lambda a_1^l, \lambda a_2^l, \lambda a_3^l, \lambda a_4^l; w^l\right), \left(\lambda a_1^u, \lambda a_2^u, \lambda a_3^u, \lambda a_4^u; w^u\right)\right], \qquad \lambda > 0,$

$\lambda\tilde{\tilde{A}} = \left[\left(\lambda a_4^l, \lambda a_3^l, \lambda a_2^l, \lambda a_1^l; w^l\right), \left(\lambda a_4^u, \lambda a_3^u, \lambda a_2^u, \lambda a_1^u; w^u\right)\right], \qquad \lambda < 0,$

$\lambda\tilde{\tilde{A}} = \left[\left(0,0,0,0; w^l\right), \left(0,0,0,0; w^u\right)\right], \qquad \lambda = 0.$

2.2.3. The distance of interval-valued generalized trapezoidal fuzzy numbers

Suppose that $\tilde{\tilde{A}} = \left[\left(a_1^l, a_2^l, a_3^l, a_4^l; w^l\right), \left(a_1^u, a_2^u, a_3^u, a_4^u; w^u\right)\right]$, be a IVGTFN, the signed distance of $\tilde{\tilde{A}}$ from $\tilde{\tilde{1}}_1$(y-axis at x=1) are as follows [6]:

$$d\left(\tilde{\tilde{A}}, \tilde{\tilde{1}}_1\right) = \frac{1}{8}(a_1^l + a_2^l + a_3^l + a_4^l + 4a_1^u + 2a_2^u + 2a_3^u + 4a_4^u + 3(a_2^u + a_3^u - a_1^u - a_4^u)\frac{w^l}{w^u} - 16).$$

From this, we can conclude that signed distance $d\left(\tilde{\tilde{A}}, \tilde{\tilde{1}}_1\right) \leq 0.$ because the values of the IVGTFN $\tilde{\tilde{A}}$ are between zero and one.

Definition 4: Let $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ be two IVGTFNs. The ranking of $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ by the signed distances $d\left(\tilde{\tilde{A}}, \tilde{\tilde{1}}_1\right)$ and $d\left(\tilde{\tilde{B}}, \tilde{\tilde{1}}_1\right)$ can be defined as follows:

i) $d\left(\tilde{\tilde{A}}, \tilde{\tilde{1}}_1\right) > d\left(\tilde{\tilde{B}}, \tilde{\tilde{1}}_1\right)$ iff $\tilde{\tilde{A}} \succ \tilde{\tilde{B}}$,

ii) $d\left(\tilde{\tilde{A}}, \tilde{\tilde{1}}_1\right) = d\left(\tilde{\tilde{B}}, \tilde{\tilde{1}}_1\right)$ iff $\tilde{\tilde{A}} \sim \tilde{\tilde{B}}$,

iii) $d\left(\tilde{\tilde{A}}, \tilde{\tilde{1}}_1\right) < d\left(\tilde{\tilde{B}}, \tilde{\tilde{1}}_1\right)$ iff $\tilde{\tilde{A}} \prec \tilde{\tilde{B}}$.

3. Linear programming with interval-valued generalized trapezoidal fuzzy numbers

In conventional linear programming problems of the parameters must be well defined and precise. But, in many applications of linear programming problems this assumption is not true. To deal with such situations, the parameters of linear programming problems may be represented as IVGTFNs. Here, we let all parameters of the linear programming problem are IVGTFNs, while the decision variables are crisp. So, a linear programming problem with IVGTFNs is defined as:

$$\begin{aligned} &Max\ \tilde{\tilde{Z}} \approx \tilde{\tilde{C}}X \\ &\text{subject to} \\ &\tilde{\tilde{A}}X \approx \tilde{\tilde{B}} \\ &X \geq 0 \end{aligned} \qquad (1)$$

Where $\tilde{\tilde{C}} = [\tilde{C}^L, \tilde{C}^U]$, $\tilde{\tilde{A}} = [\tilde{A}^L, \tilde{A}^U]$, and $\tilde{\tilde{B}} = [\tilde{B}^L, \tilde{B}^U]$ are GIVTFNs.

Definition 5: We say that a vector $X \in \mathbb{R}^n$ is a feasible solution to (1) if it satisfies the

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following conditions:

i) $\tilde{\tilde{A}}\,X \approx \tilde{\tilde{B}}$, ii) $X \geq 0$.

Definition 6: Let vector $X^* \in \mathbb{R}^n$ be a feasible solution to (1). It is an optimal solution to (1), if for all feasible solution X for (1), we have $\tilde{\tilde{C}}X^* \succcurlyeq \tilde{\tilde{C}}X$.

Definition 7: Let X_1 be an optimal solution to (1). If there exist a vector X_2 such that,
i) $X_2 \geq 0$, ii) $\tilde{\tilde{A}}\,X_2 \approx \tilde{\tilde{B}}$, iii) $\tilde{\tilde{C}}X_1 \approx \tilde{\tilde{C}}X_2$,

then X_2 is said to be an alternative optimal solution for (1).

4. Proposed method to find the optimal solution

In this section, a new method is proposed to obtain the optimal solution of problem (1), in which all the parameters are represented by IVGTFNs, while the decision variables are crisp. So, the various steps to find the optimal solution of problem (1) are as follows:

Step 1: Substituting $\tilde{\tilde{C}} = [\tilde{C}_j^L, \tilde{C}_j^U]$, $\tilde{\tilde{A}} = [\tilde{A}_{ij}^L, \tilde{A}_{ij}^U]$, and $\tilde{\tilde{B}} = [\tilde{B}_i^L, \tilde{B}_i^U]$ of the problem (1) may be written as:

$$
\begin{aligned}
&Max \sum_{j=1}^{n} [\tilde{C}_j^L, \tilde{C}_j^U]\, x_j \\
&\text{subject to} \\
&\sum_{j=1}^{n} [\tilde{A}_{ij}^L, \tilde{A}_{ij}^U]\; x_j = \sum_{i=1}^{m} [\tilde{B}_i^L, \tilde{B}_i^U] \qquad i = 1,2,\dots,m \\
&x_j \geq 0 \qquad j = 1,2,\dots,n
\end{aligned}
\tag{2}
$$

Step 2: Applying signed distance d to problem (2), obtained in step 1, we have:

$$
\begin{aligned}
&Max\ d\Big(\sum_{j=1}^{n} \big[\tilde{C}_j^L, \tilde{C}_j^U\big]\, x_j\Big) \\
&\text{subject to} \\
&d\Big(\sum_{j=1}^{n} \big[\tilde{A}_{ij}^L, \tilde{A}_{ij}^U\big]\; x_j\Big) = d\Big(\sum_{i=1}^{m} \big[\tilde{B}_i^L, \tilde{B}_i^U\big]\Big) \qquad i = 1,2,\dots,m \\
&x_j \geq 0 \qquad j = 1,2,\dots,n
\end{aligned}
\tag{3}
$$

Using arithmetic operations, defined in subsection 2.2.2 and the signed distance, stated in section 2.2.3, the problem (3) is converted into an optimization problem where may be solve by the linear programming methods.

Step 3: Find the optimal solution x_j by solving the problem obtained in Step 2.

Step 4: Find the optimal value by putting the values of x_j in the objective function of the

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problem (2) where it is an interval-valued generalized trapezoidal fuzzy number.

5. Numerical example

Example 5.1: Let us consider the following linear programming problem with interval-valued generalized trapezoidal fuzzy number and solve it using the proposed method

$$\begin{aligned} &Max\ \tilde{\tilde{Z}} \approx \tilde{\tilde{C}}_1 x_1 \oplus \tilde{\tilde{C}}_2 x_2 \\ &\text{subject to} \\ &\tilde{\tilde{A}}_{11} x_1 \oplus \tilde{\tilde{A}}_{12} x_2 \approx \tilde{\tilde{B}}_1 \\ &\tilde{\tilde{A}}_{21} x_1 \oplus \tilde{\tilde{A}}_{22} x_2 \approx \tilde{\tilde{B}}_2 \\ &x_1, x_2 \geq 0. \end{aligned} \tag{4}$$

Where the values of $\tilde{\tilde{C}}_1, \tilde{\tilde{C}}_2, \tilde{\tilde{A}}_{11}, \tilde{\tilde{A}}_{12}, \tilde{\tilde{A}}_{21}, \tilde{\tilde{A}}_{22}, \tilde{\tilde{B}}_1, \tilde{\tilde{B}}_2$ are:

$\tilde{\tilde{C}}_1 = [(02,0.5,0.7,0.9; 0.3), (0.1,0.4,0.8,0.95; 0.5)]$,
$\tilde{\tilde{C}}_2 = [(0.5,0.7,0.9,1; 0.2), (0,0.6,0.95,1; 0.4)]$,
$\tilde{\tilde{A}}_{11} = [(0.3,0.7,0.85,0.9; 0.4), (0.2,0.5,0.9,1; 0.6)]$,
$\tilde{\tilde{A}}_{12} = [(0.1,0.3,0.5,0.7; 0.8), (0,0.2,0.6,0.9; 0.9)]$,
$\tilde{\tilde{A}}_{21} = [(0.5,0.6,0.7,0.8; 0.3), (0.1,0.4,0.9,1; 0.7)]$,
$\tilde{\tilde{A}}_{22} = [(0.3,0.7,0.8,0.95; 0.4), (0.1,0.5,0.9,1; 0.8)]$,
$\tilde{\tilde{B}}_1 = [(0.4,0.5,0.7,0.8; 0.5), (0,0.2,0.8,0.9; 0.7)]$,
$\tilde{\tilde{B}}_2 = [(0.5,0.6,0.7,0.8; 0.4), (0.1,0.55,0.75,0.85; 0.9)]$,

Solution: Using Step 1 and Step 2 of proposed method, the following problem is obtained for problem (4):

$$\begin{aligned} &Max\ z = 1.1406\ x_1 + 1.3781\ x_2 \\ &\text{subject to} \\ &1.3438\ x_1 + 0.85\ x_2 = 1.0268, \\ &1.4175\ x_1 + 1.292\ x_2 = 1.1833, \\ &x_1, x_2 \geq 0. \end{aligned} \tag{5}$$

By omitting fix number, that is -2, from the objective function of problem (5), we have:

$$\begin{aligned} &Max\ z_1 = 1.1406\ x_1 + 1.3781\ x_2 \\ &\text{subject to} \\ &1.3438\ x_1 + 0.85\ x_2 = 1.0268, \\ &1.4175\ x_1 + 1.292\ x_2 = 1.1833, \\ &x_1, x_2 \geq 0. \end{aligned} \tag{6}$$

The problem (6) is a linear programming problem and its optimal solution is: $x_1^* = 0.6037$, $x_2^* = 0.2534, {z_1}^* = 1.0379$. So, the optimal value for problem (5) is as follows: $z^* = 1.0379 - 2 = -0.9621$.
The optimal value of objective function for problem (4) is a generalized interval-valued

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trapezoidal fuzzy number as follows:

$\tilde{\tilde{Z}}^* \approx \tilde{\tilde{C}}_1 x_1^* \oplus \tilde{\tilde{C}}_2 x_2^* \approx$

$[(0.1207, 0.3018, 0.4226, 0.5433; 0.3), (0.0604, 0.2415, 0.483, 0.5735; 0.5)] \oplus$

$[(0.1267, 0.1774, 0.2281, 0.2534; 0.2), (0, 0.152, 0.2441, 0.2534; 0.4)] =$

$[(0.2474, 0.4792, 0.6507, 0.7967; 0.2), (0.6004, 3935, 0.7271, 0.8269)].$

Therefore we have $d\left(\tilde{\tilde{Z}}^*, \tilde{\tilde{I}}_1\right) = -0.8714 \leq 0.$

6. Conclusions

This study, has presented a new method to find the fuzzy optimal solution of linear programming problem in which, the cost vector, the technological coefficients and the right-hand side are IVGTFNs. Then, we applied the signed distance for defuzzification of this problem. So, the crisp problem obtained after the defuzzification is solved by the linear programming methods. By using the proposed method, the optimal solution of problems with IVGTFNs coefficients, occurring in real life problems, can be easily obtained. This method can be applied for solving transportation problems with IVGTFNs as transportation cost or supply and demand values.

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